STAT 472
Quiz 3
Fall 2019

September 19, 2019

1. You are given that mortality follows the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
101	0.10	0.30	0.50	103
102	0.20	0.40	0.80	104
103	0.25	0.50	1.00	105

Further, you are given that $d = 0.10$.

Let Z be the present value random variable for a whole life insurance on (102) that pays a death benefit of 2000 at the end of the year of death.

Calculate the $E[Z] + \sqrt{Var[Z]}$.

$$l_{[102]} = 1000$$

$$l_{[102]+1} = 1000(0.8) = 800$$

$$l_{[102]+2} = 800(0.6) = 480$$

$$l_{105} = 480(0.2) = 96$$

$$l_{106} = 0$$

$$2000 \cdot A_x = E[Z] = \frac{200v + 320v^2 + 384v^3 + 96v^4}{1000} \cdot 2000 = 1,564.2432$$

$$2000^2 \cdot {}^2A_x = \frac{200v^2 + 320v^4 + 384v^6 + 96v^8}{1000} \cdot 2000^2 = 2,469,400.784$$

$$Var[Z] = {}^2A_x - (A_x)^2 = 2,469,400.784 - (1,564.2432)^2 = 22,544$$

$$E[Z] + \sqrt{Var[Z]} = 1,564.2432 + \sqrt{22,544} = 1,714.39$$

2. You are given:

   i. $A_{60} = 0.500$

   ii. ${}^2A_{60} = 0.350$

   iii. $p_{59} = 0.97$

   iv. $p_{60} = 0.96$

   v. $i = 0.10$

Let Z be the present value random variable for a whole life to (59) with a death benefit of 1 paid at the end of the year of death.

Calculate the $Var[Z]$.

$$A_x = vq_x + vp_x[A_{x+1}]$$

$$^2A_x = v^2q_x + v^2p_x[^2A_{x+1}]$$

$$A_{59} = \frac{0.03}{1.1} + \frac{0.97}{1.1}(0.5) = 0.468181$$

$$^2A_{59} = \frac{0.03}{1.21} + \frac{0.97}{1.21}(0.350) = 0.30537$$

$$Var[Z] = 0.30537 - 0.468181^2 = 0.08618$$

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During the first two years, deaths are assumed to be distributed uniformly between integral ages. After two years, it is assumed that we have a constant force of interest.

Calculate ${}_{0.6}p_{[102]+1.8}$.

$$l_{102} = 1000$$
$$l_{102+1} = 1000(0.8) = 800$$
$$l_{102+2} = 800(0.6) = 480$$
$$l_{102+3} = 480(0.2) = 96$$

$${}_{0.6}p_{[102]+1.8} = \frac{l_{[102]+2.4}}{l_{[102]+1.8}}$$

$l_{104.4} = (480)^{0.6}(96)^{0.4}$ By constant force of interest

$l_{104.4} = 252.1467$

$l_{103.8} = 800(0.2) + 480(0.8)$ By uniform distribution

$l_{103.8} = 544$

$${}_{0.6}p_{[102]+1.8} = \frac{252.1467}{544} = 0.463504907$$

2. You are given the following mortality table:

x	q_x
106	0.10
107	0.20
108	0.30
109	0.40
110	0.50
111	0.60
112	0.70
113	0.80
114	0.90
115	1.00

You are also given that $i = 0.10$.

Calculate 12,000 $A^{\,1}_{106:\overline{4|}}$.

$l_{106} = 1000$
$l_{107} = 1000(0.9) = 900$
$l_{108} = 900(0.8) = 720$
$l_{109} = 720(0.7) = 504$
$l_{110} = 504(0.6) = 302.4$

$$1000 \cdot A_{\stackrel{1}{106:\overline{4|}}} = (1000-900)v + (900-720)v^2 + (720-504)v^3 + (504-302.4)v^4$$

$$1000 \cdot A_{\stackrel{1}{106:\overline{4|}}} = 100v + 180v^2 + 216v^3 + 201.6v^4$$

$$A_{\stackrel{1}{106:\overline{4|}}} = 0.539648931$$

$$\therefore 12000 \cdot A_{\stackrel{1}{106:\overline{4|}}} = 6475.787173$$