

STAT 472

Fall 2018

Quiz 3

September 18, 2018

1. You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

If $l_{[90]} = 100,000$, calculate $l_{[91]}$.

Solution:

$$l_{93} = l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} \quad \text{and} \quad l_{93} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

so

$$l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

$$(100,000)(1-0.1)(1-0.2)(1-0.3) = l_{[91]}(1-0.25)(1-0.12)$$

$$l_{[91]} = \frac{(100,000)(1-0.1)(1-0.2)(1-0.3)}{(1-0.25)(1-0.12)} = 76,363.64$$

2. You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

Deaths between integral ages follows a constant force of mortality.

Calculate ${}_{1.7}q_{[90]+0.4}$.

Solution:

$${}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}}$$

We need to create l 's.

$$l_{[90]} = 1000; l_{[90]+1} = (1000)(1 - 0.1) = 900; l_{[90]+2} = (900)(1 - 0.2) = 720; l_{[90]+3} = (720)(1 - 0.3) = 504$$

$$l_{[90]+0.4} = (1000)^{1-0.4}(900)^{0.4} = 958.73315155$$

$$l_{[90]+2.1} = (720)^{0.9}(504)^{0.1} = 694.7719885$$

$${}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}} = \frac{958.73315155 - 694.7719885}{958.73315155} = 0.27532$$

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If $e_{92} = 2.00$, calculate $e_{[90]}$.

Solution:

$$e_{[90]} = p_{[90]}(1 + e_{[90]+1})$$

$$e_{[90]+1} = p_{[91]}(1 + e_{92})$$

$$e_{[90]} = p_{[90]}[1 + p_{[90]+1}(1 + e_{92})] = (0.9)[1 + (0.8)(1 + 2.00)] = 3.06$$

2. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.1	0.25	0.18
1	0.3	0.20	0.12
2	0.5	0.15	0.10

Calculate $e_{0:\overline{3}|}$ for the robot.

Solution:

$$e_{0:\overline{3}|} = {}_1p_0 + {}_2p_0 + {}_3p_0 = p_0 + p_0 \cdot p_1 + p_0 \cdot p_1 \cdot p_2$$

$$= (1 - 0.1) + (1 - 0.1)(1 - [1 - 0.2][0.3]) + (1 - 0.1)(1 - [1 - 0.2][0.3])(1 - [1 - 0.15][1 - 0.1][0.5])$$

$$= 0.9 + 0.684 + 0.42237 = 2.00637$$