STAT 472
Test 2
Fall 2018

November 5, 2018

1. The Chung Life Insurance Company sells life insurance policies to people who are age 70 only.

   The Company uses the Standard Ultimate Life Table and 5% interest to calculate all premiums. They also assume that deaths are uniformly distributed between integral ages.

   All premiums are calculated using the Equivalence Principle.

   All policies are sold to insureds whose death is independent of the death of any other insured.

   a. (3 points) The annual net premium for a whole life policy with a death benefit of 100,000 paid at the end of the year of death is 3600 to the nearest 100. Calculate the net premium to the nearest 1.

   Solution:

   $$PVP = PVB ==> P\ddot{a}_{70} = 100,000 A_{70}$$

   $$P = \frac{(100,000)(0.42818)}{12.0083} = 3565.70$$

Let L_0^n be the loss at issue random variable based on the net premium.

   b. (8 points) For a single policy, calculate the probability that the policy generates a loss.

   Solution:

   $$L_0^n = 100,000 v^{K_{70}+1} - 3565.70 \ddot{a}_{\overline{K_{70}+1|}}$$

   $$100,000 v^{K_{70}+1} - 3565.70 \left(\frac{1 - v^{K_{70}+1}}{d} \right) = 0 ==> 1.335471691 v^{K_{70}+1} = 1 - v^{K_{70}+1}$$

   $$v^{K_{70}+1} = 0.428179 ==> K_{70} + 1 = \frac{\ln(0.3428179)}{\ln[(1.05)^{-1}]} = 17.38 ==> 17$$

   A loss is generated by an early death so probability of a loss is

   $${}_{17}q_{70} = 1 - \frac{l_{87}}{l_{70}} = 1 - \frac{53,934.7}{91,082.4} = 0.40785$$

c. (5 points) Calculate the $\sqrt{Var[L_0^n]}$ for this whole life. It is 31,000 to the nearest 1000. Calculate it to the nearest 1.

Solution:

$$Var[L_0^n] = \left(100,000 + \frac{3565.70}{(0.05/1.05)}\right)\left({}^2A_{70} - [A_{70}]^2\right)$$

$$= (174,879.70)^2\left(0.21467 - [0.42818]^2\right)$$

$$\sqrt{Var[L_0^n]} = \sqrt{(174,879.70)^2\left(0.21467 - [0.42818]^2\right)} = 30,956$$

d. (7 points) Chung Life sells 625 whole life policies. Using the normal distribution, calculate the 90% confidence interval for L_0^n.

Solution:

$E[L_0^n] = 0$ due to the equivalence principle

$$CI = 0 \pm 1.645\sqrt{625}(30,956) ==> (-1,273,024;1,273,024)$$

e. (7 points) Calculate the monthly net premium for a 20 year term insurance policy with a death benefit of 1,000,000 paid at the moment of death.

Solution:

$$PVP = PVB \Longrightarrow 12P\ddot{a}^{(12)}_{70:\overline{20}|} = 1{,}000{,}000\bar{A}^{\,1}_{70:\overline{20}|}$$

$$12P\left(\alpha(12)\ddot{a}_{70} - \beta(12) - {}_{20}E_{70}\left[\alpha(12)\ddot{a}_{90} - \beta(12)\right]\right)$$
$$= 1{,}000{,}000\left(\frac{i}{\delta}\right)A^{\,1}_{70:\overline{20}|} = 1{,}000{,}000\left(\frac{i}{\delta}\right)(A_{70:\overline{20}|} - {}_{20}E_{70})$$

$$12P\left(\alpha(12)\ddot{a}_{70:\overline{20}|} - \beta(12)(1 - {}_{20}E_{70})\right) = 1{,}000{,}000\left(\frac{i}{\delta}\right)(A_{70:\overline{20}|} - {}_{20}E_{70})$$
$$12P\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right) = 1{,}000{,}000(1.02480)(0.47091 - 0.17313)$$

$$P = \frac{11{,}000{,}000(1.02480)(0.47091 - 0.17313)}{12\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right)} = 2370.60$$

f. (2 points) Without doing any calculations, state whether the annual premium of the policy in Part e would be more or less than 12 times the monthly premium. Explain why.

Solution:

The annual premium would be less than 12 times the monthly premium. This is for two reasons. The primary reason is that if premiums are paid annually, the insurance company can earn more interest during the year since they have the premium for the whole year whereas under monthly premiums, the premiums are spread throughout the year. Secondly, in the year of death, the annual premium is paid for the full year since it is paid at the start of the year of death. However, with monthly premiums, only the premiums prior to death in the year of death are collected.

2. The Arissa Assurance Company sells annuities to people who are age 61 only.

The Company uses the Standard Ultimate Life Table and 5% interest to calculate all premiums.

Jeff, (61), buys two annuities from Arissa.

The first annuity is a whole life annuity due that pays 1000 at the beginning of every year. Let Y_{WL} be the present value random variable for the annuity.

a. (7 points) Calculate the $Var[Y_{WL}]$.

Solution:

$$Var[Y_{WL}] = (1000)^2 \left[\frac{{}^2A_{61} - (A_{61})^2}{d^2} \right] = 441,000,000[0.11644 - (0.30243)^2]$$

$$= 11,014,458$$

The second annuity is a 10 year certain and life annuity due with monthly payments of 500. The first 10 years of payments are guaranteed.

b. (8 points) Using the two factor Woolhouse formula, the actuarial present value of this annuity is 86,300 to the nearest 100. Calculate the actuarial present value to the nearest 1.

Solution:

$$APV = (500)(12)\ddot{a}^{(12)}_{\overline{61:\overline{10}|}} = (500)(12)(\ddot{a}^{(12)}_{\overline{10}|} + {}_{10}E_{61}\ddot{a}^{(12)}_{71})$$

$$= 6000\left(\frac{1-(1.05)^{-10}}{d^{(12)}} + (0.57457)(11.6803 - 11/24) \right)$$

$$= 6000\left(\frac{1-(1.05)^{-10}}{0.04869} + (0.57457)(11.6803 - 11/24) \right) = 86,264$$

3. (9 points) Molly is (x) and buys a special three year term life annuity due with annual payments. The payment at the beginning of the first year is 50,000. The payment at the beginning of the second year is 30,000. The payment at the beginning of the third year is 10,000. Let $Y_{Special}$ be the present value random variable for the annuity.

   You are given that:

   - $v = 0.94$
   - $q_{x+t} = 0.08 + 0.04t \qquad \text{for } t = 0,1,2,3,4,5$

   Calculate the $Var[Y_{Special}]$.

   Solution:

   Since the payments are not level, we must use first principles.

Cases	Present Value	Probability
Die Year 1	$50,000$	0.08
Die Year 2	$50,000 + 30,000(0.94) = 78,200$	(0.92)(0.12)=0.1104
Live 2 Years	$50{,}000+30{,}000(0.94)+10{,}000(0.94)^2 = 87,036$	$(0.92)(0.88) = 0.8096$

$$E[Y_{Special}] = (50,000)(0.08) + (78,200)(0.1104) + (87,036)(0.8096) = 83,097.63$$

$$E[(Y_{Special})^2] = (50,000)^2(0.08) + (78,200)^2(0.1104) + (87,036)^2(0.8096) = 7,008,057,280$$

$$Var[(Y_{Special})^2] = 7,008,057,280 - (83,097.63)^{22} = 102,841,849$$

4. The Lewis Life Insurance Company sells a whole life insurance policy to (75). The policy has a death benefit of 20,000 paid at the end of the year of death. The premiums are paid annually for the life of the policy.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   The policy has the following expenses. All expenses occur at the beginning of the policy year.

   - Commissions of 50% of premium the first year and 8% of premiums in year 2 and later.
   - Issue expenses of 500 per policy in the first year only.
   - Maintenance expenses of 40 per policy in all years.
   - Termination expense of 1000 per policy paid at the end of the year of death.

   a. (6 points) Calculate the annual gross premium using the equivalence principle.

      Solution:

      $$PVP = PVB + PVE$$

      $$P\ddot{a}_{75} = 21{,}000A_{75} + 500 + 40\ddot{a}_{75} + 0.42P + 0.08P\ddot{a}_{75}$$

      $$P = \frac{(21{,}000)(0.50868) + 500 + 40(10.3177)}{0.92(10.3178) - 0.42} = 1278.05$$

Lewis decides to set the annual gross premium equal to 1325. (Note this is not the gross premium determined part a.) The loss at issue random variable L_0^g is calculated using the gross premium of 1325. L_0^g can be written in the form of $Av^{K_{75}+1} + B + C\ddot{a}_{\overline{K_{75}+1}|}$.

b. (8 points) Determine $A,\ B,$ and C .

Solution:

$$L_0^g = 21,000v^{K_{75}+1} + 500 + (0.42)(1325) + (40 - (0.92)(1325))\ddot{a}_{\overline{K_{75}+1}|}$$

$$= 21,000v^{K_{75}+1} + 1056.50 - (1179)\ddot{a}_{\overline{K_{75}+1}|}$$

$$==> A = 21,000; B = 1056.50; C = -1179$$

c. (5 points) Calculate $E[L_0^g]$.

Solution:

$$E[L_0^g] = E[21,000v^{K_{75}+1} + 1056.501 - (1179)\ddot{a}_{\overline{K_{75}+1}|}]$$

$$= 21,000A_{75} + 1056.50 - 1179\ddot{a}_{75}$$

$$= 21,000(0.50868) + 1056.50 - 1179(10.3178) = -425.91$$

d. (8 points) Calculate $\sqrt{Var[L_0^g]}$

Solution:

$$Var[L_0^g] = Var[21,000v^{K_{75}+1} + 1056.501 - (1179)\ddot{a}_{\overline{K_{75}+1}|}]$$

$$= Var\left[21,000v^{K_{75}+1} - (1179)\left(\frac{1-v^{K_{75}+1}}{d}\right)\right]$$

$$= Var\left[\left(21,000 + \frac{1179}{0.05/1.05}\right)v^{K_{75}+1}\right] = \left(21,000 + \frac{1179}{0.05/1.05}\right)^2 Var\left[v^{K_{75}+1}\right]$$

$$= \left(21,000 + \frac{1179}{0.05/1.05}\right)^2 \left({}^2A_{75} - (A_{75})^2\right)$$

$$\sqrt{Var[L_0^g]} = \left(21,000 + \frac{1179}{0.05/1.05}\right)\sqrt{0.29079 - (0.50868)^2} = 8190.05$$

5. (8 points) A special whole life insurance policy to (65) pays the death benefit at the moment of death. The death benefit is 100,000 for the first 10 years of the policy. The death benefit is 250,000 for death during the second 10 years of the policy. The death benefit is 50,000 for death after 20 years.

   The annual net premium for this policy is paid for 10 years and determined using the Equivalence Principle.

   You are given that mortality follows the Standard Ultimate Life Table with $i = 5\%$. Deaths are assumed to be uniformly distributed between integral ages.

   Determine the annual net premium.

   Solution:

$$PVP = PVB$$

$$P\ddot{a}_{65:\overline{10}|} = 100,000\bar{A}_{65} + 150,000\,{}_{10}E_{65}\bar{A}_{75} - 200,000\,{}_{20}E_{65}\bar{A}_{85}$$

$$P = \left(\frac{100,000(0.35477) + 150,000(0.55305)(0.50868) - 200,000(0.24381)(0.67622)}{7.8435}\right)(1.02480)$$

$$= 5840.58$$

6. (9 points) You are given:

   a. $\ddot{a}_{50} = 11$

   b. $i = 0.05$

   c. $q_{50} = 0.01$

   d. Deaths are uniformly distributed between integral ages.

   Let P be the net annual premium for a whole life policy on **(51)** with a death benefit of 30,000 paid at the moment of death.

   Determine P.

   Solution:

   $$PVP = PVB$$

   $$P\ddot{a}_{51} = 30,000\bar{A}_{51}$$

   $$\ddot{a}_{50} = 1 + vp_{50}\ddot{a}_{51} ==> \ddot{a}_{51} = \frac{\ddot{a}_{50} - 1}{vp_{50}} = \frac{11-1}{(1.05)^{-1}(1-0.01)} = 10.6060606$$

   $$A_{51} = 1 - d\ddot{a}_{51} = 1 - (0.05/1.05)(10.6060606) = 0.4949495$$

   $$P = \frac{(30,000)(1.02480)(0.4949495)}{10.6060606} = 1434.72$$