STAT 472
Test 2
Fall 2019

November 5, 2019

1. Jeff is (72). He has 800,000 and wants to buy an annuity to fund his retirement.

   Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Further, assume that deaths are uniformly distributed between integral ages.

   a. (4 points) If Jeff decides to buy a whole life annuity due with annual payments, the annual payment will be 70,500 to the nearest 100. Calculate it to the nearest 1.

   Solution:

   $$800,000 = P\ddot{a}_{72}$$

   $$P = \frac{800,000}{11.3468} = 70,504$$

   b. (6 points) If Y is the present value random variable for the annuity in a., determine the $\sqrt{Var[Y]}$.

   Solution:

   $$\sqrt{Var[Y]} = \sqrt{(70,504)^2 \frac{{}^2A_{72} - A_{72}^2}{d^2}} = \sqrt{(70,504)^2 \frac{0.24324 - 0.45968^2}{\left(\frac{0.05}{1.05}\right)^2}} = 264,582.88$$

c. (8 points) Jeff is also considering a certain and life annuity due with annual payments. The payments for first 15 years are guaranteed to be made. Payments after 15 years will continue if Jeff is alive. Determine the annual payment under this annuity.

Solution:

$$800,000 = P\ddot{a}_{\overline{\overline{72:\overline{15|}}}}$$

$$P = \frac{800,000}{\ddot{a}_{\overline{15|}} + {}_{15}E_{72}\ddot{a}_{87}}$$

$$P = \frac{800,000}{\frac{1-(1.05)^{-15}}{\left(\frac{0.05}{1.05}\right)} + (1.05)^{-15}\left(\frac{53,934.7}{89,082.1}\right)(6.1308)}$$

$$P = 63,070.97$$

d. (4 points) The annual payment under the annuity in c. is less than the annual payment under the annuity in a. Given that this is that case, why would Jeff consider annuity in c.

Solution:

The annuity payments in C are guaranteed for 15 years, so Jeff would receive the first 15 payments even if he dies; whereas in part A, if he dies within the first 15 years the payments will stop. So, if Jeff thinks he may within 15 years he might consider the annuity in C to ensure he gets at least 15 annuity payments.

e. (8 points) Jeff decides he wants to maximize his income over the next twelve years so he buys a 12 year term life annuity due with monthly payments. Determine the amount of the monthly payment.

Solution:

$$800,000 = 12P\ddot{a}^{(12)}_{72:\overline{12|}} \implies P = \frac{800,000}{12\ddot{a}^{(12)}_{72:\overline{12|}}}$$

$$\ddot{a}^{(12)}_{72:\overline{12|}} = \ddot{a}^{(12)}_{72} - {}_{12}E_{72}\ddot{a}^{(12)}_{84} \implies \ddot{a}^{(12)}_{72:\overline{12|}} = \left[\alpha(12)\ddot{a}_{72} - \beta(12)\right] - {}_{12}E_{72}\left[\alpha(12)\ddot{a}_{84} - \beta(12)\right]$$

$$= [(1.0002)(11.3468) - 0.46651] - (1.05)^{-12}\left(\frac{64,506.5}{89,082.1}\right)[(1.0002)(7.1421) - 0.46651]$$

$$= 10.88256 - 2.6923 = 8.190256$$

$$P = \frac{800,000}{12(8.190256)} = 8,139.75$$

2. (10 points) Bailey is (92) and buys a whole life annuity with non-level annual payments. A payment of 1000 is made at age 92 if Bailey is alive. A payment of 2000 is made at age 93 if Bailey is alive. A payment of 3000 is made at age 94 if Bailey is alive. The payments continue to increase in the same pattern for the rest of Bailey's life.

   You are given:

   a. $v = 0.9$

   b. $q_{92+t} = \dfrac{(t+1)}{3}$ for $t = 0,1,2$ and $q_{92+t} = 1$ for $t > 2$

   If Y is the present value random variable for Bailey's annuity, determine the $Var[Y]$.

   Solution:

Case	Y	Probability
Die Year 1	1000	1/3=3/9
Die Year 2	1000+2000v=2800	(2/3)(2/3)=4/9
Die Year 3	1000+2000v+3000v^2=5230	(2/3)(1/3)(1)=2/9

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y] = 1000\left(\frac{3}{9}\right) + 2800\left(\frac{4}{9}\right) + 5230\left(\frac{2}{9}\right) = 2740$$

$$E[Y^2] = 1000^2\left(\frac{3}{9}\right) + 2800^2\left(\frac{4}{9}\right) + 5230^2\left(\frac{2}{9}\right) = 9{,}896{,}200$$

$$Var[Y] = 9{,}896{,}200 - 2740^2 = 2{,}388{,}600$$

3. (10 points) Ranya is (60) and purchases a 20 year term insurance. The term insurance has a death benefit of 100,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

   You are given:

   a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.

   b. Monthly annuities are determined using the two term Woolhouse formula.

   Calculate the monthly premium for this policy.

Solution:

$$12P\ddot{a}^{(12)}_{60:\overline{5}|} = 100{,}000A^{\,1}_{60:\overline{20}|} \implies P = \frac{100{,}000A^{\,1}_{60:\overline{20}|}}{12\ddot{a}^{(12)}_{60:\overline{5}|}}$$

$$A^{\,1}_{60:\overline{12}|} = A_{60:\overline{20}|} - {}_{20}E_{60} = 0.41040 - 0.29508 = 0.11532$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = \ddot{a}^{(12)}_{60} - {}_{5}E_{60}\ddot{a}^{(12)}_{65}$$

$$\ddot{a}^{(12)}_{60} = \ddot{a}_{60} - \frac{12-1}{12(2)} = 14.9041 - \frac{11}{24} = 14.44576667$$

$$\ddot{a}^{(12)}_{65} = \ddot{a}_{65} - \frac{12-1}{12(2)} = 13.5498 - \frac{11}{24} = 13.09146667$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = (14.44576667) - (0.76687)(13.09146667) = 4.406313625$$

$$P = \frac{100{,}000(0.11532)}{12(4.406313625)} = 218.10$$

4. (8 points) Taylen is (70) and buys a whole life insurance policy with a death benefit of 50,000 to be paid at the end of the year of death. She will pay premiums annually for life.

You are given:

a. $v = 0.95$

b. $q_{70} = 0.010$

c. $q_{71} = 0.012$

d. $\ddot{a}_{71} = 11.5$

Calculate the premium for Taylen's policy.

Solution:

$$P\ddot{a}_{70} = 50,000A_{70} ==> P = \frac{50,000A_{70}}{\ddot{a}_{70}}$$

$$\ddot{a}_{70} = 1 + v \cdot p_{70} \cdot \ddot{a}_{71} = 1 + (0.95)(1 - .01)(11.5) = 11.81575$$

$$A_{70} = 1 - d \cdot \ddot{a}_{70} = 1 - (1 - 0.95)(11.81575) = 0.4092125$$

$$P = \frac{50,000(0.4092125)}{11.81575} = 1,731.64$$

5. Beau is (45) and buys a whole life policy with a death benefit of 70,000 paid at the end of the year of death. Gross premiums are paid annually.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   The expenses for Beau's policy are:

   i. Commissions of 60% of gross premium in the first year and 8% of gross premium in all years after the first;

   ii. Issue expense of 74.

   iii. Maintenance expense of 34 at the beginning of all years including the first year.

   iv. A claim expense of 750 paid at the end of the year of death.

   a. (8 points) The gross premium based on the equivalence principle is 720 to the nearest 10. Calculate the gross premium based on 1.

   Solution:

$$P\ddot{a}_{45} = 70,000A_{45} + 0.52P + 0.08P\ddot{a}_{45} + 74 + 34\ddot{a}_{45} + 750A_{45}$$

$$P = \frac{70,750A_{45} + 74 + 34\ddot{a}_{45}}{0.92a_{45} - 0.52} = \frac{(70,750)(0.15161) + 74 + (34)(17.8162)}{(0.92)(17.8162) - 0.52} = 718.68$$

   b. (6 points) If the gross premium was 750, determine the expected gain or loss at issue of the policy. Be sure to state whether it is a gain or loss.

   Solution:

$$L_0^g = 70,750A_{45} + 0.52(750) + 0.08(750)\ddot{a}_{45} + 74 + 34\ddot{a}_{45} - 750\ddot{a}_{45}$$

$$= (70,750)(0.15161) + (0.52)(750) + (0.08)(750)(17.8162)$$
$$+ 74 + (34)(17.8162) - (750)(17.8162)$$

$$= -497.02$$

   Gain of $497.02

c. (8 points) Let L_0^G be the loss at issue random variable based on a gross premium of 750. It is possible to write an expression for L_0^G as $Av^{K_{45}+1} + B\ddot{a}_{\overline{K_{45}+1|}} + C$. Determine $A, B,$ and $C.$

Solution:

$$L_0^G = 70,750v^{k_{45}+1} + (0.52)(750) + (0.08)(750)\ddot{a}_{\overline{k_{45}+1|}} + 74 + 34\ddot{a}_{\overline{k_{45}+1|}} - 750\ddot{a}_{\overline{k_{45}+1|}}$$

$$L_0^G = 70,750v^{k_{45}+1} - 656\ddot{a}_{\overline{k_{45}+1|}} + 464$$

$$A = 70,750$$
$$B = -656$$
$$C = 464$$

d. (8 points) Determine the $\sqrt{Var[L_0^G]}$.

Solution:

$$Var[L_0^G] = Var\left[70,750v^{k_{45}+1} - 656\ddot{a}_{\overline{k_{45}+1|}} + 464\right] = Var\left[70,750v^{k_{45}+1} - 656\left(\frac{1-v^{k_{45}+1}}{d}\right)\right]$$

$$= Var\left[70,750v^{k_{45}+1} + \frac{656}{d}v^{k_{45}+1}\right] = \left(70,750 + \frac{656}{d}\right)^2 Var[v^{k_{45}+1}]$$

$$= \left(70,750 + \frac{656}{d}\right)^2 [{}^2A_{45} - (A_{45})^2] = 831,915,156.62$$

$$\sqrt{Var[L_0^G]} = 9,121.14$$

e. Without doing any additional calculations, state whether the $Var[L_0^G]$ would increase, decrease or remain the same, if each of the following occurred without any other changes to the policy assumptions or premiums?

i. (2 points) The gross premium was increased.

Solution:

Increase

ii. (2 points) The claims expense was decreased.

Solution:

Decrease

iii. (2 points) The first year commission was increased to 65%

Solution:

remain the same

6 points) Brett is (35) and is receiving a whole life annuity due with annual payments. The payments at the beginning of the first 10 years are 10,000. The payments during the second 10 years are 20,000. After 20 years, the payments are 5000 for as long as Brett is alive.

You are given that mortality follows the Standard Ultimate Life Table with interest equal to 5%.

Determine the actuarial present value of Brett's annuity.

Solution:

$$APV = 10,000\ddot{a}_{35} + 10,000 \, {}_{10}E_{35}\ddot{a}_{45} - 15,000 \, {}_{20}E_{35}\ddot{a}_{55}$$

$$= (10,000)(18.9728) + (10,000)(0.61069)(17.8162) - (15,000)(0.37041)(16.0599)$$

$$= 209,298.54$$

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1. (10 points) Maddie is (50) and purchases a 20 year endowment insurance. The endowment insurance has a death benefit of 50,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

   You are given:

   a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.

   b. Deaths are uniformly distributed between integral ages.

   Calculate the monthly premium for this policy.

Solution:

$$50,000A_{50:\overline{20}|} = 12P\ddot{a}^{(12)}_{50:\overline{5}|} ==> P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}}$$

$$\ddot{a}^{(12)}_{50:\overline{5}|} = \ddot{a}^{(12)}_{50} - {}_5E_{50}\ddot{a}^{(12)}_{55} = \left[\alpha(12)\ddot{a}_{50} - \beta(12)\right] - {}_5E_{50}\left[\alpha(12)\ddot{a}_{55} - \beta(12)\right]$$

$$[(1.0002)(17.0245) - 0.46651] - 0.77772[(1.0002)(16.0599) - 0.46651] = 4.431605608$$

$$P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}} = \frac{(50,000)(0.38844)}{(12)(4.431605608)} = 365.22$$

2. (8 points) Lorenzo is (71) and buys a whole life insurance policy with a death benefit of 50,000 that will be paid at the end of the year of death. He will pay premiums annually for life.

   You are given:

   a. $v = 0.94$

   b. $q_{70} = 0.010$

   c. $q_{71} = 0.012$

   d. $\ddot{a}_{70} = 11$

   Calculate the premium for Lorenzo's policy.

Solutions:

$$P\ddot{a}_{71} = 50,000A_{71}$$

$$\ddot{a}_{70} = 1 + v \cdot p_{70} \cdot \ddot{a}_{71} \Rightarrow \ddot{a}_{71} = \frac{a_{70} - 1}{v \cdot p_{70}} = 10.74575543$$

$$A_{71} = 1 - d \cdot \ddot{a}_{71} = 1 - (1 - 0.94)(10.74575573) = 0.355254674$$

$$P = \frac{50,000(0.355254674)}{10.74575543} = 1,653$$

3. Amir is (55) and buys a whole life policy with a death benefit of 100,000 paid at the end of the year of death. Gross premiums are paid annually.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   The expenses for Amir's policy are:

   i. Commissions of 50% of gross premium in the first year and 12% of gross premium in all years after the first;

   ii. Issue Expense of 78;

   iii. Maintenance expense of 48 at the beginning of all years including the first year; and

   iv. A claim expense of 570 paid at the end of the year of death.

   a. (8 points) The gross premium based on the equivalence principle is 1780 to the nearest 10. Calculate the gross premium to the nearest 1.

      Solution:

$$P\ddot{a}_{55} = 100,570A_{55} + 0.38P + 0.12P\ddot{a}_{55} + 78 + 48\ddot{a}_{55}$$

$$P = \frac{100,570A_{55} + 78 + 48\ddot{a}_{55}}{0.88a_{55} - 0.38} = \frac{24506.962}{13.752712} = 1,782$$

   b. (6 points) If the gross premium was 1800, determine the expected gain or loss at issue of the policy. State whether it is a gain or a loss.

      Solution:

$$L_0^g = 100,570A_{55} + 0.38(1800) + 48\ddot{a}_{55} + 78 + 0.88(1800)\ddot{a}_{55}$$

$$= 25,190.962 - 25,438.8816 = -247.92$$

      Gain of 247.92

c. (8 points) Let L_0^G be the loss at issue random variable based on a gross premium of 1800. It is possible to write an expression for L_0^G as $Av^{K_{55}+1} + B\ddot{a}_{\overline{K_{55}+1}|} + C$. Determine $A, B,$ and $C.$

Solution:

$$L_0^G = 100,570v^{k_{55}+1} + 0.38(1800) + 0.12(1800)\ddot{a}_{\overline{k_{55}+1}|} + 78 + 48\ddot{a}_{\overline{k_{55}+1}|} - (1800)\ddot{a}_{\overline{k_{55}+1}|}$$

$$L_0^G = 100,570v^{k_{55}+1} - 1532\ddot{a}_{\overline{k_{55}+1}|} + 762$$

$$A = 100,570$$
$$B = -1,536$$
$$C = 762$$

d. (8 points) Determine the $\sqrt{Var[L_0^G]}$.

Solution:

$$Var[L_0^G] = Var\left[100,570v^{k_{55}+1} - 1536\ddot{a}_{\overline{k_{55}+1}|} + 762\right]$$

$$= Var\left[100,570v^{k_{55}+1} - 1536\left(\frac{1-v^{k_{55}+1}}{d}\right)\right]$$

$$= Var\left[100,570v^{k_{55}+1} + \frac{1536}{d}v^{k_{55}+1}\right] = \left(100,570 + \frac{1536}{d}\right)^2 Var\left[v^{k_{55}+1}\right]$$

$$= \left(100,570 + \frac{1536}{d}\right)^2 ({}^2A_{55} - A_{55})^2 = 343,894,922.8$$

$$\sqrt{Var[L_0^G]} = 18,544.4$$

e. Without doing any additional calculations, state whether the $Var[L_0^G]$ would increase, decrease or remain the same, if each of the following occurred without any other changes to the policy assumptions or premiums?

i. (2 points) The gross premium was increased.

Solution:

Increase

ii. (2 points) The claims expense was decreased.

Solution:

Decrease

iii. (2 points) The first year commission was increased to 55%

Solution:

Remain the same

4. Jeff is (68). He has 1,200,000 and wants to purchase an annuity.

   Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Use the two term Woolhouse formula to calculate the monthly annuities.

   a. (4 points) If Jeff decides to buy a whole life annuity due with annual payments, the annual payment will be 95,000 to the nearest 1000. Calculate it to the nearest 1.

      Solution:

      $$1,200,000 = P\ddot{a}_{68}$$

      $$P = \frac{1,200,000}{12.6456} = 94,895$$

   b. (6 points) If Y is the present value random variable for the annuity in a., determine the $\sqrt{Var[Y]}$.

      Solution:

      $$\sqrt{Var[Y]} = \sqrt{94,845^2 \frac{{}^2A_{68} - A_{68}^2}{d^2}} = \sqrt{94,845^2 \frac{0.03035129}{\left(\frac{0.05}{1.05}\right)^2}} = 347,177.21$$

c. (8 points) Jeff is also considering a certain and life annuity due with annual payments. The payments for first 15 years are guaranteed to be made. Payments after 15 years will continue if Jeff is alive. Determine the annual payment under this annuity.

Solution:

$$1,200,000 = P\ddot{a}_{\overline{\overline{68:\overline{15}|}}} \implies P = \frac{1,200,000}{\ddot{a}_{\overline{15}|} + {}_{15}E_{68}\ddot{a}_{83}}$$

$$P = \frac{1,200,000}{\dfrac{1-(1.05)^{-15}}{\left(\dfrac{0.05}{1.05}\right)} + (0.52781)\left(0.66217\right)\left(7.4873\right)} = 88,717.57$$

d. (8 points) Jeff decides he wants to purchase a deferred annuity due. The deferred annuity will beginning making monthly payments at age 80. Determine the amount of the monthly payment.

Solution:

$$1,200,000 = 12P \cdot {}_{12|}\ddot{a}_{68}^{(12)} = 12P\left({}_{12}E_{68}\ddot{a}_{80}^{(12)}\right)$$

$$P = \frac{1,200,000}{12\left(\frac{l_{80}}{l_{68}}v^{12}\right)\left(a_{80} - \frac{11}{24}\right)} = \frac{1,200,000}{44.11677} = 27,200.54$$

e. (4 points) From Jeff's perspective, state one possible disadvantage of a deferred annuity and one possible advantage of a deferred annuity.

Solution:

A disadvantage is that you might die before you begin to receive the payments

An advantage is that premiums are less expensive

5. (10 points) Mallory is (92) and buys a whole life annuity with non-level annual payments. A payment of 2000 is made at age 92 if Mallory is alive. A payment of 4000 is made at age 93 if Mallory is alive. A payment of 6000 is made at age 94 if Mallory is alive. The payments continue to increase in the same pattern for the rest of Mallory's life.

   You are given:

   a. $v = 0.9$

   b. $q_{92} = 0.3$
      $q_{93} = 0.7$
      $q_{94} = 1$

   If Y is the present value random variable for Bailey's annuity, determine the $Var[Y]$.

   Solution:

Case	Y	Probability
Die Year 1	2000	0.3
Die Year 2	2000+4000v=5600	(1-0.3)(0.7)=0.49
Die Year 3	2000+4000v+6000v^2=10,460	(1-0.3)(1-0.7)(1)=0.21

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y] = 2000(0.3) + 5600(0.49) + 10460(0.21) = 5540.60$$

$$E[Y^2] = 2000^2(0.3) + 5600^2(0.49) + 10460^2(0.21) = 39,542,836$$

$$Var[Y] = 39,542,836 - (5540.60)^2 = 8,844,587.64$$

6. (6 points) Michael (45) buys a whole life annuity due with annual payments. The payments for the first 10 years are 20,000. The payments for the next 10 years are 10,000. The payments after 20 years are 15,000 for as long as Michael is alive.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Calculate the actuarial present value of the annuity.

Solution:

$$APV = 20{,}000\ddot{a}_{45} - 10{,}000\,{}_{10}E_{45}\ddot{a}_{55} + 5{,}000\,{}_{20}E_{45}\ddot{a}_{65}$$

$$APV = 20{,}000(17.8162) - 10{,}000(0.60655)(16.0599) + 5{,}000(0.35994)(13.5478)$$

$$APV = 283{,}298.2516$$