STAT 472
Test 3
Fall 2020
November 24, 2020

1. A whole life insurance policy to (60) pays a death benefit of 100,000 at the end of the year of death. The gross annual premium is 2400 payable for the life of the insured. It was not calculated using the equivalence principle.

   You are given the following reserve basis:

   i. Mortality follows the Standard Ultimate Life Table

   ii. $i = 0.05$

   iii. Expenses as follows:

      1. Issue Expense at time 0 of 925.
      2. Maintenance expense of 37 at the start of every year including the first year.
      3. Termination expense of 1500 paid at the end of the year of death.
      4. Commissions of 40% in the first year and 6.5% thereafter

   a. (9 points) Calculate the gross premium reserve at time 10.

      Solution:

      $${}_{10}V = PVFB + PVFE - PVFP$$

      $$= 100,000A_{70} + 37\ddot{a}_{70} + 1500A_{70} + 0.065(2400)\ddot{a}_{70} - (2400)\ddot{a}_{70}$$

      $$= (101,500)(0.42818) - \left[(0.935)(2400) - 37\right](12.0083) = 16,957.95$$

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1. **Continued**

b. (6 points) Let the first year Full Preliminary Term premium be P_1^{FPT} and the premiums for years after the first be P_{x+1}^{FPT}. Calculate $P_{x+1}^{FPT} - P_1^{FPT}$.

Solution:

$$P_1^{FPT} = Svq_{60} = (100,000)(1.05)^{-1}(0.003398) = 323.62$$

$$P_{x+1}^{FPT} = \frac{100,000A_{61}}{\ddot{a}_{61}} = \frac{(100,000)(0.30243)}{14.6491} = 2064.50$$

$$P_{x+1}^{FPT} - P_1^{FPT} = 2064.50 - 323.62 = 1740.88$$

c. (6 points) Calculate the Full Preliminary Term reserve at time 10.

Solution:

$${}_{10}V^{FPT} = PVFB - PVFP = 100,000A_{70} - 2064.50\ddot{a}_{70}$$

$$= (100,000)(0.42818) - (2064.50)(12.0083) = 18,026.86$$

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2. A 20 year term insurance policy issued to (55) pays a death benefit of 500,000 at the end of the year of death. You are given the following information about the reserve basis:

   i. The annual net premium is 8500.
   ii. Mortality follows the Standard Ultimate Life Table
   iii. $i = 0.07$
   iv. Expenses as follows:
      1. Issue expenses of 1000 at time 0
      2. Maintenance expense of 21 at the beginning of every year including the first year.
      3. Commissions of 38% of premiums in the first year and 4% thereafter.
      4. A termination expense of 152 paid at the end of the year of death.

The net premium reserve at the end of the 9th year is 36,500.

The expense reserve at the end of the 9th year is – 2600. Note that this is a negative reserve.

The expense reserve at the end of the 10 year is – 2423.24. Note that this is a negative reserve.

a. (8 points) Calculate the net premium reserve at the end of the 10th year.

Solution:

$$({}_9V^n + P)(1+i) = (S_{10})q_{x+9} + {}_{10}V^n \cdot p_{x+9}$$

$$(36,500+8500)(1.07) = (500,000)(0.005288) + {}_{10}V^n(1-0.005288)$$

$${}_{10}V^n = \frac{(36,500+8500)(1.07)-(500,000)(0.005288)}{1-0.005288} = 45,77.91$$

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b. (10 points) Calculate the gross premium.

Solution:

$$ {}_{10}V^{e} = {}_{10}V^{g} - {}_{10}V^{n} ==> -2423.24 = {}_{10}V^{g} - 45,747.91 ==> {}_{10}V^{g} = 43,324.67 $$

$$ {}_{9}V^{e} = {}_{9}V^{g} - {}_{9}V^{n} ==> -2600.00 = {}_{9}V^{g} - 36,500 ==> {}_{9}V^{g} = 33,900 $$

$$ ({}_{9}V^{g} + P - e_9 - X_9^{BOY})(1+i) = (S_{10} + E_{10})q_{x+9} + {}_{10}V^{g}(1-q_{x+9}) $$

$$ 33,900 + 0.96P - 21)(1.07) = (500,000 + 152)(0.005288) + (43,324.67)(1-0.005288) $$

$$ P = \frac{\left[(500,000+152)(0.005288) + (43,324.67)(1-0.005288)\right](1.07)^{-1} - 33,879}{0.96} = 9238.55 $$

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3. In pre-historic times, the actuarial profession was a very risky profession. The Actuarial Preservation Club determined that the survivorship of actuaries could be modeled using a multi-state model. The model has three states:
   i. State 0 is Healthy
   ii. State 1 is an Sick
   iii. State 2 is Dead

   State 0 can transition to State 1 or State 2. State 1 can transition to State 2. State 2 cannot transition.

   You are given the following force of transitions:

   i. $\mu_{x+t}^{01} = 0.10$
   ii. $\mu_{x+t}^{02} = 0.15$
   iii. $\mu_{x+t}^{12} = 0.20$

   a. (10 points) The value of ${}_7 p_x^{02}$ is 0.7 to the nearest 0.1. Calculate it to five decimal places.

   Solution:

$${}_t p_x^{00} = e^{-\int_0^t (\mu_{x+s}^{01} + \mu_{x+s}^{02}) ds} = e^{-\int_0^t (0.10+0.150) dt} = e^{-0.25(t)}$$

$${}_t p_x^{01} = \int_0^t {}_s p_x^{00} \cdot \mu_{x+s}^{01} \cdot {}_{t-s} p_{x+s}^{11} \cdot ds$$

$$= \int_0^t e^{-0.25(s)} \cdot (0.10) \cdot e^{-0.20(t-s)} \cdot ds = 0.10 e^{-0.20(t)} \int_0^t e^{-0.25(s)} e^{0.20(s)} \cdot ds =$$

$$0.10 e^{-0.20(t)} \left[\frac{1 - e^{-0.05(s)}}{0.05} \right]_0^t = 20(e^{-0.20(t)} - e^{-0.25(t)})$$

$${}_7 p_x^{02} = 1 - {}_t p_x^{00} - {}_t p_x^{01} = 1 - e^{-0.25(7)} - 20(e^{-0.20(7)} - e^{-0.25(7)}) = 0.68058$$

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3. Continued

The Actuarial Preservation Club wanted to encourage people to be actuaries. They agreed to provide the following benefits to anyone who becomes an actuary:

1. Benefit 1 is a **whole life** annuity due to each actuary. The annuity will make continuous payments at an annual rate of 10,000 per year that the actuary is healthy. (This was a lot of money in pre-historic times.)
2. Benefit 2 is a lump sum payment of 5000 if an actuary becomes sick during the next seven years.
3. Benefit 3 is a lump sum endowment benefit of 20,000 paid at the end of seven years for any actuary that is not dead at the end of seven years.

All calculations should be done using $\delta = 0.04$.

b. (7 points) Calculate the actuarial present value of Benefit 1.

Solution:

$$APV = \int_0^\infty 10,000 \cdot v^t \cdot {}_t p_{x+t}^{00} \cdot dt = \int_0^\infty 10,000 \cdot e^{-0.04t} \cdot e^{-0.25t} \cdot dt$$

$$= 10,000 \int_0^\infty e^{-0.29t} dt = 10,000 \left[\frac{1-0}{0.29} \right] = 34,482.76$$

(7 points) Calculate the actuarial present value of Benefit 2.

Solution:

$$APV = \int_0^7 5000 \cdot v^t \cdot {}_t p_{x+t}^{00} \cdot \mu_{x+t}^{01} \cdot dt = \int_0^\infty 5000 \cdot e^{-0.04t} \cdot e^{-0.25t} (0.1) \cdot dt$$

$$= 500 \int_0^\infty e^{-0.29t} dt = 500 \left[\frac{1-0}{0.29} \right] = 1497.70$$

c. (5 points) Calculate the actuarial present value of Benefit 3.

Solution:

$$APV = 20,000 \cdot v^7 \cdot (1 - {}_7 p_x^{02}) = (20,000)(e^{-0.28})(1 - 0.68058) = 4828.25$$

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4. Lives insured for nursing home coverage can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is In A Nursing Home
   iii. State 2 is Dead

   State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

   You are given the following matrix of annual transition probabilities.

$$\begin{bmatrix} 0.60 & 0.30 & 0.10 \\ 0.15 & 0.40 & 0.45 \\ 0 & 0 & 1 \end{bmatrix}$$

   An insurance company decides to issue a special two year insurance policy which pays a benefit of 10,000 at the end of the year of death. The policy will also pay 120,000 at the end of each year that the insured is in a nursing home. Finally, the policy will pay 50,000 to everyone who is healthy at the end of 2 years.

   Annual premiums will be paid only by healthy lives during the two years. The premium is determined using the equivalence principle.

   You are given that $v = 0.92$.

   a. (10 points) The annual premium is 53,700 to the nearest 100. Calculate the annual premium to the nearest 1.

   Solution:

	Time 1	Time 2	Time 3
State 0	1	0.6	(0.6)(0.6)+(0.3)(0.4)=0.405
State 1	0	0.3	(0.6)(0.3)+(0.3)(0.4)=0.300
State 2	0	0.1	0.1+(0.6)(0.1)+(0.3)(0.45)=0.295

$$PVP = PVB$$

$$P(1+0.6v) = (10,000)(0.1v + (0.245 - 0.1)v^2) + (120,000)(0.3v + 0.3v^2) + (50,000)(0.405v^2)$$

$$P = 53,673$$

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4. Continued

b. (8 points) Calculate the reserve for those in State (0) at the end of one year.

Solution:

$${}_1V^{(0)} = PVFB - PVFP$$

$$= 10,000(0.1v) + 120,000(0.3v) + 50,000(0.6v) - 53,673(1) = 7967$$

c. (8 points) Calculate the reserve for those in State (1) at the end of one year.

Solution:

$${}_1V^{(1)} = PVFB - PVFP$$

$$= 10,000(0.45v) + 120,000(0.4v) + 50,000(0.15v) - 53,673(0) = 55,200$$

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5. (10 points) Lives insured for disability income policies can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is Disabled
   iii. State 2 is Dead

State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

You are given:

   i. $\mu_x^{01} = 0.10t + 0.02$

   ii. $\mu_x^{02} = 0.12$

   iii. $\mu_x^{12} = 0.2t + 0.02$

   iv. $\mu_x^{10} = 0.15$

An insurance company sells 10,000 policies to independent healthy lives. Using the Euler method with h=1/2, determine how many lives will be healthy at the end of 1 year.

Solution:

$${}_{0.5}p_x^{00} = {}_0p_x^{00} - {}_0p_x^{00}(h)(\mu_x^{01} + \mu_x^{02}) + {}_0p_x^{01}(h)(\mu_x^{10}) = 1 - (1)(0.5)[(0.1)(0) + 0.02 + 0.12] + 0 = 0.93$$

$${}_{0.5}p_x^{01} = {}_0p_x^{01} - {}_0p_x^{01}(h)(\mu_x^{10} + \mu_x^{12}) + {}_0p_x^{00}(h)(\mu_x^{01})$$

$$= 0 - (0)(0.5)[0.15 + (0.2)(0) + 0.02] + (1)(0.5)[(0.1)(0) + 0.02] = 0.01$$

$${}_{0.5}p_x^{02} = {}_0p_x^{02} + {}_0p_x^{00}(h)(\mu_x^{02}) + {}_0p_x^{01}h)(\mu_x^{12}) = 0 + (1)(0.5)(0.12) + 0 = 0.06$$

$${}_1p_x^{00} = {}_{0.5}p_x^{00} - {}_{0.5}p_x^{00}(h)(\mu_{x+0.5}^{01} + \mu_{x+0.5}^{02}) + {}_{0.5}p_x^{01}(h)(\mu_{x+0.5}^{10})$$

$$= 0.93 - (0.93)(0.5)[(0.1)(0.5) + 0.02 + 0.12] + 0.1(0.5)(0.15) = 0.8424$$

$$\text{Number of lives } = (10,000)(0.8424) = 8424$$

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6. A disability income policy follows the Standard Sickness-Death model with $i = 0.05$.

The policy pays premiums continuously at a rate of P when the insured is in State 0.

The policy pays a lump sum death benefit at the moment of death of 60,000. The policy also pays a disability annuity benefit at a rate of 40,000 per year paid continuously for life while the insured is in state 1.

An insured purchases the policy at age 55 and is Healthy.

a. (7 points) Determine the premium using the equivalence principle.

Solution:

$$PVP = PVB ==> P\bar{a}_{55}^{00} = 60,000\bar{A}_{55}^{02} + 40,000\bar{a}_{55}^{01}$$

$$P(10.1228) = (60,000)(0.39366) + (40,000)(2.3057)$$

$$P = 11,444.22$$

b. (7 points) Determine ${}_{10}V^{(1)}$.

Solution:

$${}_{10}V^{(1)} = PVFB - PVFP = 60,000\bar{A}_{65}^{12} + 40,000\bar{a}_{65}^{11} - 11,444.22\bar{a}_{65}^{10}$$

$$(60,000)(0.5681) + (40,000)(8.8123) - (11,444.22)(0.0395) = 386,125.95$$

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7. (8 points) Using the Standard Sickness-Death model with $i = 0.05$, calculate $\bar{a}^{00}_{50:\overline{10}|}$.

Solution:

$$\bar{a}^{00}_{50:\overline{10}|} = \bar{a}^{00}_{50} - v^{10} \cdot {}_{10}p^{00}_{50} \cdot \bar{a}^{00}_{60} - v^{10} \cdot {}_{10}p^{01}_{50} \cdot \bar{a}^{10}_{60}$$

$$= 11.7454 - (1.05)^{-10}[(0.83936)(8.3908) + (0.06554)(0.1082)]$$

$$= 7.41762$$

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8. A whole life insurance policy to (70) pays a death benefit of 105,000 at the end of the year of death. The policy has a gross annual premium of 4470.76 determined using the equivalence principle.

You are given the following reserve basis:

i. Mortality follows the Standard Ultimate Life Table
ii. $i = 0.05$
iii. Expenses as follows:
    1. Issue Expense at time 0 of 925.
    2. Maintenance expense of 37 at the start of every year including the first year.
    3. Termination expense of 1500 paid at the end of the year of death.
    4. Commissions of 50% in the first year and 10% thereafter

Actual experience during the 12th year is:

i. Mortality is 105% the Standard Ultimate Life Table.
ii. $i = 0.055$
iii. Expenses as follows:
    1. Maintenance expense of 45 at the start of the year.
    2. Termination expense of 1200 paid at the end of the year of death.
    3. Commissions of 9%.

The reserve at the end of the 11th year is 32,283.50.

a. (12 points) Calculate the total profit in the 12th year.

Solution:

$$ {}_{12}V = \frac{({}_{11}V + P - e - X)(1+i) - (S+E)q_{81}}{1-q_{81}} $$

$$ = \frac{(32,283.50 + 4470.76(0.9) - 37) - (105,000 + 1500)(0.036607)}{1-0.036607} = 35,484.01 $$

$$ \text{Profit} = ({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81}) $$

$$ = (32,283.50 + 4470.76(0.91) - 45)(1.055) - (105,000 + 1200)(1.05)(0.036607) $$
$$ - (35,484.01)[1-(1.05)(0.036607)] = 101.63 $$

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8. Continued

b. (12 points) The company allocates profits to interest, expenses and mortality in that order. Calculate the profit allocated to expenses.

Solution:

Gain Interest = $({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81})$

$$= (32,283.50 + 4470.76(0.90) - 37)(1.055) - (105,000 + 1500)(0.036607) - (35,484.01)[1-(0.036607)] = 181.35$$

Gain Interest and Expense = $({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81})$

$$= (32,283.50 + 4470.76(0.91) - 45)(1.055) - (105,000 + 1200)(0.036607) - (35,484.01)[1-(0.036607)] = 231.06$$

Gain from Expense = Gain from Interest and Expense - Gain from Interest

$= 231.06 - 181.35 = 49.71$

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