STAT 472
Quiz 2
Fall 2019
September 5, 2018

1. You are given:

   a. $e_{45} = 40$

   b. $e_{55} = 32$

   c. $e_{45:\overline{10}|} = 9$

   d. ${}_{9|1}q_{45} = 0.005$

   Calculate q_{54}.

Solution:

$e_x = e_{x:\overline{n}|} + {}_np_x \cdot e_{x+n}$ where x = 45 and n=10

$e_{45} = e_{45:\overline{10}|} + {}_{10}p_{45} \cdot e_{45+10} \rightarrow 40 = 9 + {}_{10}p_{45} \cdot 32$

${}_{10}p_{45} = \frac{31}{32}$

${}_{9|1}q_{45} = {}_9p_{45} - {}_{10}p_{45} \rightarrow 0.005 = {}_9p_{45} - \frac{31}{32}$

${}_9p_{45} = 0.97375$

${}_{9|1}q_{45} = {}_9p_{45} \cdot q_{54} \rightarrow 0.005 = 0.97375 \cdot q_{54}$

$q_{54} = 0.0051348$

2. You are given that ${}_t q_{90} = \frac{t^3}{1000}$.

Calculate the $Var[T_{90}]$.

Solution:

${}_t p_{90} = 1 - {}_t q_{90} = 1 - \frac{t^3}{1000}$ $\omega = 1000^{\frac{1}{3}} = 10$

${}_t p_{90} = \frac{1000 - t^3}{1000}$ so $0 \le t \le 10$

$Var[T_x] = E[T_x^2] - E[T_x]^2$

$\mathring{e}_{90} = E[T_{90}] = \frac{1}{1000}\int_0^{10}(1000 - t^3)dt$

$E[T_{90}] = \frac{1}{1000}[1000t - \frac{t^4}{4}]_0^{10} = 7.5$

$E[T_{90}^2] = (2)\frac{1}{1000}\int_0^{10} t(1000 - t^3)dt$

$E[T_{90}^2] = \frac{1}{500}[500t^2 - \frac{t^5}{5}]_0^{10} = 60$

$Var[T_{90}] = 60 - 7.5^2 = 3.75$

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1. You are given that ${}_t q_{95} = \frac{t^3}{125}$.

   Calculate the $Var[K_{90}]$.

   Solution:

   ${}_t p_{95} = 1 - {}_t q_{95} = 1 - \frac{t^3}{125}$ $\qquad \omega = 125^{\frac{1}{3}} = 5$

   ${}_t p_{95} = \frac{125 - t^3}{125}$ $\qquad$ so $0 \le t \le 5$

   $Var[K_x] = E[K_x^2] - E[K_x]^2$

   $E[K_{95}] = \sum_{K=1}^{5} {}_k p_x$

   $= \frac{125-1^3}{125} + \frac{125-2^3}{125} + \frac{125-3^3}{125} + \frac{125-4^3}{125} + \frac{125-5^3}{125}$

   $= 3.2$

   $E[K_{95}^2] = 2\sum_{k=1}^{5} k \cdot {}_k p_x$ - $E[K_{95}]$

   $= 2[(1)\frac{125-1^3}{125} + (2)\frac{125-2^3}{125} + (3)\frac{125-3^3}{125} + (4)\frac{125-4^3}{125}] - 3.2$

   $E[K_{95}^2] = 11.136$

   $Var[K_{95}] = 11.136 - 3.2^2 = 0.896$

2. You are given:

   a. ${}_5q_{60} = 0.10$

   b. ${}_5q_{65} = 0.2$

   c. ${}_5q_{70} = 0.3$

   d. ${}_5q_{75} = 0.4$

   Calculate ${}_{5|15}q_{60}$.

Solution:

$${}_{5|15}q_{60} = \frac{l_{65} - l_{80}}{l_{60}}$$

where:

$${}_5q_{60} = 0.1 = 1 - \frac{l_{65}}{l_{60}} \quad \rightarrow \quad 0.9 \cdot l_{60} = l_{65}$$

$${}_5q_{65} = 0.2 = 1 - \frac{l_{70}}{l_{65}} \quad \rightarrow \quad 0.8 \cdot l_{65} = l_{70}$$

$${}_5q_{70} = 0.3 = 1 - \frac{l_{75}}{l_{70}} \quad \rightarrow \quad 0.7 \cdot l_{70} = l_{75}$$

$${}_5q_{75} = 0.4 = 1 - \frac{l_{80}}{l_{75}} \quad \rightarrow \quad 0.6 \cdot l_{75} = l_{80}$$

$${}_{5|15}q_{60} = \frac{l_{65} - l_{80}}{l_{60}}$$

$$= \frac{0.9l_{60} - 0.6(0.7(0.8(0.9)))l_{60}}{l_{60}}$$

$$= \frac{0.9 - 0.6(0.7(0.8(0.9)))}{1} = 0.9 - 0.3024$$

$${}_{5|15}q_{60} = 0.5976$$