There are three versions of this quiz. Please find your version.

STAT 472
Quiz 4
Fall 2020

October 14, 2020

1. (10 points) Abishek is (25) and purchases a special term policy with a non-level death benefit. The death benefit paid at the end of the year of death is 50,000 if he dies between ages 25 and 40. The death benefit is 100,000 if he dies between ages 40 and 60. For death between ages 60 and 75, the death benefit is 25,000. No death benefit is payable after age 75 is Abishek lives 50 years.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%.

   Calculate the present value of this life insurance policy.

   Solution:

$$EPV = 50,000A_{25} + 50,000\,{}_{15}E_{25} \cdot A_{40} - 75,000\,{}_{35}E_{25} \cdot A_{60} - 25,000\,{}_{50}E_{25} \cdot A_{75}$$

$$= 50,000A_{25} + 50,000\,{}_{5}E_{25} \cdot {}_{10}E_{30} \cdot A_{40}$$
$$- 70,000\,{}_{20}E_{25} \cdot {}_{10}E_{45} \cdot {}_{5}E_{55} \cdot A_{60} - 25,000\,{}_{20}E_{25} \cdot {}_{20}E_{45} \cdot {}_{10}E_{65} \cdot A_{70}$$

$$= (50,000)(0.06147) + (50,000)(0.78240)(0.61152)(0.12106)$$
$$-(75,000)(0.37373)(0.60655)(0.77382)(0.29028)$$
$$-(25,000)(0.37373)(0.35994)(0.55305)(0.50868)$$

$$= 1204.54$$

2. Michelle, (55), purchases a 20 year term life annuity due. The annuity makes annual payments of 10,000.

   Let Y be the present value random variable for this annuity.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%.

   a. (2 points) Calculate the Actuarial Present Value of this annuity.

      Solution:

      $$EPV = 10,000\ddot{a}_{55:\overline{20|}} = (10,000)(12.6737) = 126,737$$

      Note that the value of $\ddot{a}_{55:\overline{20|}}$ is in the tables.

   b. (8 points) Calculate the $Var[Y]$.

      Solution:

      $$Var[Y] = (10,000)^2 \left[\frac{{}^2A_{55:\overline{20|}} - (A_{55:\overline{20|}})^2}{d^2} \right]$$

      Note that $A_{55:\overline{20|}}$ is in the tables as is the value of d.

      $${}^2A_{55:\overline{20|}} = {}^2A_{55} - v^{20}\,{}_{20}E_{55} \cdot {}^2A_{75} + v^{20}\,{}_{20}E_{55}$$

      $$= 0.07483 - (1.05)^{-20}(0.32819)(0.29079) + (1.05)^{-20}(0.32819) = 0.16255$$

      $$Var[Y] = (10,000)^2 \left[\frac{0.016255 - (0.39649)^2}{(0.04762)^2} \right] = 235,893,040$$

STAT 472
Quiz 4
Fall 2020
October 14, 2020

1. (10 points) Lauren is (35) and purchases a special whole life policy with a non-level death benefit. The death benefit paid at the end of the year of death is 50,000 if she dies between ages 35 and 50. The death benefit is 100,000 if she dies between ages 50 and 65. For death after age 65, the death benefit is 30,000.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%.

   Calculate the present value of this life insurance policy.

   Solution:

   $$EPV = 50,000A_{35} + 50,000\,{}_{15}E_{35} \cdot A_{50} - 70,000\,{}_{30}E_{35} \cdot A_{65}$$

   $$= 50,000A_{35} + 50,000\,{}_{5}E_{35} \cdot {}_{10}E_{40} \cdot A_{50} - 70,000\,{}_{10}E_{35} \cdot {}_{20}E_{45} \cdot A_{65}$$

   $$= (50,000)(0.09653) + (50,000)(0.78181)(0.60920)(0.18931) - (70,000)(0.61069)(0.35994)(0.35477)$$

   $$= 3875.93$$

2. Eli, (65), purchases a continuous whole life annuity. The annuity pays at an annual rate of 10,000.

   Let Y be the present value random variable for this annuity.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%. You are also given that deaths are uniformly distributed between integral ages. Finally, you are given that ${}^2\bar{A}_{65} = 0.16197$.

   a. (3 points) Calculate the Actuarial Present Value of this annuity.

      Solution:

      $$APV = 10,000\bar{a}_{65} = (10,000)\left(\frac{1-(i/\delta)A_{65}}{\delta}\right)$$

      $$= (10,000)\left(\frac{1-(1.02480)(0.35477)}{0.04879}\right) = 130,443.06$$

   b. (7 points) Calculate the $Var[Y]$.

      Solution:

      $$Var[Y] = (10,000)^2\left[\frac{{}^2\bar{A}_{65} - (\bar{A}_{65})^2}{\delta^2}\right]$$

      $$= (10,000)^2\left[\frac{0.16197 - [(1.02480)(0.35477)]^2}{(0.04879)^2}\right] = 1,251,356,580$$

STAT 472
Quiz 4
Fall 2020

October 14, 2020

1. (10 points) Abishek is (25) and purchases a special term policy with a non-level death benefit. The death benefit paid at the end of the year of death is 50,000 if he dies between ages 25 and 40. The death benefit is 100,000 if he dies between ages 40 and 60. For death between ages 60 and 75, the death benefit is 25,000. No death benefit is payable after age 75 if Abishek lives 50 years.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%.

   Calculate the present value of this life insurance policy.

   Solution:

$$EPV = 50,000A_{25} + 50,000\,{}_{15}E_{25} \cdot A_{40} - 75,000\,{}_{35}E_{25} \cdot A_{60} - 25,000\,{}_{50}E_{25} \cdot A_{75}$$

$$= 50,000A_{25} + 50,000\,{}_{5}E_{25} \cdot {}_{10}E_{30} \cdot A_{40}$$
$$- 70,000\,{}_{20}E_{25} \cdot {}_{10}E_{45} \cdot {}_{5}E_{55} \cdot A_{60} - 25,000\,{}_{20}E_{25} \cdot {}_{20}E_{45} \cdot {}_{10}E_{65} \cdot A_{70}$$

$$= (50,000)(0.06147) + (50,000)(0.78240)(0.61152)(0.12106)$$
$$-(75,000)(0.37373)(0.60655)(0.77382)(0.29028)$$
$$-(25,000)(0.37373)(0.35994)(0.55305)(0.50868)$$

$$= 1204.54$$

2. Michelle, (65), purchases a continuous whole life annuity. The annuity pays at an annual rate of 10,000.

   Let Y be the present value random variable for this annuity.

   You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%. You are also given that deaths are uniformly distributed between integral ages. Finally, you are given that ${}^2\bar{A}_{65} = 0.16197$.

   a. (3 points) Calculate the Actuarial Present Value of this annuity.

      Solution:

$$APV = 10,000\bar{a}_{65} = (10,000)\left(\frac{1-(i/\delta)A_{65}}{\delta}\right)$$

$$= (10,000)\left(\frac{1-(1.02480)(0.35477)}{0.04879}\right) = 130,443.06$$

   b. (7 points) Calculate the $Var[Y]$.

      Solution:

$$Var[Y] = (10,000)^2\left[\frac{{}^2\bar{A}_{65} - (\bar{A}_{65})^2}{\delta^2}\right]$$

$$= (10,000)^2\left[\frac{0.16197 - [(1.02480)(0.35477)]^2}{(0.04879)^2}\right] = 1,251,356,580$$