STAT 472
Test 3
Fall 2018

December 10, 2018

1. Richard buys a whole life policy when he is (70). The policy pays a death benefit of 200,000 at the end of the year of death. The premiums are paid annually as long as Richard is alive.

   You are given that mortality follows the Standard Ultimate Mortality Table with interest at 5%.

   a. (2 points) The net premium is 7100 to the nearest 100. Calculate the net premium to the nearest 1.

$$P_{70} = \frac{200,000A_{70}}{\ddot{a}_{70}} = \frac{200,000(0.42818)}{12.0083} = 7,131.4$$

   b. (3 points) Calculate the net premium reserve at the end of the 10th year.

$${}_{10}V^{n} = 200,000A_{80} - P\ddot{a}_{80} = 200,000(0.59293) - 7131.4(8.5484) = 57,623.94$$

$$or$$

$${}_{10}V^{n} = 200,000\left(1 - \frac{\ddot{a}_{80}}{\ddot{a}_{70}}\right) = 200,000\left(1 - \frac{8.5484}{12.0083}\right) = 57,625.14$$

The gross premium for this policy is 8308.22. The premium was determined using the equivalence principal.

The reserve basis for the gross premium reserves is:

- Mortality follows the Standard Ultimate Mortality Table
- $i = 0.05$
- Commissions are 65% of premiums in the first year and 8% thereafter
- Issue Expenses are 400 per policy and 1.00 per thousand
- Maintenance expenses are 50 at the beginning of each year including the first year.
- Termination expense of 500 paid at the end of the year of death

c. (4 points) The gross premium reserve at the end of the first year is 284.81. Determine the gross premium reserve after 9 months.

$$_{0.75}V^g = (1-0.75)(_0V^g + P^g - Expenses) + 0.75(_1V^g)$$

$$_0V^g = 0 \text{ because of the equivalence principle}$$

$$_{0.75}V^g = 0.25\left(0 + 8308.22 - 0.65(8308.22) - 400 - 50 - \frac{200,000}{1,000}\right) + 0.75(284.81) = 778.08$$

d. (4 points) Determine the gross premium reserve at the end of the 10th year.

$$_{10}V^g = PVFB + PVFE - PVFP =$$

$$(200,000 + 500)A_{80} - (8308.22 - 0.08(8308.22) - 50)\ddot{a}_{80}$$

$$= 200,500(0.59293) - 7593.5624(8.5484) = 53,969.66$$

e. (2 points) Determine the expense reserve at the end of the 10th year.

$$_{10}V^e = {}_{10}V^g - {}_{10}V^n = 53,969.66 - 57,625.14 = -3655.48$$

2. Jaden who is (25) buys a whole life policy with a death benefit of 75,000 paid at the end of the year of death. The policy has annual gross premiums.

   The reserve basis for the gross premium reserves is:

   - Mortality follows the Standard Ultimate Mortality Table
   - $i = 0.06$ ← Note that this is not $i = 0.05$.
   - Expenses are 10% of premium and 40 per policy at the start of each year.
   - Termination expense of 250 paid at the end of the year of death

   The gross premium reserves for the 9th, 10th, and 11th year are given in the following table:

Time	Gross Premium Reserve
9	1000.00
10	1271.51
11	1558.02

   a. (5 points) The gross premium is 300 to the nearest 25. Determine the gross premium to the nearest 1

$$_{10}V = \frac{({}_9V + P - .1P - 40)(1.06) - (75000 + 250)q_{34}}{p_{34}}$$

$$1271.51 = \frac{(960 + 0.9P)(1.06) - (75,250)(0.000372)}{1 - 0.000372}$$

$$.9P = 265.5 ==> P = 295$$

The actual experience during the 10th year of this policy is:

- Mortality is 50% of the Standard Ultimate Mortality Table
- $i = 0.07$
- Expenses are 8% of premium and 50 per policy at the start of the year.
- Termination expense of 275 paid at the end of the year of death

b. (4 points) Determine the total gain for the 10th year of this policy.

$$G = ({}_9V + P - 0.08P - 50)(1.07) = (75,000 + 275)(0.5)q_{34} + {}_{10}V[1-(0.5)q_{34}]$$

$$G = (1000 + 295(.92) - 50)(1.07) - 75,275(0.5)(0.000372) - 1271.51[1-(0.5)(0.000391)]$$

$$G = 21.62$$

Gains by source are determined first for expenses, then for interest, and finally for mortality.

c. (4 points) Determine the gain from expenses.

$$G^e = (1000 + 295(0.92) - 50)(1.06) - 75,275(0.000372) - 1271.51[1-(0.000391)]$$

$$= -4.36$$

d. (4 points) Determine the gain from interest.

$$G^{i\&e} = (1000 + 295(.92) - 50)(1.06) - 75,275(0.000372) - 1271.51[1-(0.000391)]$$

$$G^{i\&e} = 7.86$$

$$G^{i\&e} = G^e + G^i ==> 7.86 = -4.36 + G^i ==> G^i = 12.22$$

e. (4 points) Determine the gain from mortality.

$$G^m = G^{Total} - G^i - G^e$$
$$= 21.62 - 12.22 - (-4.36)$$
$$= 13.76$$

3. Jacqueline is (44) and purchases an Endowment at Age 65. In other words, this is an endowment that ends in 21 years. The death benefit for this policy is 50,000.

   Reserves are calculated using the Full Preliminary Term method with mortality based on the Standard Ultimate Life Table and $i = 0.05$.

   Let ${}_tV^{FPT}$ be the Full Preliminary Term reserve at time t on this policy.

   a. (3 points) Calculate the net premium for the first year of this policy.

   $${}_1P^{FPT} = S \cdot v \cdot q_x = 50,000(1.05)^{-1}(0.000710) = 33.81$$

   b. (3 points) Calculate ${}_{0.8}V^{FPT}$.

   $$\begin{aligned} {}_{0.8}V &= ({}_0V + {}_1P^{FPT})(1-0.8) + ({}_1V)(0.8) \\ &= (0+33.81)(0.2) + (0)(0.8) \\ &= 6.76 \end{aligned}$$

   c. (4 points) Calculate the net premium for year 2 and later for this policy.

   $$P^{FPT}_{45:\overline{20}|} = \frac{50,000A_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}} = \frac{50,000(A_{45} - {}_{20}E_{45}A_{65} + {}_{20}E_{45})}{\ddot{a}_{45} - {}_{20}E_{45}\ddot{a}_{65}}$$

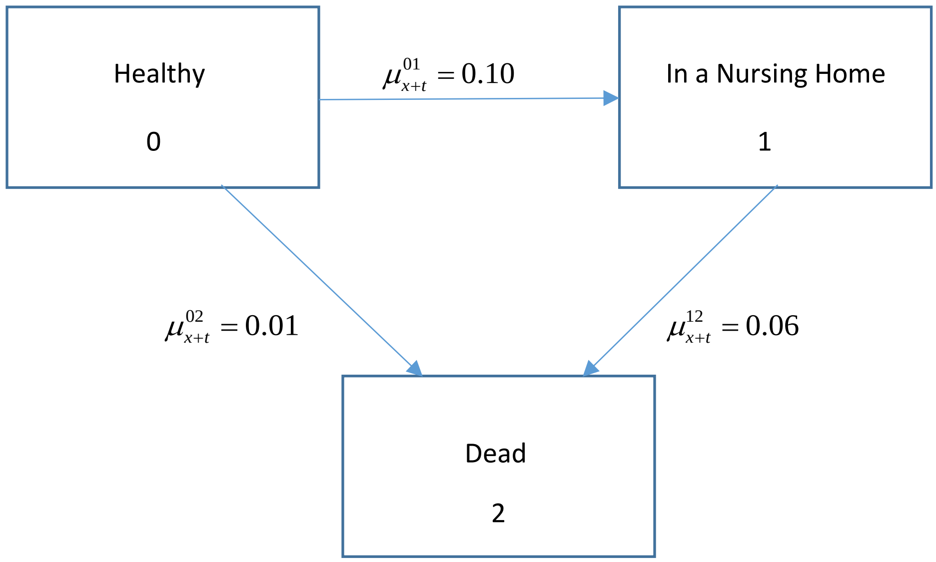
   $$= \frac{50,000(0.15161 - 0.35994 \cdot 0.35477 + 0.35994)}{17.8162 - 0.35994 \cdot 13.5498} = 1,483.31$$

   d. (5 points) Calculate ${}_{11}V^{FPT}$.

   $${}_{11}V^{FPT} = 50,000A_{55:\overline{10}|} - 1,483.31\ddot{a}_{55:\overline{10}|}$$

   $$= 50,000(0.61813) - 1,483.31(8.0192) = 19,011.54$$

The following information is for questions 4-10:



The above mutli-state model is used for a long term care policy which has a term of 7 years. Savanah who is age x purchases this policy.

The policy pays six benefits:

- Benefit 1 is continuous annuity at an annual rate of 15,000 per year while a person is in State 0.
- Benefit 2 is a lump sum benefit of 20,000 at the moment of transition from State 0 to State 1.
- Benefit 3 is a lump sum benefit of 30,000 at the moment of transition from State 0 to State 2.
- Benefit 4 is a lump sum benefit of 10,000 at the moment of transition from State 1 to State 2.
- Benefit 5 is a continuous annuity at an annual rate of 25,000 per year while a person is in State 1.
- Benefit 6 is a return of the premium paid (without any interest) if Savanah remains in State 0 the entire 7 years.

The premium for this policy is a single premium payable at time zero. The premium was determined using the equivalence principle.

You are given that $\delta = 0.05$ and the Actuarial Present Value of Benefit 3 is 1263.

4. (4 points) Savanah determines the probability that she remains in State 0 for all seven years is 0.5 to one decimal place. Calculate ${}_7p_x^{00}$ to three decimal places.

$$ {}_7p_x^{00} = e^{-\int_0^7 (\mu_{x+t}^{01} + \mu_{x+t}^{02})dt} = -e^{-0.11t|_0^7} = e^{-0.77} = 0.463 $$

5. (6 points) Show that ${}_t p_x^{01} = 2\left(e^{-0.06t} - e^{-0.11t}\right)$.

$${}_t p_x^{01} = \int_0^t {}_s p_x^{00} \mu_{x+s}^{01} p_{x+s}^{11} \, ds = \int_0^t e^{-0.11s} (0.10) e^{-(0.06)(t-s)} ds$$

$$= 0.10 \int_0^t e^{-0.11s} e^{-0.06t} e^{0.06s} ds = 0.10 e^{-0.06t} \int_0^t e^{-0.05s} ds$$

$$= \frac{0.10}{0.05} e^{-0.06t} (1 - e^{-0.05t}) = 2(e^{-0.06t} - e^{-0.11t})$$

6. (4 points) Calculate the actuarial present value of Benefit 1

$$PV = 15,000 \int_0^7 v^t ({}_t p_{x+t}^{00}) dt = 15,000 \int_0^7 e^{-0.05t} e^{-0.11t} dt$$

$$= \frac{15,000}{0.16} (1 - e^{-0.16(7)}) = 63,161.27$$

7. (4 points) Calculate the actuarial present value of Benefit 2

$$PV = 20,000 \int_0^7 v^t {}_t p_{x+t}^{00} \mu_{x+t}^{01} dt = 20,000 \int_0^7 e^{-0.05t} e^{-0.11t} (0.10) dt$$

$$= 20,000(0.10) \left(\frac{1 - e^{-0.16(7)}}{0.16} \right) = 8421.50$$

8. (6 points) Calculate the actuarial present value of Benefit 4

$$PV = 10,000 \int_0^7 v^t \cdot {}_t p_{x+t}^{01} \cdot \mu_{x+t}^{12} = 10,000 \int_0^7 e^{-0.05t} \cdot 2\left(e^{-0.06t} - e^{-0.11t}\right)(0.06) dt$$

$$= 1200 \int_0^7 (e^{-0.11t} - e^{-0.16t}) dt = 1,200 \left(\frac{1 - e^{-0.11(7)}}{0.11} - \frac{1 - e^{-0.16(7)}}{0.16} \right) = 805.14$$

9. (4 points) Calculate the actuarial present value of Benefit 5

$$PV = 25{,}000\int_0^7 v^t \cdot {}_t p_{x+t}^{01} dt = 25{,}000\int_0^7 e^{-0.05t} \cdot 2\left(e^{-0.06t} - e^{-0.11t}\right)dt$$

$$= 50{,}000\int_0^7 \left(e^{-0.11t} - e^{-0.16t}\right)dt = 50{,}000\left(\frac{1-e^{-0.11(7)}}{0.11} - \frac{1-e^{-0.16(7)}}{0.16}\right) = 33{,}547.40$$

10. (6 points) Calculate the single premium for this policy. Do not forget to include Benefit 6.

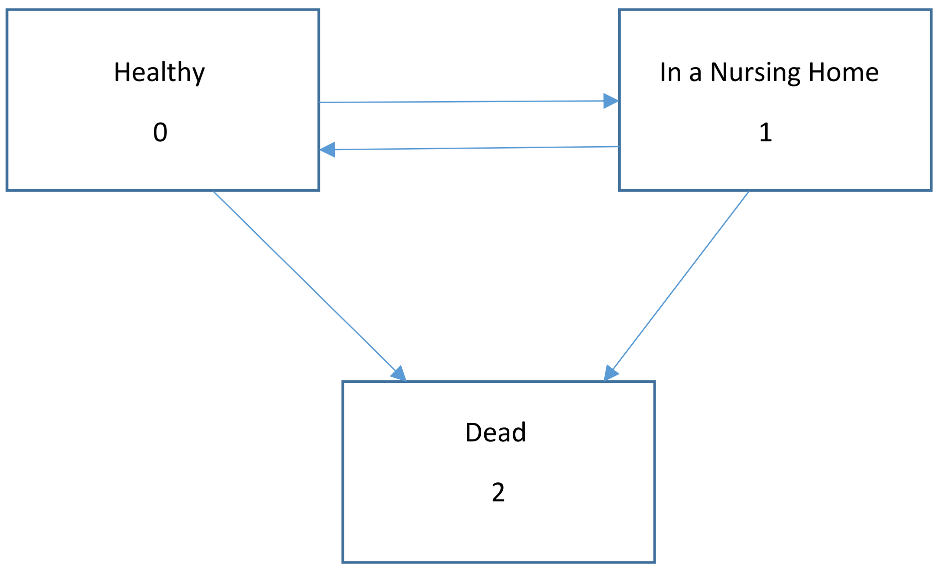
$$PVP = PVB$$
$$P = B_1 + B_2 + B_3 + B_4 + B_5 + B_6$$

$$P = 63{,}161.27 + 8{,}421.50 + 1263 + 805.14 + 33{,}547.40 + {}_7 p_x^{00} \cdot v^7 \cdot P$$

$$P = \frac{107{,}198.31}{1-e^{-0.11(7)}e^{-0.05(7)}} = 159{,}113.99$$

The following information is for questions 11-13:

For a Long Term Care Policy sold by Tan Insurance Company, you are given the following multi-state model:



You are also given that $v = 0.92$ and the following transition matrix of annual probabilities of transition from one state to another state:

$$\begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0.60 & 0.30 & 0.10 \\ 0 & 0 & 1 \end{bmatrix}$$

A three year long term care insurance policy pays 250,000 at the end of the year of death. It also pays 100,000 at the end of the year if you are in a nursing home at that time.

Premiums are payable annually at the beginning of each year if you are healthy.

11. (10 points) The net benefit premium for this policy is 38,000 to the nearest 100. Calculate the net benefit premium to the nearest 1.

Year 0	Year 1	Year 2	Year 3
1	**0.75**	0.75(0.75)+0.20(0.60)=**0.6825**	0.6825(0.75)+0.21(0.60)=**0.63875**
0	**0.20**	0.75(0.20)+0.20(0.30)=**0.21**	0.6825(0.20)+0.21(0.30)=**0.1995**
0	**0.05**	0.05+0.75(0.05)+0.20(0.10)=**0.1075**	0.1075+0.6825(0.05)+0.21(0.10)=**0.162625**

$$P = \frac{250,000 A_{x:\overline{3}|} + 100,000 A_{x:\overline{3}|}}{a_{x:\overline{3}|}}$$

$$250,000[0.05(0.92) + (0.1075 - 0.05)(0.92)^2 + (0.162625 - 0.1075)(0.92)^3] = \$34,398.294$$

$$100,000[0.20(0.92) + 0.21(0.92)^2 + 0.1995(.92)^3] = \$51,709.2256$$

$$P = \frac{34,398.294 + 51,709.2256}{1 + 0.75(0.92) + .6825(0.92)^2} = 37,971.84$$

12. (6 points) Calculate ${}_1V$ which is the reserve for this policy for all insureds that are alive at the end of one year.

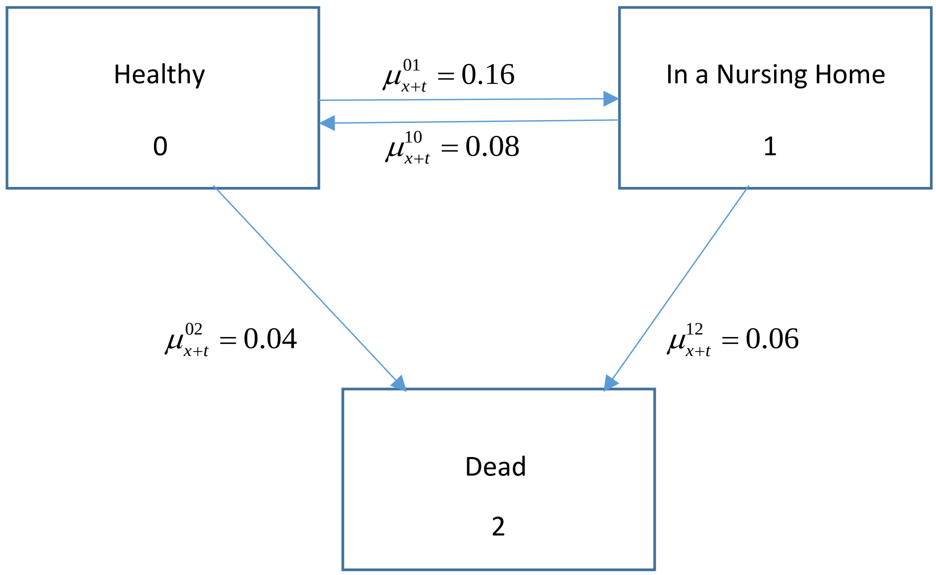
$${}_1V = \frac{(0 + 37,971.84)\left(\frac{1}{0.92}\right) - 250,000(0.05) - 100,000(0.20)}{0.75 + 0.2} = 9235.70$$

13. (6 points) Calculate ${}_1V^{(0)}$ which is the reserve for this policy for all insureds that are in State 0 at the end of one year.

$${}_1V^{(0)} = 250,000(.05v + .0575v^2) + 100,000(.20v + .21v^2) - 37,971.84(1 + .75v)$$

$$= -4331.28$$

The following information is for questions 14 and 15:



14. (9 points) You are given the model above for a Long Term Care policy. Using the Euler method with h=0.5, complete the following table. Please show all your work.

Time	$t = 0.5$	$t = 1.0$
${}_t p_x^{00}$	$= 1 - (0.5)(\mu_x^{01}) - (0.5)(\mu_x^{02})$ $= 1 - 0.5(0.16) - 0.5(0.04)$ $= 0.9$	${}_{0.5} p_x^{00}[1 - 0.5(\mu_{x+0.5}^{01}) - 0.5(\mu_{x+0.5}^{02})] + {}_{0.5} p_x^{01}(0.5)(\mu_{x+0.5}^{10})$ $= 0.9[1 - 0.5(0.16) - 0.5(0.04)] + 0.08(0.5)(0.08)$ $= 0.8132$
${}_t p_x^{01}$	$= (0.5)(\mu_x^{01})$ $= 0.5(0.16)$ $= 0.08$	${}_{0.5} p_x^{01}[1 - 0.5(\mu_{x+0.5}^{10}) - 0.5(\mu_{x+0.5}^{12})] + {}_{0.5} p_x^{00}(0.5)(\mu_{x+0.5}^{01})$ $= 0.08[1 - 0.5(0.08) - 0.5(0.06)] + 0.9(0.5)(0.16)$ $= 0.1464$
${}_t p_x^{02}$	$= (0.5)(\mu_x^{02})$ $= 0.5(0.04)$ $= 0.02$	${}_{0.5} p_x^{02} + {}_{0.5} p_x^{00}(0.5)(\mu_{x+0.5}^{02}) + {}_{0.5} p_x^{01}(0.5)(\mu_{x+0.5}^{12})$ $= 0.02 + 0.9(0.5)(0.04) + 0.08(0.5)(0.06)$ $= 0.0404$

15. (4 points) Let N be the random variable which represents that number of people in a nursing home after one year assuming that 100,000 independent lives buy a Long Term Care policy.

Calculate the $Var(N)$.

This is a binomial distribution so:

$$
\begin{aligned}
&Var(N) = 100,000 \cdot {}_1 p_x^{01} \cdot {}_1 q_x^{01} \\
&= 100,000(0.1464)(1 - 0.1464) \\
&= 12,496.704
\end{aligned}
$$