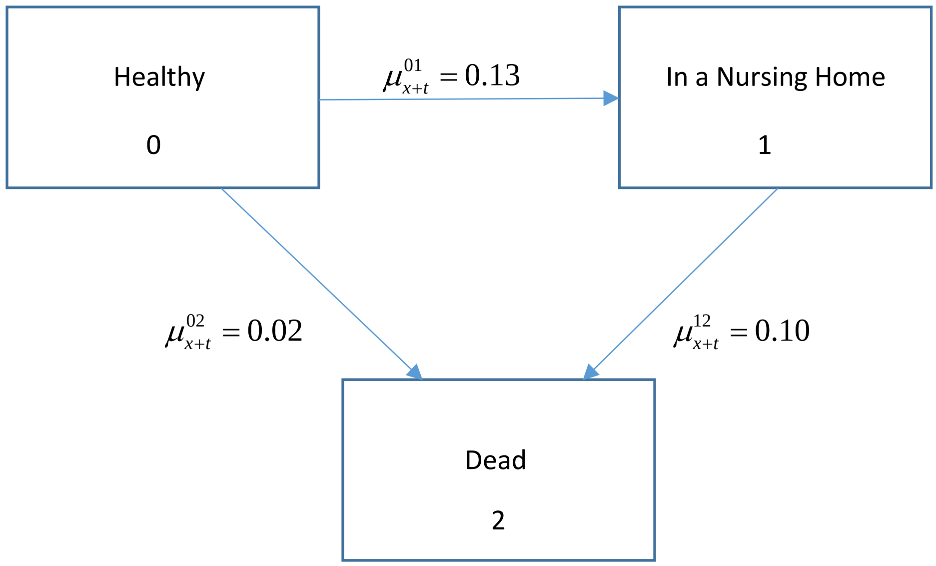

The above multi-state model is used for a long term care policy which has a term of 12 years. Jeff who is age x purchases this policy.

The policy pays four benefits:

- Benefit 1 is a lump sum benefit of 100,000 at the moment of transition from State 0 to State 1.
- Benefit 2 is a lump sum benefit of 200,000 at the moment of transition from State 0 to State 2.
- Benefit 3 is a lump sum benefit of 75,000 at the moment of transition from State 1 to State 2.
- Benefit 4 is a continuous annuity at an annual rate of 25,000 per year while a person is in State 1.

You are given that $\delta = 0.05$ and ${}_t p_x^{01} = (2.6)\left(e^{-0.10t} - e^{-0.15t}\right)$.

1. Calculate the probability that Jeff will receive Benefit 2.

   Solution:

$$\Pr = \int_0^{12} {}_t p_x^{00} \cdot \mu_{x+t}^{02} dt = \int_0^{12} e^{-\int_0^t 0.15 r \cdot dr} \cdot 0.02 dt = \int_0^{12} e^{-0.15t} \cdot 0.02 dt = 0.02 \frac{1 - e^{-0.15(12)}}{0.15} = 0.1113$$

2. Calculate the actuarial present value of Benefit 3.

Solution:

$$APV = (75,000)\int_0^{12} v^t \cdot {}_t p_x^{01} \cdot \mu_{x+t}^{12} \cdot dt = (75,000)\int_0^{12} e^{-0.05t} \cdot 2.6(e^{-0.1t} - e^{-0.15t})(0.10)dt$$

$$= (75,000)(0.26)\int_0^{12} (e^{-0.15t} - e^{-0.2t})dt = 19,500\left[\frac{1-e^{-0.15(12)}}{0.15} - \frac{1-e^{-0.20(12)}}{0.2}\right] = 19856.14$$

STAT 472
Quiz 6
Fall 2019

November 21, 2019

1. Lives insured can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is Disabled
   iii. State 2 is Dead

   State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

   You are given the following matrix of annual transition probabilities.

$$\begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.30 & 0.60 & 0.10 \\ 0 & 0 & 1 \end{bmatrix}$$

   An insurance company decides to issue a three year term insurance policy which pay a benefit of 10,000 at the end of the year of death. The policy will also pay 50,000 at the end of each year that the insured is disabled. Annual premiums will be paid only by healthy lives. The premium is determined using the equivalence principle.

   If $v = 0.95$, calculate the annual premium.

   Solution:

Time	**0**	**1**		**1**
State				
0	**1**	**0.80**	**0.685**	**0.61100**
1	**0**	**0.15**	**0.210**	**0.22875**
2	**0**	**0.05**	**0.105**	**0.16025**

$$P(1+0.8v+0.685v^2)$$
$$=(50,000)(0.15v+0.21v^2+0.22875v^3)+(10,000)(0.05v+(0.105-0.05)v^2+(0.16025-0.105)v^3$$

$$P=\frac{27,852.55}{2.378}=11,712.59$$

2. Actuarial science students can be modeled using a multi-state model. The model has three states:
   i. State 0 is actively taking exams
   ii. State 1 is having stopped taking exams but are still in the actuarial profession.
   iii. State 2 is has left the actuarial profession.

State 0 can transition to State 1 or State 2. State 1 can transition to State 2. State 2 cannot transition.

You are given the following force of transitions:

i. $\mu_{x+t}^{01} = 0.3$

ii. $\mu_{x+t}^{02} = 0.1$

iii. $\mu_{x+t}^{12} = 0.6$

Calculate ${}_8 p_x^{02}$.

Solution:

$$ {}_8 p_x^{00} = e^{-\int_0^8 (0.3+0.1)dt} = e^{(-0.4)(8)} = 0.04076 $$

$$ {}_8 p_x^{01} = \int_0^8 {}_t p_x^{00} \cdot \mu_{x+t}^{01} \cdot {}_{8-t} p_{x+t}^{11} \cdot dt = \int_0^8 e^{-0.4t}(0.3)e^{-0.6(8-t)}dt = (0.3)e^{-4.8}\int_0^8 e^{-0.4t} \cdot e^{0.6t}dt $$

$$ = (0.3)e^{-4.8}\int_0^8 e^{0.2t}dt = (0.3)e^{-4.8}\left[\frac{e^{(0.2)(8)}-1}{0.2}\right] = 0.04880 $$

$$ {}_8 p_x^{02} = 1 - {}_8 p_x^{00} - {}_8 p_x^{02} = 0.91044 $$