STAT 472
Test 2
Fall 2020
October 27, 2020

1. Megan is (45). She purchases a whole life policy with death benefit of 250,000 payable at the moment of death.

   You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$. You are also given that deaths are uniformly distributed between integral ages.

   a. (4 points) The actuarial present value of Megan's death benefit is 38,800 to the nearest 100. Calculate it to the nearest 1.

   Solution:

   $$APV = 250{,}000\bar{A}_{45} = (250{,}000)\left(\frac{i}{\delta}\right)A_{45} = (250{,}000)\left(\frac{0.05}{\ln(1.05)}\right)(0.15161) = 38{,}842$$

   Megan wants to explore different premium payment patterns. All premiums are calculated using the equivalence principle.

   b. (4 points) Calculate the net annual premium if premiums are paid annually during her lifetime.

   Solution:

   $$PVP = PVB \implies P\ddot{a}_{45} = 38{,}842$$

   $$P = \frac{38{,}842}{17.8162} = 2180.15$$

   c. (7 points) Calculate the net annual premium if premiums are only payable for 15 years.

   Solution:

   $$PVP = PVB \implies P\ddot{a}_{45:\overline{15|}} = 38{,}842$$

   $$P = \frac{38{,}842}{\ddot{a}_{45} - {}_{15}E_{45} \cdot \ddot{a}_{60}} = \frac{38{,}842}{17.8162 - (1.05)^{-15}\left(\frac{96{,}634.1}{99{,}033.9}\right)(14.9041)} = 3589.57$$

d. (7 points) Calculate the net quarterly premium if premiums are payable for life.

Solution:

$$PVP = PVB ==> 4P\ddot{a}_{45}^{(4)} = 38,842$$

$$P = \frac{38,842}{4P\ddot{a}_{45}^{(4)}} = \frac{38,842}{4\{\alpha(4)\ddot{a}_{45} - \beta(4)\}} = \frac{38,842}{4\{(1.00019)(17.8162) - 0.38272\}} = 556.89$$

e. (4 points) List two reasons that the annual premium in part b is less than four times the quarterly premium in part d.

Solution:

One reason is the time value of money. The annual premium is paid at the beginning of the year and therefore can earn interest during the entire year. The quarter premiums are paid at the start of each quarter. The premium paid at the beginning of the first quarter earns interest for the whole year. However the premium paid at the start of the second quarter only earns ¾ of a year of interest. The premium paid at the start of the third quarter only earns interest for ½ year. The last quarterly premium only earns interest for ¼ year. Therefore, the quarterly premiums earn less interest so they must be larger.

In the year of death, the entire annual premium will be collected but 4 quarterly premiums may not be collected as premiums are not collected after the death of the insured. For example, if the insured dies during the first quarter the entire annual premium would be collected but if the premiums were quarterly, only one quarterly premium would be collected.

f. (7 points) Megan decides that she wants to pay annual premiums for 30 years. The premiums will not be level. The premiums for the first 10 years will be P. The premiums during the second 10 years will be $2P$. The premiums during the final 10 years will be $3P$.

Determine P.

Solution:

$$PVP = PVB$$

$$P(\ddot{a}_{45} + {}_{10}E_{45} \cdot \ddot{a}_{55} + {}_{20}E_{45} \cdot \ddot{a}_{65} - 3{}_{30}E_{45} \cdot \ddot{a}_{75}) = 38{,}842$$

$$P = \frac{38{,}842}{17.8162 + (0.60655)(16.0599) + (0.35994)(13.5498) - (3)(0.35994)(0.55305)(10.3178)}$$

$$= 1478.42$$

2. Kaitlyn who is (70) buys a whole life insurance policy with a death benefit of 500,000 which is payable at the end of the year of death.

   You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

   The expenses associated with Kaitlyn's policy are:

   i. Commissions of 55% of premium in year 1 and 8% of premium thereafter;
   ii. Issue Expense of 300 per policy in the first year only;
   iii. Maintenance expenses of 40 per policy in all years including the first;
   iv. Termination expense of 1000 paid at the end of the year of death.

   a. (6 points) The gross premium based on the equivalence principle is 20,350 to the nearest 10. Calculate it to the nearest 0.01.

      Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{70} = 500,000A_{70} + 0.47P + 0.08P\ddot{a}_{70} + 300 + 40\ddot{a}_{70} + 1000A_{70}$$

$$P = \frac{501,000A_{70} + 300 + 40\ddot{a}_{70}}{0.92\ddot{a}_{70} - 0.47} = \frac{(501,000)(0.42818) + 300 + 40(12.0083)}{0.92(12.0083) - 0.47} = 20,354.12$$

b. (7 points) The loss at issue random variable is L_0^g which can be expressed as $Av^{K_{70}+1} + B + C\ddot{a}_{\overline{K_{70}+1|}}$. Determine $A,\ B,$ and C.

Solution:

$$L_0^g = 500{,}000v^{K_{70}+1} + (0.47)(20{,}354.12) + 300 + 1000v^{K_{70}+1} + [40 - 0.92(20{,}354.12)]a_{\overline{K_{70}+1|}}$$

$$= 501{,}000v^{K_{70}+1} + 9866.4364 - 18{,}685.7904a_{\overline{K_{70}+1|}}$$

$$A = 501{,}000$$
$$B = 9866.4364$$
$$C = -18{,}685.7904$$

c. (7 points) Calculate the $\sqrt{Var[L_0^g]}$.

Solution:

$$Var[L_0^g] = Var[501{,}000v^{K_{70}+1} + 9866.4364 - 18{,}685.7904a_{\overline{K_{70}+1|}}]$$

$$= Var\left[501{,}000v^{K_{70}+1} + 9866.4364 - 18{,}685.7904\left(\frac{1 - v^{K_{70}+1}}{d}\right)\right]$$

$$= Var\left[\left(501{,}000 + \frac{18{,}685.7904}{d}\right)v^{K_{70}+1} + 9866.4364 - \frac{18{,}685.7904}{d}\right]$$

$$Var\left[\left(501{,}000 + \frac{18{,}685.7904}{d}\right)v^{K_{70}+1}\right] = \left(501{,}000 + \frac{18{,}685.7904}{d}\right)^2 Var\left[v^{K_{70}+1}\right]$$

$$\left(501{,}000 + \frac{18{,}685.7904}{d}\right)^2 \left({}^2A_{70} - (A_{70})^2\right)$$

$$\sqrt{Var[L_0^g]} = \left(501{,}000 + \frac{18{,}685.7904}{(0.05/1.05)}\right)\sqrt{0.21467 - (0.42818)^2} = 158{,}139.37$$

3. Ian who is (90) buys a special 3 year life annuity due with non-level payments. The payment in the first year is 50,000. The payment in the second year is 40,000. The payment in the third year is 20,000.

You are given that $v = 0.94$. You are given that $q_{89+t} = 0.1t$ for $t = 1,\ 2,\ 3,\ 4,\ and\ 5$.

Let Y be the present value random variable for this annuity.

a. (5 points) The $E[Y] = 97,000$ to the nearest 1000. Calculate the $E[Y]$ to the nearest 1.

Solution:

Case	Y	**Probability**
Die Year 1	50,000	0.10
Die Year 2	50,000+40,000v = 87,600	(0.9)(0.2) = 0.18
Live 2 Years	50,000+40,000v+20,000v^2 = 105,272	(0.9)(0.8) = 0.72

$$E[Y] = (50,000)(0.1) + (87,600)(0.18) + (105,272)(0.72) = 96,563.84$$

b. (7 points) Calculate the $Var[Y]$.

Solution:

$$Var[Y] = E[Y^2] - (E[Y])^2$$

$$E[Y] = 96,563.84$$

$$E[Y] = (50,000)^2(0.1) + (87,600)^2(0.18) + (105,272)^2(0.72) = 9,610,456,468$$

$$Var[Y] = 9,610,456,468 - (96,563.84)^2 = 285,850,372$$

c. (4 points) If the payments were reversed (year 1 of 20,000, year 2 of 40,000 and year 3 of 50,000), explain why the $E[Y]$ is lower and the $Var[Y]$ is higher.

Solution:

The payment of 50,000 is now only made for those who are alive at the end of 2 years so it gets discounted for both two years of interest and for the probability that Ian is alive at that time. The second payment has the exact same present value. The payment of 20,000 now being paid at time 0 has a higher present value but since it is a smaller amount (substantially less than 50,000), it will not increase in value as much the 50,000 payment will decrease in value.

The variance increases because the largest payment is no longer certain and this adds variability to the present value whereas before the largest payment was certain to be made so it added no variance.

4. (7 points) Let Y be the present value random variable for a whole life annuity due with annual payments of 100 issued to (84).

You are given:

i. $\ddot{a}_{85} = 8$

ii. ${}^2A_{84} = 0.4$

iii. $i = 0.05$

iv. $q_{84} = 0.040$

v. $q_{85} = 0.045$

Calculate the $Var[Y]$.

Solution:

$$Var[Y] = (100)^2 \left(\frac{{}^2A_{84} - \{A_{84}\}^2}{d^2} \right)$$

$$\ddot{a}_{84} = 1 + vp_{84}\ddot{a}_{85} = 1 + (1.05)^{-1}(1-0.04)(8) = 8.314285714$$

$$A_{84} = 1 - d\ddot{a}_{84} = 1 - \left(\frac{0.05}{1.05} \right)(8.314285714) = 0.604081633$$

$$Var[Y] = (100)^2 \left(\frac{0.4 - \{0.60401633\}^2}{(0.05/1.05)^2} \right) = 154,727$$

5. Jeff retires from Purdue at age 65. He is entitled to a pension payout of 100,000 annually at the beginning of the each year. The standard payout is based on a 10 year certain and life thereafter payout.

   You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

   a. (7 points) Calculate the Actuarial Present Value of Jeff's pension payments.

      Solution:

      $$APV = 100,000\ddot{a}_{\overline{65:\overline{10|}}} = (100,000)(\ddot{a}_{\overline{10|}} + {}_{10}E_{65} \cdot \ddot{a}_{75})$$

      $$= (100,000)\left(\frac{1-(1.05)^{-10}}{(0.05/1.05)} + (0.55305)(10.3178)\right) = 1,381,408.10$$

Jeff has the option to receive a 20 year deferred whole life annuity due with no certain period instead of the 10 year certain and life payout in part a. This annuity would begin making level payments to Jeff at age 85. If Jeff died prior to age 85, no benefits would be paid. The payment under the deferred annuity option will be actuarially equivalent (which means it will have the same actuarial present value).

b. (7 points) Calculate the payment if Jeff chooses this option.

Solution:

$$1{,}381{,}408.10 = P \cdot {}_{20|}\ddot{a}_{65} = P \cdot {}_{20}E_{65} \cdot \ddot{a}_{85}$$

$$P = \frac{1{,}381{,}408.10}{(0.24381)(6.7993)} = 833{,}309.40$$

c. (4 points) List two advantages and one disadvantage to the option in part b.

Solution

Advantages

- Jeff receives a much larger payment if he survives 20 years.
- A deferred annuity such as this provides longevity insurance as it assures Jeff that he will not outlive his savings

Disadvantage

- If Jeff dies in the next 20 years, he will not receive any payments.

6. (6 points) A special whole life insurance policy to (70) provides a death benefit payable at the moment of death. The death benefit for the first 10 years is 100,000. The death benefit for death between ages 80 and 90 is 75,000. The death benefit after age 90 is 25,000.

   You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$. You are also given that deaths are uniformly distributed between integral ages.

   Calculate the Expected Present Value of this special whole life.

   Solution:

$$APV = 100,000\bar{A}_{70} - 25,000\,{}_{10}E_{70} \cdot \bar{A}_{80} - 50,000\,{}_{20}E_{70} \cdot \bar{A}_{90}$$

$$= 100,000\left(\frac{i}{\delta}\right)A_{70} - 25,000\,{}_{10}E_{70}\left(\frac{i}{\delta}\right)A_{80} - 50,000\,{}_{20}E_{70}\left(\frac{i}{\delta}\right)A_{90}$$

$$= (100,000)(1.0248)(0.42818)$$
$$-(25,000)(0.50994)(1.0248)(0.59293) - (50,000)(0.17313)(1.0248)(0.75317)$$

$$= 29,452$$