STAT 472
Test 3
Fall 2019
December 11, 2019

1. Ranya who is (21) purchases a whole life insurance policy with a death benefit of 100,000 payable at the end of the year of death. The policy has annual premiums. **The gross premium for this policy is 360.**

   You are given:

   i. Mortality follows that Standard Ultimate Life Table.

   ii. $i = 0.05$

   iii. Deaths are uniformly distributed between integral ages.

   a. (3 points) The net premium is 260 to the nearest 10. Calculate the net premium to the nearest 0.01.

   Solution:

   $$PVP = PVB$$

   $$P^n \ddot{a}_{21} == 100{,}000 A_{21} ==> P^n = \frac{(100{,}000)(0.051441)}{19.9197} = 258.24$$

   b. (4 points) Calculated the net premium reserve at the end of 20 years.

   Solution:

   $${}_{20}V^n = PVFB - PVFP^n = 100{,}000 A_{41} - 258.24 \ddot{a}_{41}$$

   $$= (100{,}000)(0.12665) - 258.24(18.3403) = 7928.80$$

   Or

   $${}_{20}V^n = (100{,}000)\left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{21}}\right) = (100{,}000)\left(1 - \frac{18.3403}{19.9197}\right) = 7928.83$$

Question 1 Continued . . .

This policy has the following expenses:

i. First year expense of 400 per policy plus 53% of premium

ii. Expense of 50 per policy plus 5% of premium in years 2+

iii. Claim expense of 500 incurred at the end of the year of death

Per policy expenses are incurred at the beginning of the policy year.

c. (4 points) The gross premium reserve at the end of 20 years is 7370 to the nearest 10. Calculate the reserve to the nearest 0.1. Remember that the gross premium is 360.

Solution:

$$ {}_{20}V^{g} = PVFB + PVFE - PVFP $$

$$ = 100,000A_{41} + 50\ddot{a}_{41} + 0.05P\ddot{a}_{41} + 500A_{41} - P\ddot{a}_{41} $$

$$ = (100,500)(0.12665) - [(0.95)(360) - 50](18.3403) = 7372.96 $$

d. (4 points) Use the recursive formula to find the gross premium reserve at the end of 21 years.

Solution:

$$ {}_{21}V^{g} = \frac{({}_{20}V^{g} + 0.95P - 50)(1.05) - (100,500)q_{41}}{1 - q_{41}} $$

$$ = \frac{(7372.96 + (0.95)(360) - 50)(1.05) - (100,500)(0.000565)}{1 - 0.000565} = 7995.94 $$

e. (4 points) Calculate the gross premium reserve at time 20.7 years.

Solution:

$$ {}_{20.7}V = (0.3)(7372.96 + 0.95P - 50) + (0.7)(7995.94) = 7896.65 $$

Question 1 Continued . . .

f. (2 points) Calculate the expense reserve at the end of 20 years.

Solution:

$$ {}_{20}V^{e} = {}_{20}V^{g} - {}_{20}V^{n} = 7372.96 - 7928.80 = -555.76 $$

g. (3 points) Explain why the expense reserve is negative.

Solution:

Expenses are front loaded. That means that the expenses in the first year are higher than the expenses in the second year and later. However, the premium for expenses is level. Initially the present value of the expense premiums is equal to the present value of the expenses. However after the first year, the present value of future expense premiums is greater than the present value of future expenses so the reserve is negative. This occurs because the expenses in the first year are greater than the premium for expenses.

Question 1 Continued . . .

During the 21st year of Ranya's policy, the insurance company has actual experience as follows:

i. Mortality is 110% of the Standard Ultimate Life Table

ii. $i = 0.06$

iii. Expenses are 40 per policy, 6% of premium, and 700 per claim paid.

h. (6 points) Determine the total profit or loss on this policy during the 21st year. Be sure to state if it is a profit or loss.

Solution:

$$Total\ \ Gain = (7372.96 + 360 - 0.06(360) - 40)(1.06)$$
$$-(100,700)(1.1)(0.000565) - (7995.95)[1-(1.1)(0.000565)] = 78.08$$

Profit

The company wants to allocate the gain or loss to the source. The company allocates gains and losses in the following order – First to interest, then to expenses, and finally to mortality.

i. (6 points) Determine the gain or loss from expenses. Be sure to state if it is a gain or loss.

Solution:

$$Gain\ Int = (7372.96 + 360 - 0.05(360) - 50)(1.06)$$
$$-(100,500)(0.000565) - (7995.95)[1-(0.000565)] = 76.65$$

$$Gain\ Int\ \&\ Expense = (7372.96 + 360 - 0.06(360) - 40)(1.06)$$
$$-(100,700)(0.000565) - (7995.95)[1-(0.000565)] = 83.32$$

$$Gain\ Expense = 83.32 - 76.65 = 6.67 \text{ of Gain}$$

Question 1 Continued . . .

The insurance company decides to hold modified net premium reserves by holding Full Preliminary Term (FPT) reserves. Under FPT reserves, the difference between the FPT premium in years 2 and later and the FPT premium in the first year is called the expense allowance.

j. (4 points) Calculate the expense allowance $(P_{x+1}^{FPT} - {}_1P^{FPT})$ for Ranya's policy.

Solution:

$${}_1P^{FPT} = S \cdot v \cdot q = (100,000)(1.05)^{-1}(0.000253) = 24.10$$

$$P_{x+1}^{FPT} = \frac{100,000A_{22}}{\ddot{a}_{22}} = \frac{(100,000)(0.05378)}{19.8707} = 270.65$$

$$P_{x+1}^{FPT} - {}_1P^{FPT} = 270.65 - 24.10 = 246.55$$

k. (4 points) Calculate the FPT reserve at the end of 20 years on Ranya's policy.

Solution:

$${}_{20}V^{FPT} = 100,000A_{41} - P_{x+1}^{FPT}\ddot{a}_{41} = (100,000)(0.12665) - (270.65)(18.3403) = 7701.20$$

or

$${}_{20}V^{FPT} = 100,000\left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{21}}\right) = (100,000)\left(1 - \frac{18.3403}{19.8707}\right) = 7701.79$$

l. (3 points) List two reasons that FPT reserves are preferable to net premium reserves or gross premium reserves.

Solution:

FPT reserves a better reflection of economic reserves because some allowance is permitted for expenses.

The FPT reserve maintains the simplicity of the net premium calculations compared to the complexity of the gross premium reserve calculations.

2. Jake who is (70) purchases a 30 year term insurance policy with a death benefit of 500,000 paid at the moment of death. For this policy, premiums are paid quarterly for 20 years.

   You are given:

   i. Mortality follows that Standard Ultimate Life Table.

   ii. $i = 0.05$

   iii. Deaths are uniformly distributed between integral ages.

   a. (6 points) The quarterly net premium is 4920 to the nearest 10. Calculate the quarterly net premium to the nearest 0.01 .

      Solution:

$$PVP = PVB ==> 4P\ddot{a}^{(4)}_{70:\overline{20|}} = 500,000\bar{A}^{1}_{70:\overline{30|}}$$

$$\ddot{a}^{(4)}_{70:\overline{20|}} = \ddot{a}^{(4)}_{70} - {}_{20}E_{70}\cdot\ddot{a}^{(4)}_{90} = [\alpha(4)\cdot\ddot{a}_{70} - \beta(4)] - {}_{20}E_{70}[\alpha(4)\cdot\ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(12.0083) - 0.38272] - (0.38272)[(1.00019)(5.1835) - 0.38272]$$

$$= 10.79653$$

$$\bar{A}^{1}_{70:\overline{30|}} = \left(\frac{i}{\delta}\right)\left(A_{70} - {}_{30}E_{70}\cdot A_{100}\right) = \left(\frac{0.05}{\ln(1.05)}\right)\left(0.42818 - (0.17313)(0.09168)(0.87068)\right)$$

$$= 0.42463$$

$$P = \frac{500,000(0.42463)}{4(10.79653)} = 4916.33$$

b. (5 points) Calculate the net premium reserve at the end of 10 years.

Solution:

$${}_{10}V = PVB - PVP ==> 500,000\bar{A}^{1}_{80:\overline{20|}} - 4P\ddot{a}^{(4)}_{80:\overline{10|}}$$

$$\ddot{a}^{(4)}_{80:\overline{10|}} = \ddot{a}^{(4)}_{80} - {}_{10}E_{80} \cdot \ddot{a}^{(4)}_{90} = [\alpha(4) \cdot \ddot{a}_{80} - \beta(4)] - {}_{10}E_{80}[\alpha(4) \cdot \ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(8.5484) - 0.38272] - (0.33952)[(1.00019)(5.1838) - 0.38272]$$

$$= 6.53701$$

$$\bar{A}^{1}_{80:\overline{20|}} = \left(\frac{i}{\delta}\right)\left(A_{80} - {}_{20}E_{80} \cdot A_{100}\right) = \left(\frac{0.05}{\ln(1.05)}\right)\left(0.59293 - (0.03113)(0.87068)\right)$$

$$= 0.57986$$

$${}_{10}V = (500,000)(0.57986) - (4)(4916.33)(6.53701) = 161,377.61$$

c. (2 points) Calculate the net premium reserve at the end of 20 years.

Solution:

Solution:

$${}_{20}V = PVB - PVP ==> 500,000\bar{A}^{1}_{90:\overline{20|}} - 0$$

$$500,000\bar{A}^{1}_{90:\overline{10|}} = (500,000)\left(\frac{i}{\delta}\right)\left(A_{90} - {}_{10}E_{80} \cdot A_{100}\right)$$

$$= (500,000)\left(\frac{0.05}{\ln(1.05)}\right)\left(0.75317 - (0.9168)(0.87068)\right) = 345,021$$

3. A disability income policy issued to Ryan who is (60) follows the Standard Sickness-Death Model with $i = 0.05$.

   The policy pays premiums continuously at a rate of P when the insured is in state 0. The premium is determined using the equivalence principle.

   The policy pays the following benefits:

   i. Benefit 1 is a lump sum death benefit at the moment of death of 50,000

   ii. Benefit 2 is a disability annuity benefit of 48,000 per year paid continuously while the insured is in state 1.

   a. (5 points) The premium for this policy is 17,800 to the nearest 10. Calculate it to the nearest 0.01 .

   Solution:

$$PVP = PVB$$

$$P\bar{a}_{60}^{00} = 50,000\bar{A}_{60}^{02} + 48,000\bar{a}_{60}^{01}$$

$$P = \frac{(50,000)(0.46236) + (48,000)(2.6295)}{8.3908} = 17,797.35$$

   b. (5 points) Determine ${}_{10}V^{(0)}$, the reserve at time 10 for a policy in state 0.

   Solution:

$${}_{10}V^{(0)} = PVFB - PVFP = 50,000\bar{A}_{70}^{02} + 48,000\bar{a}_{70}^{01} - P\bar{a}_{70}^{00} =$$

$$(50,000)(0.61070) + (48,000)(3.0177) - (17,797.35)(4.9609) = 87,093.73$$

   c. (4 points) Determine ${}_{10}V^{(1)}$, the reserve at time 10 for a policy in state 1.

   Solution:

$${}_{10}V^{(1)} = PVFB - PVFP = 50,000\bar{A}_{70}^{12} + 48,000\bar{a}_{70}^{11} - P\bar{a}_{70}^{10} =$$

$$(50,000)(0.63947) + (48,000)(7.3744) - (17,797.35)(0.0134) = 385,706.22$$

Question 3 Continued . . .

The insurance company also offers the above benefits in a plan where the premiums are only paid for 10 years.

d. (6 points) Calculate the premium for this policy. Please note that the benefits do not change.

Solution:

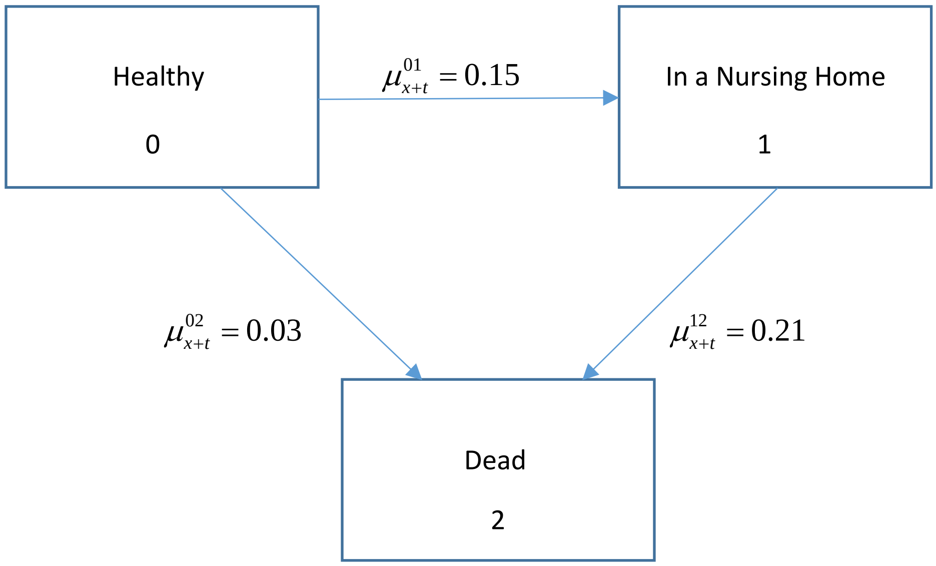
$$PVP = PVB$$

$$P\bar{a}^{00}_{60:\overline{10|}} = 50,000\bar{A}^{02}_{60} + 48,000\bar{a}^{01}_{60}$$

$$P = \frac{(50,000)(0.46236) + (48,000)(2.6295)}{\bar{a}^{00}_{60} - v^{10}\,{}_{10}p^{00}_{60} \cdot \bar{a}^{00}_{70} - v^{10}\,{}_{10}p^{01}_{60} \cdot \bar{a}^{10}_{70}}$$

$$= \frac{149,334}{8.3908 - (1.05)^{-10}(0.59115)(4.9609) - (1.05)^{-10}(0.19589)(0.0134)} = 22,664.81$$

4.



The above multi-state model is used for a long term care policy which has a term of 10 years. Jeff who is age x purchases this policy.

The policy pays four benefits:

- Benefit 1 is a lump sum benefit of 50,000 at the moment of transition from State 0 to State 1.
- Benefit 2 is a lump sum benefit of 140,000 at the moment of transition from State 0 to State 2. **The Actuarial Present Value of this benefit is 16,430.06.**
- Benefit 3 is a lump sum benefit of 80,000 at the moment of transition from State 1 to State 2.
- Benefit 4 is a continuous annuity at an annual rate of 30,000 per year while a person is in State 1.

You are given that $\delta = 0.05$ and ${}_t p_x^{01} = (5)\left(e^{-0.18t} - e^{-0.21t}\right)$.

a. (4 points) Calculate the probability that Jeff will receive Benefit 2.

Solution:

$$\text{Prob} = \int_0^{10} {}_t p_x^{00} \cdot \mu_{x+t}^{02} \cdot dt = \int_0^{10} e^{-\int_0^t (\mu_{x+s}^{01} + \mu_{x+s}^{02}) ds} \cdot \mu_{x+t}^{02} \cdot dt = \int_0^{10} e^{-0.18t} \cdot 0.03 \cdot dt$$

$$= 0.03 \frac{1 - e^{-0.18(10)}}{0.18} = 0.13912$$

Question 4 Continued . . .

b. (4 points) Calculate the actuarial present value of Benefit 1.

Solution:

$$\text{APV} = 50,000\int_0^{10} e^{-\delta t} \cdot {}_t p_x^{00} \cdot \mu_{x+t}^{01} \cdot dt = 50,000\int_0^{10} e^{-0.05t} \cdot e^{-0.18t} \cdot 0.15 \cdot dt$$

$$= (50,000)(0.15)\left(\frac{1-e^{-0.23(10)}}{0.23}\right) = 29,339.69$$

c. (4 points) Calculate the actuarial present value of Benefit 3

Solution:

$$\text{APV} = 80,000\int_0^{10} e^{-\delta t} \cdot {}_t p_x^{01} \cdot \mu_{x+t}^{12} \cdot dt = 80,000\int_0^{10} e^{-0.05t} \cdot (5)\left(e^{-0.18t} - e^{-0.21t}\right) \cdot 0.25 \cdot dt$$

$$= (80,000)(5)(0.21)\left(\frac{1-e^{-0.23(10)}}{0.23} - \frac{1-e^{-0.26(10)}}{0.26}\right) = 29,520.27$$

d. (4 points) Calculate the actuarial present value of Benefit 4

Solution:

$$APV = 30,000\bar{a}_x^{01} = 30,000\int_0^{10} e^{-\delta t} \cdot {}_t p_x^{01} \cdot dt = 30,000\int_0^{10} e^{-0.05t} \cdot (5)\left(e^{-0.18t} - e^{-0.21t}\right) \cdot dt$$

$$= (30,000)(5)\left(\frac{1-e^{-0.23(10)}}{0.23} - \frac{1-e^{-0.26(10)}}{0.26}\right) = 52,714.77$$

Question 4 Continued . . .

e. (4 points) Using the equivalence principle, calculate the net premium for this coverage. The net premium is paid continuously while in State 0. The present value of benefits for all four benefits is 128,000 to the nearest 1000. If you cannot get the right present value of benefits, then use 128,000 to get the premium.

Solution:

$$PVP = PVB$$

$$P\bar{a}_x^{00} = 16,430.06 + 29,339.39 + 52,714.77 + 29,520.27 = 128,004.49$$

$$\bar{a}_x^{00} = \int_0^{10} e^{-0.05t} e^{0.18t}\, dt = \left(\frac{1 - e^{-0.23(10)}}{0.23} \right) = 3.911918$$

$$P = \frac{128,004.49}{3.911918} = 32,721.67$$

5. Lives insured can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is Disabled
   iii. State 2 is Dead

State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

You are given the following matrix of annual transition probabilities.

$$\begin{bmatrix} 0.90 & 0.08 & 0.02 \\ 0.40 & 0.50 & 0.10 \\ 0 & 0 & 1 \end{bmatrix}$$

An insurance company decides to issue a two year term insurance policy which pays a benefit of 100,000 at the end of the year of death. The policy will also pay 40,000 at the end of each year that the insured is disabled. Annual premiums will be paid only by healthy lives. The premium is determined using the equivalence principle.

You are given that $v = 0.92$.

a. (5 points) The net annual premium using the equivalence principle is 5900 to the nearest 100. Calculate the net premium to the nearest 1.

Solution:

	Time		
State	**0**	**1**	**2**
0	1	0.9	(0.9)(0.9)+(0.08)(0.4) = 0.842
1	0	0.08	(0.9)(0.08)+(0.08)(0.5) = 0.112
2	0	0.02	(0.9)(0.02)+(0.08)(0.1)+(0.02)(1) = 0.46

$$PVP = PVB$$

$$P(1+0.9v) = (40,000)(0.08v + 0.112v^2) + (100,000)(0.02v + [0.046 - 0.02]v^2)$$

$$P = 5895.25$$

Question 5 Continued . . .

b. (3 points) Calculate ${}_1V^{(0)}$.

Solution:

$${}_1V^{(0)} = PVFB - PVFP = 40,000(0.08v) + (100,000)(0.02v) - 5895.25$$

$$= -1,111.25$$

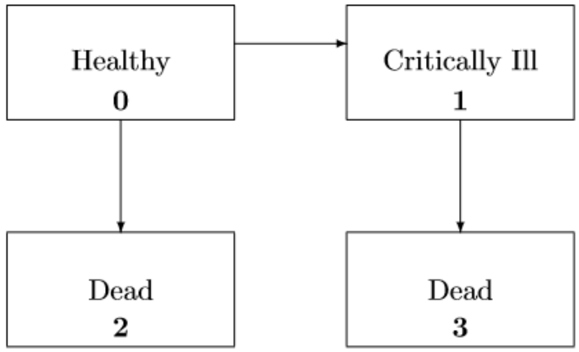
c. (3 points) Calculate ${}_1V^{(1)}$.

Solution:

$${}_1V^{(0)} = PVFB - PVFP = 40,000(0.5v) + (100,000)(0.1v) - 0$$

$$= 27,600$$

6. (9 points) Critical illness policies use the following multi-state model:



Keyi Critical Illness Insurance Company sells critical illness insurance policies. Under Keyi's policies, the premiums are paid annually. The policies have the following benefits:

i. Pay a lump sum of 50,000 at the end of the year during which an insured transfers from Healthy to Critically Ill.

ii. Pay a lump sum payment of 40,000 is made at the end of the year during which an insured transfers from critically ill to dead.

iii. Pay a payment of 10,000 at the end of each year when the policyholder is in State 1. This means that in the year that an insured moves from State 0 to State 1, a total payment of 60,000 will be paid -- The 50,000 from Benefit i and the 10,000 from this benefit.

iv. Pay a lump sum benefit 90,000 is paid at the end of the year if the insured transfers from Healthy to Dead.

For a policy issued to age 60, the annual premium is 8000 and paid while the insured is in State 0. You are also given the following information:

$i = 0.05$ $\qquad {}_{10}V^{(0)} = 70,000$ $\qquad$ and $\qquad {}_{10}V^{(1)} = 120,000$

$p_{70}^{00} = 0.969 \quad p_{70}^{01} = 0.019 \quad p_{70}^{02} = 0.012 \quad p_{70}^{03} = 0 \quad p_{70}^{11} = 0.94 \quad p_{70}^{13} = 0.06$

Calculate the reserve for a policy at the end of the 11th year for both policies in state 0 and in state 1.

THIS IS NOT AN APPLICABLE QUESTION.

Question 6 Continued . . . (This page is intentionally left blank)