**There are three versions of the quiz.**
**Please find your version.**

# STAT 472
# Quiz 3
# Fall 2020
**September 17, 2020**

1. You are given that Mortality follows the Standard Ultimate Life Table.

   You are also given that mortality between 90 and 91 and between 91 and 92 follows a constant force of mortality while deaths are uniformly distributed between 92 and 93 and between 93 and 94.

   Calculate ${}_{1.6\,|\,1.2}q_{90.8}$ .

   **Solution:**

$$ {}_{1.6\,|\,1.2}q_{90.8} = \frac{l_{92.4} - l_{93.6}}{l_{90.8}} $$

$$ l_{90.8} = (l_{90})^{0.2}(l_{91})^{0.8} = (41{,}841.1)^{0.2}(37{,}618.6)^{0.8} = 38{,}427.55 $$

$$ l_{92.4} = (0.6)(33{,}379.9) + (0.4)(29{,}183.8) = 31{,}701.46 $$

$$ l_{93.6} = (0.4)(29{,}183.8) + (0.6)(25{,}094.3) = 26{,}730.1 $$

$$ {}_{1.6\,|\,1.2}q_{90.8} = \frac{31{,}701.46 - 26{,}730.1}{38{,}427.55} = 0.12937 $$

2. You are also given that mortality rates for 2020 follow the Standard Ultimate Life Table. In other words, $q(x,0) = q_x$ in the standard ultimate life table.

   During 2020, a vaccine for COVID is developed which significantly reduces death rates for older persons. The vaccine is available on January 1, 2021. With this vaccine, the mortality improvement factors which vary by age and time are:

| $x$ | $\varphi(x,1)$ | $\varphi(x,2)$ | $\varphi(x,3)$ |
|---|---|---|---|
| 95 | 0.08 | 0.07 | 0.02 |
| 96 | 0.09 | 0.06 | 0.02 |
| 97 | 0.10 | 0.05 | 0.02 |
| 98 | 0.12 | 0.04 | 0.02 |
| 99 | 0.15 | 0.03 | 0.02 |

   Lauren is (96) on January 1, 2020. She receives a vaccine on January 1, 2021.

   Calculate the probability that Lauren is alive at age 99.

**Solution:**

$${}_3p_{96} = [1-q(96,0)][1-q(97,0)\{1-\varphi(97,1)\}][1-q(98,0)\{1-\varphi(98,1)\}\{1-\varphi(98,2)\}]$$

$$= [1-0.192877][1-0.214030\{1-0.10\}][1-0.237134\{1-0.12\}\{1-0.04\}]$$

$$= 0.52110$$

# STAT 472
# Quiz 3
# Fall 2020

**September 17, 2020**

1. You are given the following values from a mortality table:

$$q_{90} = 0.25 \qquad q_{91} = 0.40 \qquad q_{92} = 0.50$$

You are also given that deaths are uniformly distributed between 90 and 91 and between 91 and 92. Mortality between 92 and 93 follows a constant force of mortality.

Calculate ${}_{0.7\,|\,1.3}q_{90.6}$ .

**Solution:**

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{l_{91.3} - l_{92.6}}{l_{90.6}}$$

$$l_{90} = 1000 \text{ ==> } l_{91} = (1000)(1-0.25) = 750 \text{ ==> } l_{92} = 750(1-0.4) = 450 \text{ ==> } l_{93} = (450)(1-0.5) = 225$$

$$l_{90.6} = (0.4)(1000) + (0.6)(750) = 850$$

$$l_{91.3} = (0.7)(750) + (0.3)(450) = 660$$

$$l_{92.6} = (450)^{0.4}(225)^{0.6} = 296.88928$$

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{660 - 296.8828}{850} = 0.42719$$

2. The probability that a new iPhone must be replaced over the next three years based on existing data is given in the table below. However, software improvements will result in an extended lifetime with a reduced “death” rate. The iPhone’s mortality rates and improvement factors are given below:

| $x$ | $q(x,0)$ | $\varphi(x,1)$ | $\varphi(x,2)$ | $\varphi(x,3)$ |
|---|---|---|---|---|
| 0 | 0.25 | 0.20 | 0.12 | 0.06 |
| 1 | 0.50 | 0.15 | 0.08 | 0.04 |
| 2 | 0.75 | 0.10 | 0.04 | 0.02 |

Calculate the probability that an iPhone placed into service today is still functioning at the end of three years if all software updates have been applied.

**Solution:**

$${}_3p_x = [1-q(0,0)][1-q(1,0)\{1-\varphi(1,1)\}][1-q(2,0)\{1-\varphi(2,1)\}\{1-\varphi(2,2)\}]$$

$$= [1-0.25][1-0.5\{1-0.15\}][1-0.75\{1-0.1\}\{1-0.04\}]$$

$$= 0.1518$$

# STAT 472
# Quiz 3
# Fall 2020

**September 17, 2020**

1. You are given that mortality follows the following select and ultimate mortality table.

| $[x]$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | $q_{x+3}$ | $x$ |
|---|---|---|---|---|---|
| 93 | 0.14 | 0.24 | 0.36 | 0.50 | 96 |
| 94 | 0.21 | 0.32 | 0.45 | 0.70 | 97 |
| 98 | 0.28 | 0.40 | 0.63 | 0.90 | 98 |
| 99 | 0.36 | 0.56 | 0.81 | 1.00 | 99 |

Further, you are given that deaths are uniformly distributed during the first two years following underwriting. Mortality after the first two years following underwriting follows a constant force of mortality.

Calculate ${}_{1.5\,|\,2.3}q_{[93]+0.4}$

**Solution:**

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{l_{[93]+0.4+1.5} - l_{[93]+0.4+1.5+2.3}}{l_{[93]+0.4}} = \frac{l_{[93]+1.9} - l_{97.2}}{l_{[93]+0.4}}$$

$$l_{[93]} = 1000 \text{ ==> } l_{[93]+1} = (1000)(1-0.14) = 860 \text{ ==> } l_{[93]+2} = 860(1-0.24) = 653.6$$

$$\text{==> } l_{96} = (653.6)(1-0.36) = 418.304 \text{ ==> } l_{97} = (418.304)(1-0.5) = 209.152$$

$$\text{==> } l_{98} = (209.152)(1-0.7) = 62.7456$$

$$l_{[93]+0.4} = (0.6)(1000) + (0.4)(860) = 944$$
$$l_{[93]+1.9} = (0.1)(860) + (0.9)(653.6) = 674.24$$
$$l_{97.2} = (209.152)^{0.8}(62.7456)^{0.2} = 164.3941174$$

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{674.24 - 164.3941174}{944} = 0.54009$$

2. The probability that a new iPhone must be replaced over the next three years based on existing information is given in the column labeled $q(x,0)$ in the table below. However, software improvements will result in an extended lifetime with a reduced "death" rate. The iPhone's mortality rates and improvement factors are given below:

| $x$ | $q(x,0)$ | $\varphi(x,1)$ | $\varphi(x,2)$ | $\varphi(x,3)$ |
|---|---|---|---|---|
| 0 | 0.25 | 0.20 | 0.12 | 0.06 |
| 1 | 0.50 | 0.15 | 0.08 | 0.04 |
| 2 | 0.75 | 0.10 | 0.04 | 0.02 |

Calculate the 3 year expected curtate lifetime ( $e_{0:\overline{3}|}$ ) of an iPhone placed into service today if all software updates are applied.

**Solution:**

$$e_{0:\overline{3}|} = {}_1p_0 + {}_2p_0 + {}_3p_0$$

$$= (1-q(0,0)) + (1-q(0,0))(1-q(1,0)[1-\varphi(1,1)])$$
$$+ (1-q(0,0))(1-q(1,0)[1-\varphi(1,1)])(1-q(2,0)[1-\varphi(2,1)][1-\varphi(2,2)])$$

$$= (1-0.25) + (1-0.25)(1-0.50[1-0.15]) + (1-0.25)(1-0.50[1-0.15])(1-0.75[1-0.1][1-0.04])$$

$$= 1.33305$$