STAT 472
Quiz 5
Fall 2018

October 29, 2018

1. Mindy is (65) and is retiring. She has 1,000,000 in her retirement fund that will be used to purchase an annuity for her retirement.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

   a. (3 points) If Mindy buys a life annuity due with annual payments, her annual payment will be 73,800 to the nearest 100. Calculate the annual payment to the nearest 1.

   Solution:

   $$(Payment)\ddot{a}_{65} = 1,000,000$$

   $$Payment = \frac{1,000,000}{13.5498} = 73,802$$

   b. (5 points) If Y is the present value random variable for Mindy's life annuity due, calculate the $Var(Y)$.

   Solution:

   $$Var[73,802Y] = (73,802)^2 Var[Y] = (73,802)^2 \frac{{}^2A_{65} - (A_{65})^2}{d^2}$$

   $$= (73,802)^2 \frac{0.15420 - (0.35477)^2}{\left(\frac{0.05}{1.05}\right)^2} = 68,068,759,290$$

c. (5 points) If Mindy buys a life annuity due with monthly payments, calculate the monthly payment that Mindy would receive.

Solution:

$$1,000,000 = (Payment)(12)\ddot{a}_{65}^{(12)} = (Payment)(12)[\alpha(12)\cdot\ddot{a}_{65} - \beta(12)]$$

$$Payment = \frac{1,000,000}{(12)[(1.00020)(13.5498) - 0.46651]} = 6368.13$$

d. (5 points) If Mindy buys a 10 year certain life annuity due with annual payments, then the first 10 payments are guaranteed to be made even if Mindy dies during the first ten years. Calculate the annual payment under such an annuity.

Solution:

$$1,000,000 = (Payment)\ddot{a}_{\overline{65:\overline{10|}}} = (Payment)(\ddot{a}_{\overline{10|}} + {}_{10}E_{65}\cdot\ddot{a}_{75})$$

$$Payment = \frac{1,000,000}{\frac{1-(1.05)^{-10}}{0.05/1.05} + (0.55305)(10.3178)} = 72,389.90$$

e. (2 points) Explain why the annual payment in Part a is less than the annual payment (which is twelve times the monthly payment) in Part c.

Solution:

When the company pays the annual payment (Part a), then they cannot earn interest on that money during the year. When then make a payment monthly, they can earn additional interest throughout the year on the money that has not been paid. This additional interest can be used to pay more money to Mindy on a monthly basis (Part c).

The second reason that the monthly payments can be larger is that in the year of death, we will not make a full year of payments to Mindy whereas if we are paying annually at the beginning of the year, Mindy will get a full payment in the year of death. Therefore, we can pay more each month so that present value of the payments is the same.

STAT 472
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Fall 2018

October 29, 2018

1. Madison is (60) and is retiring. She has 1,000,000 in her retirement fund that will be used to purchase an annuity for her retirement.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   a. (3 points) If Madison buys a life annuity due with annual payments, her annual payment will be 67,100 to the nearest 100. Calculate the annual payment to the nearest 1.

   Solution:

   $$(Payment)\ddot{a}_{65} = 1{,}000{,}000$$

   $$Payment = \frac{1{,}000{,}000}{14.9041} = 67{,}096$$

   b. (5 points) If Y is the present value random variable for Madison's life annuity due, calculate the $Var(Y)$.

   Solution:

   $$Var[67{,}096Y] = (67{,}096)^2 Var[Y] = (67{,}096)^2 \frac{{}^2A_{60} - (A_{60})^2}{d^2}$$

   $$= (67{,}096)^2 \frac{0.10834 - (0.29028)^2}{\left(\frac{0.05}{1.05}\right)^2} = 47{,}801{,}731{,}770$$

c. (5 points) If Madison buys a life annuity due with monthly payments, calculate the monthly payment that Madison would receive. Use the three term Woolhouse formula to calculate the monthly benefit.

Solution:

$$1,000,000 = (Payment)(12)\ddot{a}_{60}^{(12)} = (Payment)(12)\left[\ddot{a}_{60} - \frac{11}{24} - \frac{143}{12^3}(\delta + \mu_{60})\right]$$

$$Payment = \frac{1,000,000}{(12)\left[14.9041 - \frac{11}{24} - \frac{143}{12^3}(\ln(1.05) - 0.5\left[\ln(1-0.003048) + \ln(1+0.003398)\right]\right]}$$

$$= 5770.42$$

d. (5 points) Madison decides to purchase an annual life annuity due with annual payments. The first fifteen payments will be two times the payments made in thereafter. Determine Madison's first payment.

Solution:

$$1,000,000 = 2P\ddot{a}_{60} - P \cdot {}_{15}E_{60} \cdot \ddot{a}_{75} = 2P(14.9041) - P(0.57864)(0.73295)(10.3178)$$

$$P = \frac{1,000,000}{2(14.9041) - (0.57864)(0.73295)(10.3178)} = 39,320.11$$

$$Payment = 2p = 78,640.23$$

e. (2 points) Explain why the annual payment in Part a is less than the annual payment (which is twelve times the monthly payment) in Part c.

Solution:

When the company pays the annual payment (Part a), then they cannot earn interest on that money during the year. When then make a payment monthly, they can earn additional interest throughout the year on the money that has not been paid. This additional interest can be used to pay more money to Madison on a monthly basis (Part c).

The second reason that the monthly payments can be larger is that in the year of death, we will not make a full year of payments to Madison whereas if we are paying annually at the beginning of the year, Madison will get a full payment in the year of death. Therefore, we can pay more each month so that present value of the payments is the same.