STAT 472
Fall 2018
Test 1
October 2, 2018

1. (10 points) You are given that $F_{50}(t) = 0.0004t^2 \quad \text{for} \;\; 0 \le t \le 50$ and $\delta = 0.05$.

   Calculate $1000\bar{A}_{50}$.

2. (10 points) You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.7
94	1.0

For ages 90 to 91 and 91 to 92, deaths are uniformly distributed between integral ages.

For ages 92 to 93 and 93 to 94, there is a constant force of mortality between integral ages.

Calculate ${}_{0.8|1.5}q_{90.6}$.

3. (10 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.70	102
101	0.30	0.60	1.00	103

Calculate $Var[K_{[100]}]$

4. (6 points) Compare and contrast aggregate survival models with select and ultimate mortality models. In other words, state how they are the same and how they are different. Also, state when each should be used.

5. Knapp Industries uses robots to produce their products. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.1	0.30	0.22
1	0.2	0.25	0.15
2	0.4	0.10	0.08

Knapp buys 1000 robots.

a. (10 points) Complete the following table being sure to show your work:

x	l_x
0	1000
1	
2	
3	

Knapp decides to buy a three year warranty on each new robot that will pay a benefit of 100,000 at the end of the year if a robot dies. (This is equivalent to a 3 year term policy on the robot's life.)

b. (10 points) Calculate the Expected Present Value of the three year warranty using $d = 0.10$. This value should be for each robot, not for all 1000 robots.

6. You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

    a. (10 points) Calculate the Expected Present Value for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death**.

    b. (10 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death.**

c. (10 points) Calculate the Expected Present Value for a 30 year endowment insurance issued to (50) with a death benefit of 10,000 **paid at the moment of death**.

d. (4 points) Explain why the Expected Present Value in c. is greater than the Expected Present Value in a. Provide two reasons.

7. (10 points) Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

   You are given:

   a. $A_{50} = 0.300$

   b. $Var[Z_{50}] = 0.110$

   c. $q_{50} = 0.01$

   d. $v = 0.93$

   Calculate the $Var[Z_{51}]$.

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1. (10 points) Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

   You are given:

   a. $A_{50} = 0.400$

   b. $Var[Z_{50}] = 0.140$

   c. $q_{50} = 0.015$

   d. $v = 0.93$

   Calculate the $Var[Z_{51}]$.

2. (10 points) You are given that $F_{50}(t) = 0.0004t^2$ for $0 \le t \le 50$ and $\delta = 0.06$.

Calculate $1000\bar{A}_{50}$.

3. You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

    a. (10 points) Calculate the Expected Present Value for a 30 year term insurance issued to (65) with a death benefit of 10,000 paid **at the end of the year of death**.

    b. (10 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (65) with a death benefit of 10,000 paid **at the end of the year of death.**

c. (10 points) Calculate the Expected Present Value for a 30 year endowment insurance issued to (65) with a death benefit of 10,000 **paid at the moment of death**.

d. (4 points) Explain why the Expected Present Value in c. is greater than the Expected Present Value in a. Provide two reasons.

4. (10 points) You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.8
94	1.0

For ages 90 to 91 and 91 to 92, there is a constant force of mortality between integral ages.

For ages 92 to 93 and 93 to 94, deaths are uniformly distributed between integral ages.

Calculate ${}_{1.5|0.5}q_{90.8}$.

5. (10 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.60	102
101	0.30	0.50	1.00	103

Calculate $Var[K_{[100]}]$

6. (3 points) Explain why population tables are not appropriate to use with life insurance products.

7. (3 points) Explain why the mortality during the select period is less than the mortality during ultimate period in a select and ultimate mortality table.

8. Knapp Industries uses robots to produce their products. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.1	0.25	0.22
1	0.2	0.20	0.17
2	0.4	0.15	0.10

Knapp buys 1000 robots.

a. (10 points) Complete the following table being sure to show your work:

x	l_x
0	1000
1	
2	
3	

Knapp decides to buy a three year warranty on each new robot that will pay a benefit of 100,000 at the end of the year if a robot dies. (This is equivalent to a 3 year term policy on the robot's life.)

(10 points) Calculate the Expected Present Value of the three year warranty using $d = 0.10$. This value should be for each robot, not for all 1000 robots.