STAT 472
Quiz 2
Fall 2020

September 3, 2020

1. You are given that $\mu_x = 0.001x + 0.01$.

   Calculate ${}_{10}q_{50}$.

   Solution:

$$S_0(x) = e^{-\int_0^x \mu_r dr} = e^{-\int_0^x (0.001r + 0.01)\,dr} = e^{-[0.0005r^2 + 0.01r]_0^x} = e^{-0.0005\mathrm{x}^2 - 0.01\mathrm{x}}$$

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)} = \frac{e^{-0.0005(\mathrm{x+t})^2 - 0.01(\mathrm{x+t})}}{e^{-0.0005(\mathrm{x})^2 - 0.01(\mathrm{x})}}$$

$${}_{10}q_{50} = 1 - {}_{10}p_{50}$$

$${}_tp_x = \frac{e^{-0.0005(\mathrm{x+t})^2 - 0.01(\mathrm{x+t})}}{e^{-0.0005(\mathrm{x})^2 - 0.01(\mathrm{x})}}$$

$${}_{10}p_{50} = \frac{e^{-0.0005(60)^2 - 0.01(60)}}{e^{-0.0005(50)^2 - 0.01(50)}} = \frac{0.090718}{0.173774} = 0.52205$$

$${}_{10}q_{50} = 1 - 0.52205 = 0.47795$$

2. You are given that ${}_t p_x = 1 - \frac{t^3}{n^3}$ for $0 \le t \le n$.

You are also given that $\overset{\circ}{e}_x = 4.5$

Calculate n.

Solution:

$$\overset{\circ}{e}_x = \int_0^n {}_t p_x dt = \int_0^n \left(1 - \frac{t^3}{n^3}\right) dt = \left[t - \frac{t^4}{4n^3}\right]_0^n = n - \frac{n}{4} = 0.75n = 4.5 ==> n = \frac{4.5}{0.75} = 6$$

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1. You are given that $S_0(x) = 1 - \frac{x^2}{6400}$ for $0 \le x \le 80$.

   If $\mu_x = \frac{17}{222}$, determine x.

   Solution:

$$\mu_x = \frac{-\frac{d}{dx} S_0(x)}{S_0(x)}$$

$$\mu_x = \frac{17}{222} = \frac{-\frac{d}{dx}\left(1 - \frac{x^2}{6400}\right)}{1 - \frac{x^2}{6400}} \text{ ==> } \frac{17}{222} = \frac{2x}{6400 - x^2} \text{ ==> } (17)(6400 - x^2) = (222)(2x)$$

$$\therefore 17x^2 + 444x - 108{,}800 = 0$$

$$x = \frac{-444 \pm \sqrt{444^2 - 4(17)(-108{,}000)}}{2(17)} = \frac{-444 \pm 2756}{34} \to x = 68, -94.12$$

$$x = 68$$

2. You are given that $\mu_x = 0.002x + 0.01$.

Calculate $e_{20:\overline{5}|}$.

Solution:

$$S_0(x) = e^{-\int_0^x \mu_r dr} = e^{-\int_0^x (0.002r+0.01)\,\mathrm{d}r} = e^{-[0.001r^2+0.01r]_0^x} = e^{-0.001\mathrm{x}^2-0.01\mathrm{x}}$$

$$S_{20}(t) = {}_t\, p_{20} = \frac{S_0(20+t)}{S_0(20)} = \frac{e^{-0.001(20+\mathrm{t})^2-0.01(20+\mathrm{t})}}{e^{-0.001(20)^2-0.01(20)}} = e^{-0.001(40t+t^2)-0.01\mathrm{t}} = e^{-0.05t-0.001t^2}$$

$$e_{20:\overline{5}|} = \sum_{n=1}^{5} {}_t\, p_{20} =$$

$$e^{-0.05(1)+0.001(1^2)} + e^{-0.05(2)+0.001(2^2)} + e^{-0.05(3)+0.001(3^2)} + e^{-0.05(4)+0.001(4^2)} + e^{-0.055(5)+0.001(5^2)}$$

$$= 4.26981$$

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The n was provided in the email instructions for the quiz.

1. You are given that ${}_t p_x = 1 - \frac{t^3}{n^3}$ for $0 \le t \le n$. Calculate:

   a. ${}_{3|2} q_x$

   Solution:

$$ {}_{3|2} q_x = {}_3 p_x - {}_5 p_x = \left(1 - \frac{3^3}{n^3}\right) - \left(1 - \frac{5^3}{n^3}\right) = \frac{125 - 27}{n^3} $$

$n = 6 ==> 0.45370$ $\quad$ $n = 7 ==> 0.28571$ $\quad$ $n = 8 ==> 0.19141$

   b. e_x

   Solution:

$$ e_x = \sum_{t=1}^{n} {}_t p_x = \sum_{t=1}^{n} \left(1 - \frac{t^3}{n^3}\right) = $$

$$ \left(1 - \frac{1^3}{n^3}\right) + \left(1 - \frac{2^3}{n^3}\right) + \left(1 - \frac{3^3}{n^3}\right) + \ldots + + \left(1 - \frac{(n-1)^3}{216}\right) + \left(1 - \frac{n^3}{216}\right) = $$

$n = 6 ==> 3.95833$ $\quad$ $n = 7 ==> 4.71429$ $\quad$ $n = 8 ==> 5.46875$

c. $Var[T_x]$

Solution:

$$Var[T_x] = \mathrm{E}[\mathrm{T}_x{}^2] - \left(\mathring{e}_x\right)^2$$

$$\mathring{e}_x = \int_0^n {}_t p_x dt = \int_0^n \left(1 - \frac{t^3}{n^3}\right) dt = t - \frac{t^4}{(n^3)(4)}\Bigg|_0^n = n - \frac{n^4}{(n^3)(4)} = 0.75n$$

$$\mathrm{E}[\mathrm{T}_x{}^2] = 2\int_0^n t \cdot {}_t\, p_x dt = 2\int_0^n t\left(1 - \frac{t^3}{n^3}\right) dt = 2\left(\frac{t^2}{2} - \frac{t^5}{(n^3)(5)}\Bigg|_0^n\right)$$

$$= 2\left(0.5n^2 - \frac{n^5}{(n^3)(5)}\right) = 0.6n^2$$

$$Var[T_x] = \mathrm{E}[\mathrm{T}_x{}^2] - \left(\mathring{e}_x\right)^2 = 0.6n^2 - \left(0.75n\right)^2 = 0.0375n^2$$

$n = 6 ==> 1.35$ $\qquad$ $n = 7 ==> 1.8375$ $\qquad$ $n = 8 ==> 2.4$

2. You are given:

x	e_x	$e_{x:\overline{10}\rvert}$
30	45	9
40	37	8
50	30	7

Calculate ${}_{20}q_{30}$.

Solution:

$${}_{20}q_{30} = 1 - {}_{20}p_{30} = 1 - {}_{10}p_{30} \bullet {}_{10}p_{40}$$

$$e_x = e_{x:\overline{10}\rvert} + {}_{10}p_x \cdot e_{x+10} ==> {}_{10}p_x = \frac{e_x - e_{x:\overline{10}\rvert}}{e_{x+10}}$$

$${}_{10}p_{30} = \frac{e_{30} - e_{30:\overline{10}\rvert}}{e_{30+10}} = \frac{45-9}{37} = \frac{36}{37} \text{ and } {}_{10}p_{40} = \frac{e_{40} - e_{40:\overline{10}\rvert}}{e_{40+10}} = \frac{37-8}{30} = \frac{29}{30}$$

$${}_{20}q_{30} = 1 - {}_{10}p_{30} \bullet {}_{10}p_{40} = 1 - \frac{36}{37} \cdot \frac{29}{30} = 0.059459$$