STAT 472
Quiz 4
Fall 2019

October 15, 2019

1. Yash is (60) and is the beneficiary of a whole life annuity due with annual payments of 80,000.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that mortality is uniformly distributed between integral ages.

   a. The actuarial present value of Yash's annuity is 1,192,000 to the nearest 1000. Calculate the actuarial present value to the nearest 1.

   $$\begin{aligned} &80,000\ddot{a}_{60} \\ &= 80,000(14.9041) \\ &= 1,192,328 \end{aligned}$$

   b. Let Y be the present value random variable for the annuity in Part a. Calculate the standard deviation of Y.

   $$Var[Y] = 80,000^2\left[\frac{{}^2A_x - (A_x)^2}{d^2}\right] = 80,000^2\left[\frac{0.10834 - 0.29028^2}{0.00226}\right] = 67,956,396,960$$

   $$StdDev[Y] = \sqrt{Var[Y]} = \sqrt{67,956,396,960} = 260,684$$

c. Yash has the option of taking a whole life annuity due with annual payments where the first 15 payments are guaranteed. This annuity has the same present value as the annuity in Part a. Determine the annual payments of this annuity. (If you cannot get the answer to Part a, use the 1,192,000 as the actuarial present value.)

$$P(\ddot{a}_{\overline{15|}} + {}_{15}E_{60}\ddot{a}_{75}) = 1{,}192{,}328$$

$${}_{15}E_{60} = {}_{10}E_{60} \cdot {}_{5}E_{70}$$

$$P\left(\frac{1-1.05^{-15}}{\frac{0.05}{1.05}} + 0.57864(0.73295)(10.3178)\right) = 1{,}192{,}328$$

$$P(15.27456) = 1{,}192{,}328$$

$$P = 78{,}059.69$$

d. (Bonus) Yash decides to buy the annuity in Part a. from Rahn Life Insurance Company. With Yash's annuity, Rahn now has 625 identical annuities sold to independent lives.

Calculate the 90% confidence interval for the actuarial present value of these annuities.

$$CI = \left(625 \cdot E[Y] - \phi(0.95)\sqrt{625} \cdot SD[Y]; 625 \cdot E[Y] + \phi(0.95)\sqrt{625} \cdot SD[Y]\right)$$

$$= \left((625)(1{,}192{,}328) - (1.645)\sqrt{625}(260{,}684.48); (625)(1{,}192{,}328) + (1.645)\sqrt{625} \cdot 260{,}684.48\right)$$

$$= (734{,}484{,}350.80, 755{,}925{,}649.20)$$

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Fall 2019

October 15, 2019

1. David is (90) and is the beneficiary of a whole life annuity due with annual payments of 100,000. You are given that interest is 5% and mortality is as follows:

Age x	q_x
90	0.25
91	0.50
92	0.75
93	1

a. The actuarial present value is 215,000 to the nearest 5000. Calculate the actuarial present value of this annuity to the nearest 1.

$$l_{90} = 100,000$$
$$l_{91} = 100,000(1-.25) = 75,000$$
$$l_{92} = 75,000(1-.50) = 37,500$$
$$l_{93} = 37,500(1-.75) = 9,375$$
$$100,000\ddot{a}_{90} = 100,000 + 75,000(1.05)^{-1} + 37,500(1.05)^{-2} + 9,375(1.05)^{-3}$$
$$= 213,541$$

b. Let Y be the present value random variable for the annuity in Part a.
Calculate the variance of Y

$${}_2A_{90} = \frac{(100,000-75,000)(1.05)^{-2} + (75,000-37,500)(1.05)^{-4} + (37,500-9,375)(1.05)^{-6} + 9,375(1.05)^{-8}}{100,000}$$
$${}_2A_{90} = 0.808597568$$
$$A_{90} = \frac{(100,000-75,000)(1.05)^{-1} + (75,000-37,500)(1.05)^{-2} + (37,500-9,375)(1.05)^{-3} + 9,375(1.05)^{-4}}{100,000}$$
$$A_{90} = 0.898314$$
$$Var[Y] = P^2\left(\frac{{}_2A_{90} - (A_{90})^2}{d^2}\right)$$
$$= 100,000^2\left(\frac{0.808597568 - (0.898314)^2}{\left(\frac{0.05}{1.05}\right)^2}\right) = 7,186,124,670$$

c. David has the option of taking a whole life annuity due with quarterly payments. This annuity has the same present value as the annuity in Part a. Determine the quarterly payments of this annuity assuming that deaths are uniformly distributed between integral ages. (If you cannot get the answer to Part a, use the 215,000 as the actuarial present value.)

$$PV = 4P \cdot \ddot{a}_{90}^{(4)} = 213,541$$

$$\ddot{a}_{90}^{(4)} = \frac{1 - A_{90}^{(4)}}{d^{(4)}} = \frac{1 - \frac{i}{i^{(4)}}(A_{90})}{d^{(4)}} = \frac{1 - \frac{0.05}{0.04904}(0.89314)}{0.04849}$$

$$\ddot{a}_{90}^{(4)} = 1.753631919$$

$$4(1.75361919)P = 213,541$$

$$P = 30,442.68$$

d. (Bonus) David decides to buy the annuity in Part a. Calculate the probability that the actuarial present value will be less than 175,000.

Let Z be the # of payments he receives

$$100,000\left(\frac{1-(1.05)^{-z}}{0.05}\right) < 175,000 ==> 1-(1.05)^{-z} < 0.0875$$

$$\ln\left(1.05^{-z}\right) > \ln(1-0.0875) ==> -z\ln(1.05) > -0.0915672$$

$$z < \frac{0.0915672}{\ln(1.05)} ==> z < 1.876755$$

$$\Rightarrow \leq 1\,payment$$

$$P(APV < 175,000) = q_{90} = 0.25$$