STAT 472
Test 1
Fall 2020
September 29, 2020

1. You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.08$ which means that $v = 0.92$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate ${}_{0.8}p_{101.6}$.

b. Let Z be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the end of the year of death.

i. (4 points) Write an expression of Z.

ii. (8 points) The expected present value which is $10,000A_{100}$ is 8000 to the nearest 100. Calculate $10,000A_{100}$ to the nearest 1.

iii. (8 points) Calculate $Var[Z]$.

2. (8 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]:\overline{4}|}$ which is a four year endowment insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

3. You are given that $S_x(t) = (1 - 0.001t^3)$ for $0 \le t \le 10$.

   a. (6 points) Calculate μ_{x+5} .

   b. (6 points) Calculate p_{x+6} .

   c. (8 points) Calculate the $Var[T_x]$.

4. (8 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $i = 0.05$
3. $q_{50} = 0.0018$ and $q_{51} = 0.0020$ and $q_{52} = 0.0022$

Determine $1000A_{52}$

5. (8 points) Megan (25) buys a 20 year term policy with a death benefit of 1,000,000 payable at the end of the year of death.

   You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

   Calculate the expected present value which is $1{,}000{,}000 A^{1}_{25:\overline{20|}}$.

6. The probability that a new iPhone must be replaced over the next three years based on existing data is given in the table below. However, software improvements will result in an extended lifetime with a reduced "death" rate. The iPhone's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$	$\varphi(x,3)$
0	0.2	0.25	0.20	0.10
1	0.4	0.20	0.12	0.06
2	0.6	0.10	0.05	0.04

a. (8 points) Calculate probability that an iPhone placed into service today is still functioning at the end of three years if all software updates have been applied.

Let L_3 be the random variable representing the number of iPhones still in service at the end of 3 years assuming that 20,000 new iPhones are sold today and all software updates are applied.

b. (6 points)Using the probability from Part (a), calculate $Var[L_3]$. If you are not able to calculate the probability in Part (a), assume that it is 0.25. If you get an answer for Part (a), please use that answer.

7. (8 points) You are given that ${}_{t|1}q_x = 0.04 \quad \text{for} \;\; t = 0, 1, 2, \ldots, 24$.

Calculate ${}_{12}q_{x+5}$.

8. (8 points) You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

You are also given that $i = 0.06$.

Calculate $1000 \, {}_{10}E_{70}$.

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You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate $_{0.6}p_{101.8}$.

b. (6 points) Calculate μ_{102} .

c. Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

   i. (4 points) Write an expression of Z^{CONT}.

d. Let $Z^{Discrete}$ be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

   i. (8 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

   ii. (8 points) Calculate $Var[Z^{Discrete}]$

2. You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
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78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.08$.

a. (8 points) If $l_{[75]} = 100,000$, calculate $l_{[76]}$.

b. (8 points) Calculate $100,000A_{[77]}$ which is a whole life insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

3. You are given that $S_x(t) = (1 - 0.008t^3)$ for $0 \le t \le 5$.

   a. (6 points) Calculate p_3 .

   b. (8 points) Calculate e_0 .

4. (8 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{52} = 0.32$
2. $i = 0.05$
3. $q_{50} = 0.0018$ and $q_{51} = 0.0020$ and $q_{52} = 0.0022$

Determine $1000A_{50}$

5. (8 points) Madeline (25) buys a 32 year pure endowment contract that will pay her 1,000,000 at the end of the term provided she is alive.

   You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

   Calculate the expected present value which is $1,000,000A_{25:\overline{32|}}^{\;\;\;1}$.

6. The probability that a new iPhone must be replaced over the next three years based on existing data is given in the table below. However, software improvements will result in an extended lifetime with a reduced "death" rate. The iPhone's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$	$\varphi(x,3)$
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1	0.25	0.20	0.12	0.06
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a. (8 points) Calculate probability that an iPhone placed into service today is still functioning at the end of three years if all software updates have been applied.

Let L_3 be the random variable representing the number of iPhones still in service at the end of 3 years assuming that 20,000 new iPhones are sold today and all software updates are applied.

b. (6 points) Using the probability from Part (a), calculate $Var[L_3]$. If you are not able to calculate the probability in Part (a), assume that it is 0.4. If you get an answer for Part (a), please use that answer.

7. (8 points) You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

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Calculate $1000A^{\ 1}_{80:\overline{2}|}$.

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a. (6 points) Calculate ${}_{0.6}p_{101.8}$.

b. (6 points) Calculate μ_{102}.

c. Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

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d. Let $Z^{Discrete}$ be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

   i. (8 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

   ii. (8 points) Calculate $Var[Z^{Discrete}]$

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You are also given that $d = 0.08$.

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Determine $1000A_{50}$

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Calculate $1000A^{\,1}_{80:\overline{2}|}$.