STAT 472
Chapter 2 and 3 Cheat Sheet

$$S_0(x) = 1 - F_0(x) = {}_x p_0 = 1 - {}_x q_0 = e^{-\int_0^x \mu_s ds} = \frac{l_x}{l_0}$$

$$S_x(t) = 1 - F_x(t) = \frac{S_0(x+t)}{S_0(x)} = {}_t p_x = 1 - {}_t q_x = \frac{{}_{x+t} p_0}{{}_x p_0} = e^{-\int_x^{x+t} \mu_r dr} = e^{-\int_0^t \mu_{x+s} ds} = \frac{l_{x+t}}{l_x}$$

$$p_x = 1 - q_x = \frac{l_{x+1}}{l_x}$$

$$S_x(t+u) = S_x(t) \cdot S_{x+t}(u) = {}_{t+u} p_x = {}_t p_x \cdot {}_u p_{x+t} = \frac{l_{x+t+u}}{l_x}$$

$${}_t q_x = F_x(t) = 1 - S_x(t) = 1 - {}_t p_x = \frac{l_x - l_{x+t}}{l_x}$$

$$q_x = 1 - p_x = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x}$$

$${}_t p_x + {}_t q_x = 1$$

$$l_{x+1} = l_x \cdot p_x$$

$$d_x = l_x - l_{x+1} = l_x \cdot q_x$$

$${}_{u|t} q_x = {}_u p_x - {}_{u+t} p_x = {}_u p_x \cdot {}_t q_{x+u} = \frac{l_{x+u} - l_{x+u+t}}{l_x}$$

μ_x =force of mortality= $\dfrac{-\frac{d}{dx} S_0(x)}{S_0(x)} = -\frac{d}{dx} \ln[S_0(x)] = \dfrac{f_0(x)}{S_0(x)} = \dfrac{-\frac{d}{dx}\, {}_x p_0}{{}_x p_0}$

$$\mu_{x+t} = \frac{-\frac{d}{dt} S_x(t)}{S_x(t)} = -\frac{d}{dt} \ln[S_x(t)] = \frac{f_x(t)}{S_x(t)} = \frac{-\frac{d}{dt}\, {}_t p_x}{{}_t p_x}$$

$$f_x(t) = \frac{d}{dt} F_x(t) = -\frac{d}{dt} S_x(t) = S_x(t) \cdot \mu_{x+t} = {}_t p_x \cdot \mu_{x+t}$$

$E[T_x] = \overset{o}{e}_x =$ *complete expectation of life* $= \int_o^\infty t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = \int_0^\infty {}_t p_x \cdot dt$

$$E[T_x^2] = \int_o^\infty t^2 \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = 2\int_0^\infty t \cdot {}_t p_x \cdot dt$$

$$Var[T_x] = E[T_x^2] - (E[T_x])^2$$

$E[\min(T_x, n)] = \overset{o}{e}_{x:\overline{n}|} = n -$ *year partial or term complete expectation of life* $= \int_o^n t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = \int_0^n {}_t p_x \cdot dt$

$$E[\{\min(T_x, n)\}^2] = \int_o^n t^2 \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = 2\int_0^n t \cdot {}_t p_x \cdot dt$$

$$Prob[K_x = k] = {}_{k|} q_x = {}_k p_x - {}_{k+1} p_x = {}_k p_x \cdot q_{x+k}$$

$E[K_x] = e_x =$ *curtate expectation of life* $= \sum_{k=0}^\infty k({}_k p_x - {}_{k+1} p_x) = \sum_{k=1}^\infty {}_k p_x$

$$E[K_x{}^2] = \sum_{k=0}^\infty k^2({}_k p_x - {}_{k+1} p_x) = \{2\sum_{k=1}^\infty k \cdot {}_k p_x\} - e_x$$

$$Var[K_x] = E[K_x{}^2] - (E[K_x])^2$$

$E[\min(K_x, n)] = e_{x:\overline{n}|} = n -$ *year term curtate expectation of life* $= \sum_{k=1}^n {}_k p_x$

$$E[\{\min(K_x, n)\}^2] = \{2\sum_{k=1}^n k \cdot {}_k p_x\} - e_{x:\overline{n}|}$$

$\overset{o}{e}_x \approx e_x + 1/2$ <= This is exact under UDD and approximate otherwise.

$$e_x = {}_1 p_x (1 + e_{x+1})$$

$$e_x = e_{x:\overline{n-1}|} + {}_n p_x (1 + e_{x+n}) = e_{x:\overline{n}|} + {}_n p_x \cdot e_{x+n}$$

$$\overset{o}{e}_x = \overset{o}{e}_{x:\overline{n}|} + {}_n p_x \cdot \overset{o}{e}_{x+n}$$

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	Gompertz	**Makeham**
μ_x	Bc^x	$A+Bc^x$
${}_t p_x$	$\exp\left\{\frac{-B}{\ln(c)}(c^x)(c^t-1)\right\}$	$\exp\left\{-A\cdot t-\frac{B}{\ln(c)}(c^x)(c^t-1)\right\}$

UDD for $0\le s\le 1$	**CFM for** $0\le s\le 1$
$l_{x+s}=(1-s)l_x+(s)l_{x+1}$	$l_{x+s}=l_x\cdot(p_x)^s=(l_x)^{1-s}\cdot(l_{x+1})^s$
${}_s q_x=s\cdot q_x$	${}_s p_x=(p_x)^s$
$l_{x+s}=l_x-s\cdot d_x$	${}_s p_{x+t}=(p_x)^s$ *if* $s>0$ *and* $t+s\le 1$
UDD for $0\le s<1$	**CFM for** $0\le s<1$
$\mu_{x+s}=\frac{q_x}{1-s\cdot q_x}$	$\mu_{x+s}=-\ln[p_x]$