STAT 472
Test 1
Fall 2020
September 29, 2020

1. You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.08$ which means that $v = 0.92$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate ${}_{0.8}p_{101.6}$.

Solution:

$${}_{0.8}p_{101.6} = \frac{l_{102.4}}{l_{101.6}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$$l_{104} = (280)(1-0.75) = 70; l_{105} = 0$$

$${}_{0.8}p_{101.6} = \frac{l_{102.4}}{l_{101.6}} = \frac{(560)^{0.6}(280)^{0.4}}{(800)(0.4)+(560)(0.6)} = 0.64695$$

b. Let Z be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the end of the year of death.

i. (4 points) Write an expression of Z.

Solution:

$$Z = 10,000v^{K_{100}+1} = (10,000)(0.92)^{K_{100}+1}$$

ii. (8 points) The expected present value which is $10,000A_{100}$ is 8000 to the nearest 100. Calculate $10,000A_{100}$ to the nearest 1.

Solutions:

Using the ls from Part a:

$$1000A_{100} = 200v + 240v^2 + 280v^3 + 210v^4 + 70v^2 = 801.7469 ==> A_{100} = 0.8017469$$

$$10,000A_{100} = 8017.47$$

iii. (8 points) Calculate $Var[Z]$.

Solution:

$$1000\left({}^2A_{100}\right) = 200v^2 + 240v^4 + 280v^6 + 210v^8 + 70v^{10} ==> {}^2A_{100} = 0.64918295$$

$$Var[Z] = (10,000)^2[0.64918295 - (0.8017469)^2] = 638,486$$

2. (8 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]:\overline{4}|}$ which is a four year endowment insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]:\overline{4}|} = (100,000 - 80,000)(0.9) + (80,000 - 48,000)(0.9)^2 +$$
$$(48,000 - 9600)(0.9)^3 + 9600v^4 = 78,212.34$$

3. You are given that $S_x(t) = (1 - 0.001t^3)$ for $0 \le t \le 10$.

a. (6 points) Calculate μ_{x+5} .

Solution:

$$\mu_{x+t} = \frac{-\frac{d}{dt} S_x(t)}{S_x(t)} = \frac{-(-0.003t^2)}{(1-0.001t^3)} ==> \mu_{x+5} = \frac{-(-0.003(5)^2)}{(1-0.001(5)^3)} = 0.08571$$

b. (6 points) Calculate p_{x+6} .

Solution:

$$p_{x+6} = \frac{S_x(7)}{S_x(6)} = \frac{1-0.001(7)^3}{1-0.001(6)^3} = 0.83801$$

c. (8 points) Calculate the $Var[T_x]$.

Solution:

$$Var[T_x] = E[T_x^2] - (E[T_x])^2$$

$$E[T_x] = \int_0^{10} {}_t p_x dt = \int_0^{10} (1 - 0.001t^3) dt = \left[t - \frac{0.001t^4}{4} \right]_0^{10} = 7.5$$

$$E[T_x^2] = 2\int_0^{10} t \cdot {}_t p_x dt = 2\int_0^{10} t(1 - 0.001t^3) dt = 2\left[\frac{t^2}{2} - \frac{0.001t^5}{5} \right]_0^{10} = 60$$

$$Var[T_x] = 60 - (7.5)^2 = 3.75$$

4. (8 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $i = 0.05$
3. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine $1000A_{52}$

Solution:

$$A_{50} = vq_{50} + vp_{50} \cdot A_{51} \text{ ==> } 0.3 = (1.05)^{-1}(0.018) + (1.05)^{-1}(1-0.018)(A_{51}) \text{ ==> } A_{51} = 0.31376$$

$$A_{51} = vq_{51} + vp_{51} \cdot A_{52} \text{ ==> } 0.31376 = (1.05)^{-1}(0.002) + (1.05)^{-1}(1-0.002)(A_{52}) \text{ ==> } A_{52} = 0.32810$$

$$1000A_{52} = 328.10$$

5. (8 points) Megan (25) buys a 20 year term policy with a death benefit of 1,000,000 payable at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000A^{\,1}_{25:\overline{20}|}$.

Solution:

$$A^{\,1}_{25:\overline{20}|} = A_{25:\overline{20}|} - {}_{20}E_{25} = 0.37854 - 0.37373 = 0.00481$$

$$1,000,000A^{\,1}_{25:\overline{20}|} = (1,000,000)(0.00481) = 4810$$

6. The probability that a new iPhone must be replaced over the next three years based on existing data is given in the table below. However, software improvements will result in an extended lifetime with a reduced "death" rate. The iPhone's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$	$\varphi(x,3)$
0	0.2	0.25	0.20	0.10
1	0.4	0.20	0.12	0.06
2	0.6	0.10	0.05	0.04

a. (8 points) Calculate probability that an iPhone placed into service today is still functioning at the end of three years if all software updates have been applied.

Solution:

$${}_3p_0 = [1-q(0,0)][1-q(1,0)\{1-\varphi(1,1)\}][1-q(2,0)\{1-\varphi(2,1)\}\{1-\varphi(2,2)\}]$$

$$=[1-0.2][1-(0.4)\{1-0.2\}][1-(0.6)\{1-0.1\}\{1-0.05\}] = 0.26493$$

Let L_3 be the random variable representing the number of iPhones still in service at the end of 3 years assuming that 20,000 new iPhones are sold today and all software updates are applied.

b. (6 points)Using the probability from Part (a), calculate $Var[L_3]$. If you are not able to calculate the probability in Part (a), assume that it is 0.25. If you get an answer for Part (a), please use that answer.

Solution:

$$Var[L_3] = npq = (20,000)(0.26493)(1-0.26493) = 3894.84$$

7. (8 points) You are given that ${}_{t|1}q_x = 0.04 \quad \text{for} \;\; t = 0,1,2,\ldots,24$.

Calculate ${}_{12}q_{x+5}$.

Solution:

$$ {}_{12}q_{x+5} = \frac{l_{x+5} - l_{x+17}}{l_{x+5}} \quad \text{and} \;\; l_x = 100 $$

$$ {}_{t|1}q_x = \frac{l_{x+t} - l_{x+t+1}}{l_x} = 0.04 \quad \text{for} \;\; t = 0,1,2,\ldots,24 $$

$$ \frac{l_x - l_{x+1}}{l_x} = 0.04 \quad \text{for} \quad t = 0 ==> \frac{100 - l_{x+1}}{100} = 0.04 ==> l_{x+1} = 96 $$

$$ \frac{l_{x+1} - l_{x+2}}{l_x} = 0.04 \quad \text{for} \quad t = 1 ==> \frac{96 - l_{x+2}}{100} = 0.04 ==> l_{x+2} = 92 $$

$$ \frac{l_{x+2} - l_{x+3}}{l_x} = 0.04 \quad \text{for} \quad t = 2 ==> \frac{92 - l_{x+3}}{100} = 0.04 ==> l_{x+3} = 92 $$

$$ \therefore l_{x+t} = 100 - 4t $$

$$ {}_{12}q_{x+5} = \frac{l_{x+5} - l_{x+17}}{l_{x+5}} = \frac{80 - 32}{80} = 0.6 $$

8. (8 points) You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

You are also given that $i = 0.06$.

Calculate $1000_{10}E_{70}$.

Solution:

$${}_{10}p_{70} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{70}(1.11^{10} - 1)\right] = 0.76930$$

$$1000_{10}E_{70} = v^{10} \cdot {}_{10}p_{70} = (1000)(1.06)^{-10}(0.76930) = 429.57$$

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101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate ${}_{0.6}p_{101.8}$.

Solution:

$${}_{0.6}p_{101.8} = \frac{l_{102.4}}{l_{101.8}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$${}_{0.6}p_{101.8} = \frac{l_{102.4}}{l_{101.8}} = \frac{(560)^{0.6}(280)^{0.4}}{(800)(0.2)+(560)(0.8)} = 0.69803$$

b. (6 points) Calculate μ_{102}.

Solution:

For a constant force of mortality, $\mu_x = -\ln[p_x]$

$$\mu_{102} = -\ln[p_{102}] = -\ln[0.5] = 0.69315$$

c. Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

i. (4 points) Write an expression of Z^{CONT}.

Solution:

$$Z^{CONT} = 10,000v^{T_{100}} = (10,000)(0.9)^{T_{100}}$$

d. Let $Z^{Discrete}$ be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

i. (8 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

Solution:

Using the ls from Part a:

$$1000A_{100:\overline{3}|} = 200v + 240v^2 + 560v^3 = 782.64 ==> A_{100:\overline{3}|} = 0.78264$$

$$10,000A_{100:\overline{3}|} = 7826.4$$

ii. (8 points) Calculate $Var[Z^{Discrete}]$

Solution:

Using the ls from Part a:

$$1000[{}^2A_{100:\overline{3}|}] = 200v^2 + 240v^4 + 560v^6 = 617.07096 ==> {}^2A_{100:\overline{3}|} = 0.61707096$$

$$Var[Z^{Discrete}] = (10,000)^2[{}^2A_{100:\overline{3}|} - (A_{100:\overline{3}|})^2]$$

$$= (10,000)^2[0.61707096 - (0.78264)^2] = 454,559.04$$

2. You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.08$.

a. (8 points) If $l_{[75]} = 100,000$, calculate $l_{[76]}$.

Solution:

$$l_{[75]+1} = (100,000)(1-0.1) = 90,000$$

$$l_{77} = (90,000)(1-0.2) = 72,000$$

$$l_{78} = (72,000)(1-0.4) = 43,200$$

$$l_{78} = l_{[76]+1}(1-0.3) ==> l_{[76]+1} = \frac{l_{78}}{0.7} = \frac{43,200}{0.7} = 61,714.28571$$

$$l_{[76]+1} = l_{[76]}(1-0.15) ==> l_{[76]} = \frac{61,714.28571}{0.85} = 72,605.04202$$

b. (8 points) Calculate $100,000A_{[77]}$ which is a whole life insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]} = (100,000-80,000)(0.92) + (80,000-48,000)(0.92)^2 + (48,000-9600)(0.92)^3 + (9600-96)v^4 + 96v^5 = 82,208.77$$

3. You are given that $S_0(t) = (1 - 0.008t^3)$ for $0 \le t \le 5$.

a. (6 points) Calculate p_3 .

Solution:

$$p_3 = \frac{S_0(4)}{S_0(3)} = \frac{(1-0.008(4)^3)}{(1-0.008(3)^3)} = 0.662245$$

b. (8 points) Calculate e_0 .

Solution:

$$e_0 = \sum_{k=1}^{5} {}_k p_0 = (1-0.008(1)^3) + (1-0.008(2)^3) + (1-0.008(3)^3)$$

$$+ (1-0.008(4)^3) + (1-0.008(5)^3) = 3.2$$

4. (8 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{52} = 0.32$
2. $i = 0.05$
3. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine $1000A_{50}$.

Solution:

$$A_{51} = vq_{51} + vp_{51} \cdot A_{52} = (1.05)^{-1}(0.002) + (1.05)^{-1}(1-0.002)(0.32) = 0.30606$$

$$A_{50} = vq_{50} + vp_{50} \cdot A_{51} = (1.05)^{-1}(0.018) + (1.05)^{-1}(1-0.018)(0.30606) = 0.29267$$

$$1000A_{50} = 292.67$$

5. (8 points) Madeline (25) buys a 32 year pure endowment contract that will pay her 1,000,000 at the end of the term provided she is alive.

   You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

   Calculate the expected present value which is $1,000,000 A_{25:\overline{32|}}^{\;\;1}$.

   Solution:

$$1,000,000 A_{25:\overline{32|}}^{\;\;1} = (1,000,000) v^{32} \frac{l_{57}}{l_{25}}$$

$$= (1,000,000)(1.05)^{-32} \frac{97,435.2}{99,871.1} = 204,747.44$$

6. The probability that a new iPhone must be replaced over the next three years based on existing data is given in the table below. However, software improvements will result in an extended lifetime with a reduced "death" rate. The iPhone's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$	$\varphi(x,3)$
0	0.10	0.25	0.20	0.10
1	0.25	0.20	0.12	0.06
2	0.50	0.10	0.05	0.04

a. (8 points) Calculate probability that an iPhone placed into service today is still functioning at the end of three years if all software updates have been applied.

Solution:

$$_3p_0 = [1-q(0,0)][1-q(1,0)\{1-\varphi(1,1)\}][1-q(2,0)\{1-\varphi(2,1)\}\{1-\varphi(2,2)\}]$$

$$= [1-0.1][1-(0.25)\{1-0.2\}][1-(0.5)\{1-0.1\}\{1-0.05\}] = 0.4122$$

Let L_3 be the random variable representing the number of iPhones still in service at the end of 3 years assuming that 20,000 new iPhones are sold today and all software updates are applied.

b. (6 points) Using the probability from Part (a), calculate $Var[L_3]$. If you are not able to calculate the probability in Part (a), assume that it is 0.4. If you get an answer for Part (a), please use that answer.

Solution:

$$Var[L_3] = npq = (20,000)(0.4122)(1-0.4122) = 4845.8232$$

7. (8 points) You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

You are also given that $i = 0.06$.

Calculate $1000A^{\,1}_{80:\overline{2}|}$.

Solution:

$$p_{80} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{80}(1.11-1)\right] = 0.95644$$

$$p_{81} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{81}(1.11-1)\right] = 0.95177$$

$$l_{80} = 1000; l_{81} = (1000)(0.95644) = 956.44; l_{82} = (956.44)(0.95177) = 910.31$$

$$1000A^{\,1}_{80:\overline{2}|} = (1000-956.44)(1.06)^{-1} + (956.44-910.31)(1.06)^{-2} = 82.15$$

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1. You are given that mortality follows the following mortality table:

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100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.09$ which means that $v = 0.91$. Further, you are given that between integral ages for ages 100 and 101 and between ages 101 and 102, mortality follows a constant force of mortality. For ages over 102, deaths are uniformly distributed between integral ages.

a. (6 points) Calculate ${}_{1.2}p_{101.6}$.

Solution:

$${}_{1.2}p_{101.6} = \frac{l_{102.8}}{l_{101.6}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$$l_{104} = (280)(1-0.75) = 70; l_{105} = 0$$

$${}_{1.2}p_{101.6} = \frac{l_{102.8}}{l_{101.6}} = \frac{(560)(0.2)+(280)(0.8)}{(800)^{0.4}(560)^{0.6}} = 0.520224$$

b. (4 points) Calculate $\mu_{102.2}$.

Solution:

$$\mu_{x+s} = \frac{q_x}{1-s\cdot q_x} \quad \text{under UDD.}$$

$$\mu_{102.2} = \frac{q_{102}}{1-(0.2)q_{102}} = \frac{0.5}{1-(0.2)(0.5)} = 0.555556$$

c. (6 points) The expected value of $E[K_{102}]$ is 0.6 to the nearest 0.1. Calculate it to the nearest 0.001.

Solution:

$$E[K_{102}] = \sum_{k=1}^{3} {}_t p_{102} = (0.5) + (0.5)(0.25) + (0.5)(0.25)(0) = 0.625$$

d. (6 points) Calculate the $Var[K_{102}]$.

Solution:

$$\text{From Part e, } E[K_{102}] = 0.625$$

$$Var[K_{102}] = E[K_{102}{}^2] - \left(E[K_{102}]\right)^2$$

$$E[K_{102}{}^2] = 2\sum_{k=1}^{3} t \cdot_t p_{102} - e_{102} = 2\left[(1)(0.5) + (2)(0.5)(0.25) + (3)(0.5)(0.25)(0)\right] - 0.625 = 0.875$$

$$Var[K_{102}] = E[K_{102}{}^2] - \left(E[K_{102}]\right)^2 = 0.875 - (0.625)^2 = 0.484375$$

e. (3 points) Calculate the complete expectation of life for a life age 102 which is $\mathring{e}_{102}$.

Solution:

$$\mathring{e}_{102} = e_{102} + \frac{1}{2} \quad \text{under UDD and we have UDD for ages 102 and above.}$$

$$\mathring{e}_{102} = 0.625 + 0.5 = 1.125$$

f. Let Z be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

i. (3 points) Write an expression of Z.

Solution:

$$Z = \begin{cases} 10,000v^{K_x+1} & \text{for } K_x < n \\ 10,000v^n & \text{for } K_x \geq n \end{cases}$$

ii. (6 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

Solution:

Using the ls from Part a:

$$1000A_{100:\overline{3}|} = 200v + 240v^2 + 560v^3 = 802.74376 ==> A_{100:\overline{3}|} = 0.80274376$$

$$10,000A_{100:\overline{3}|} = 8027.44$$

iii. (6 points) Calculate $Var[Z]$.

Solution:

Using the ls from Part a:

$$1000[{}^2A_{100:\overline{3}|}] = 200v^2 + 240v^4 + 560v^6 = 648.2066875 ==> {}^2A_{100:\overline{3}|} = 0.6482066875$$

$$Var[Z] = (10,000)^2[{}^2A_{100:\overline{3}|} - (A_{100:\overline{3}|})^2]$$

$$= (10,000)^2[0.6482066875 - (0.80274376)^2] = 380,914$$

2. (6 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]}$ which is a whole life issued to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]} = (100,000 - 80,000)(0.9) + (80,000 - 48,000)(0.9)^2 +$$
$$(48,000 - 9600)(0.9)^3 + (9600 - 96)(0.9)^4 + 96(0.9)^5 = 78,149.17$$

3. (6 points) You are given that $\mu_x = \dfrac{3}{120-x}$ for $0 \le x < 120$.

Calculate ${}_{10}p_{60}$.

Solutions:

$${}_t p_0 = e^{-\int_0^t \frac{3}{120-x}dx} = e^{\left[3\ln(120-x)\right]_0^t} = e^{3\ln(1220-t)-3\ln(120)} = e^{\ln\left(\frac{120-t}{120}\right)(3)} = \left(\frac{120-t}{120}\right)^3$$

$${}_t p_{60} = \frac{{}_{70}p_0}{{}_{60}p_0} = \left(\frac{120-70}{120-60}\right)^3 = 0.578704$$

4. (6 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $Var[Z] = 0.04$
3. $i = 0.05$
4. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine ${}^2A_{52}$ to four decimal places.

Solution:

$$Var(Z) = {}^2A_{50} - (A_{50})^2 \implies 0.04 = {}^2A_{50} - (0.3)^2 \implies {}^2A_{50} = 0.13$$

$${}^2A_{50} = v^2 q_{50} + v^2 p_{50} \, {}^2A_{51}$$

$$0.13 = (1.05)^{-2}(0.0018) + (1.05)^{-2}(1 - 0.0018)\, {}^2A_{51} \implies {}^2A_{51} = 0.141780204$$

$${}^2A_{51} = v^2 q_{51} + v^2 p_{51} \, {}^2A_{52}$$

$$0.141780204 = (1.05)^{-2}(0.002) + (1.05)^{-2}(1 - 0.002)\, {}^2A_{52} \implies {}^2A_{52} = 0.15462$$

5. (6 points) Varun (30) buys a 10 year term policy with a death benefit of 1,000,000 payable at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000 A^{1}_{30:\overline{10}|}$.

Solution:

$$A^{1}_{30:\overline{10}|} = A_{30:\overline{10}|} - {}_{10}E_{30} = 0.61447 - 0.61152 = 0.00295$$

$$1,000,000 A^{1}_{30:\overline{10}|} = (1,000,000)(0.00295) = 2950$$

6. (6 points) Becky (55) buys a 24 year pure endowment contract that will pay her 50,000 at the end of the term provided she is alive.

   You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

   Calculate the expected present value which is $50,000A_{55:\overline{24|}}^{\;\;\;1}$.

   Solution:

$$A_{55:\overline{24|}}^{\;\;\;1} = v^{24}\,{}_{24}p_{55} = (1.05)^{-24}\left(\frac{l_{79}}{l_{55}}\right) = (1.05)^{-24}\left(\frac{77,927.4}{97.846.2}\right) = 0.246946597$$

$$50,000A_{55:\overline{24|}}^{\;\;\;1} = (50,000)(0.246946597) = 12,347.33$$

7. (4 points) Explain why an n-year endowment insurance is equivalent to purchasing an n-year term policy and an n-year pure endowment.

   Solution:

   An n-year endowment pays a benefit when the insured dies during the n-year period or pays a benefit at the end of n-years if the insured is still alive.

   An n-year term pays a benefit when the insured dies during the n-year period but makes no payment if the insured is alive at the end of n years.

   An n-year pure endowment pays a benefit at the end of n-years if the insured is still alive but does not pay a benefit if the insured dies.

   If you add the benefits paid by the term insurance and the pure endowment, you will get the benefits paid the n-year endowment insurance.

8. You are also given that mortality rates for 2020 follow the Standard Ultimate Life Table. In other words, $q(x,0) = q_x$ in the Standard Ultimate Life Table.

   During 2020, a vaccine for COVID is developed which significantly reduces death rates for older persons. The vaccine is available on January 1, 2021. With this vaccine, the mortality improvement factors which vary by age and time are:

x	$\varphi(x,1)$	$\varphi(x,2)$	$\varphi(x,3)$
95	0.10	0.08	0.03
96	0.11	0.07	0.03
97	0.12	0.06	0.02
98	0.13	0.05	0.02
99	0.14	0.04	0.01

   Michelle is (96) on January 1, 2020. If she is alive, she receives a vaccine on January 1, 2021.

   (6 points) Calculate the probability that Michelle is alive at age 99.

   Solution:

$$ {}_3p_{96} = [1 - q(96,0)][1 - q(97,0)\{1 - \varphi(97,1)\}][1 - q(98,0)\{1 - \varphi(98,1)\}\{1 - \varphi(98,2)\}] $$

$$ = [1 - 0.192887][1 - (0.214030)\{1 - 0.12\}][1 - (0.237134)\{1 - 0.13\}\{1 - 0.05\}] = 0.52670 $$

9. (4 points) List two reasons why it is important to build mortality improvement into your calculations. Explain the two reasons.

   Solution:

   Including mortality improvement results in a more accurate price being charged for life insurance. The premiums will be lower as insureds will die later.

   One of the risks faced by both insurance companies and individuals is longevity risk. By building in mortality improvement, these risks can more accurately be evaluated.

10. You are given that mortality follows Makeham's Law with:

$$A = 0.004,\ B = 0.00001, \text{ and } c = 1.11$$

a. (6 points) Calculate probability that a person age 90 is still alive at age 95.

Solution:

$$_{5}p_{90} = \exp\left[-(0.004(5) - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11^{5}-1)\right] = 0.44596284$$

Let L_5 be the random variable representing the number of people alive at age 95 if there were 50,000 people alive at age 90.

b. (4 points) Using the probability from Part (a), calculate $Var[L_5]$. If you are not able to calculate the probability in Part (a), assume that it is 0.45. If you get an answer for Part (a), please use that answer.

Solution:

$$Var[L_5] = npq = (50,000)(0.44596284)(1-0.44596284) = 12,353.99927$$

c. (6 points) You are also given that $i = 0.06$. Calculate $1000A^{\,1}_{90:\overline{2|}}$.

Solution:

$$p_{90} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11-1)\right] = 0.877698382$$

$$p_{91} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{91}(1.11-1)\right] = 0.865574333$$

$$l_{90} = 1000; l_{91} = (1000)(0.877698382) = 877.698382;$$

$$l_{92} = (877.698382)(0.865574333) = 759.7131916$$

$$1000A^{\,1}_{90:\overline{2|}} = (1000 - 877.698382)(1.06)^{-1} + (877.698382 - 759.7131916)(1.06)^{-2} = 220.39$$