Chapter 3

1. Complete the following table:

x	q_x	l_x	d_x
50		20,000	800
51			
52		18,000	
53	0.100		
54	0.125	14,985	

2. You are given that mortality follows the Standard Ultimate Life Table. Calculate:
   a. p_{80}
   b. ${}_{40}p_{20}$
   c. ${}_{40}q_{20}$
   d. $\Pr(60 < T_{20} < 80)$
   e. ${}_{10}p_{75}$
   f. ${}_{10}q_{75}$
   g. ${}_{10|5}q_{75}$

3. You are given that the mortality for all employees of Purdue University follows the Standard Ultimate Life Table. There are 1000 employees who are age 58 at Purdue University.

   Let L_{22} be the random variable representing the number who will still be alive at age 80.

   a. Calculate the expected value of L_{22}.
   b. Calculate the $Var[L_{22}]$.

4. Assume that mortality follows the Standard Ultimate Life Table for integral ages. Assume that deaths are uniformly distributed (UDD) between integral ages. Calculate:
   a. ${}_{0.5}q_{80}$
   b. ${}_{0.5}p_{80}$
   c. $\mu_{80.5}$
   d. ${}_{1.5}p_{80}$
   e. ${}_{1.5}q_{80}$
   f. ${}_{0.5}q_{80.5}$
   g. ${}_{0.5}q_{80.25}$
   h. ${}_{3.2\,|\,2.4}q_{80.5}$

5. *For a certain mortality table, you are given:

   i. $\mu_{80.5} = 0.0202$
   ii. $\mu_{81.5} = 0.0408$
   iii. $\mu_{82.5} = 0.0619$
   iv. Deaths are uniformly distributed between integral ages.

   Calculate ${}_2q_{80.5}$

6. Assume that mortality follows the Standard Ultimate Life Table for integral ages. Assume that probability of survival is geometrically distributed (Constant Force) between integral ages. Calculate:

   a. ${}_{0.5}q_{80}$
   b. ${}_{0.5}p_{80}$
   c. $\mu_{80.5}$
   d. ${}_{1.5}p_{80}$
   e. ${}_{1.5}q_{80}$
   f. ${}_{0.5}q_{80.5}$
   g. ${}_{0.5}q_{80.25}$
   h. ${}_{3.2\,|\,2.4}q_{80.5}$

7. You are given $q_{80} = 0.06$ and $q_{81} = 0.09$. Calculate:

   a. ${}_{0.5}q_{80}$ given UDD
   b. ${}_{0.5}q_{80}$ given CFM
   c. ${}_{0.5}q_{80.75}$ given UDD
   d. ${}_{0.5}q_{80.75}$ given CFM

8. You are given that ${}_{t|}q_x = 0.05$ for $t = 0,1,2,\ldots,19.$

   Calculate ${}_4q_{x+8}$.

9. (SWAQ) David and Adam both work for Lauren who is the Chief Actuary at Baugh Life Insurance Company. Lauren calls both David and Adam into her office and asks them to each calculate $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$.

   David assumes uniform distribution of deaths between integral ages and calculates $\mu_{80.3} = 0.1277126$ and ${}_{0.3}q_{80.4}$.

   Adam assumes a constant force of mortality between integral ages and calculates $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$.

   i. (2 points) Calculate the value of ${}_{0.3}q_{80.4}$ determined by David.

   ii. (2 points) Calculate the value of $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$ determined by Adam.

   iii. (2 points) After David and Adam provide their answers to Lauren, she calls them into her office to explain the difference in the answers to explain which number she should use and why. What should David and Adam tell Lauren?

10. You are given the following select and ultimate mortality table of q_x 's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

Calculate:

a. $p_{[54]}$

b. $p_{[53]+1}$

c. $p_{[52]+2}$

d. $p_{[51]+3}$

e. p_{54}

f. ${}_5p_{[54]}$

g. ${}_{2|2}q_{[52]}$

h. A life policy insurance policy was issued two years ago to (52). Calculate the probability that this person will live to age 59.

i. Clair is 54 and just purchased a life insurance policy. Raf is 54 and purchased a life insurance policy at age 50. How much larger is the probability that Raf will die during the next 4 years than the probability that Clair will die.

11. *You are given:

i. $\mu_x = F + e^{2x}, x \geq 0$

ii. ${}_{0.4}p_0 = 0.5$

Calculate F .

12. You are given the following select mortality table.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
90	0.04	0.10	0.17	0.20	93
91	0.06	0.14	0.18	0.30	94
92	0.08	0.16	0.27	0.40	95
93	0.14	0.24	0.36	0.50	96
94	0.21	0.32	0.45	0.70	97
95	0.28	0.40	0.63	0.90	98
96	0.35	0.56	0.81	1.00	99

Calculate $e_{[94]}$ and e_{94} .

You are given the following select and ultimate mortality table of q_x 's to be used for Numbers 13-15.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

13. If deaths are uniformly distributed between integral ages, calculate ${}_{1.5}q_{[53]+2}$.

14. If $l_{[51]} = 100,000$, calculate $l_{[50]}$.

15. Trout Life Insurance Company has two cohorts of policyholders.

    Cohort A has 1000 insured lives who are all age 53 and were just underwritten today.

    Cohort B has 1000 insured lives who are all age 53 and were underwritten 3 years ago.

    Calculate the total number of insured lives that will still be alive after 2 years.

16. For a two year select and ultimate table, you are given:

    i. $q_{[x]} = 0.50q_x$

    ii. $q_{[x]+1} = 0.75q_{x+1}$

    Complete the following table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1473.1458		1200
107			800
108			400
109			100

17. *For a 2-year select and ultimate mortality model, you are given:

    i. $q_{[x]+1} = 0.80 q_{x+1}$

    ii. $l_{51} = 100,000$

    iii. $l_{52} = 99,000$

    Calculate $l_{[50]+1}$.

18. *You are given:

    i. $p_x = 0.95$

    ii. $p_{x+1} = 0.92$

    iii. $e_{x+1.6} = 12$

    iv. Deaths are uniformly distributed between ages x and $x+1$.

    v. The force of mortality is constant between ages $x+1$ and $x+2$.

    Calculate $e_{x+0.6}$.

19. You are given the following two year select and ultimate mortality table:

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x
70	0.01	0.03	0.06
71	0.02	0.05	0.09
72	0.04	0.07	0.12
73	0.06	0.12	0.15
74	0.10	0.18	0.20
75	0.15	0.22	0.25

    Calculate

    i. ${}_3p_{[70]}$

    ii. ${}_2q_{[71]+1}$

    iii. ${}_{1|2}q_{[70]}$

Answers

1.

x	q_x	l_x	d_x
50	0.040	20,000	800
51	0.0625	19,200	1,200
52	0.075	18,000	1,350
53	0.100	16,650	1,665
54	0.125	14,985	1,873.125

2.

a. 0.96734
b. 0.96634
c. 0.03366
d. 0.69409
e. 0.71810
f. 0.28190
g. 0.22703

3.

a. 778.40
b. 172.49

4.

a. 0.01633
b. 0.98367
c. 0.03320
d. 0.94964
e. 0.05036
f. 0.01660
g. 0.01646
h. 0.10957

5. 0.0782

6.

a. 0.01647
b. 0.98353
c. 0.03320
d. 0.94947
e. 0.05053
f. 0.01647
g. 0.01647
h. 0.10953

7.
    a. 0.03
    b. 0.030464
    c. 0.03785
    d. 0.03829
8. $1/3$
9. Answer not given
10.
    a. 0.960
    b. 0.951
    c. 0.943
    d. 0.935
    e. 0.935
    f. 0.63944
    g. 0.11594
    h. 0.60680
    i. 0.04393
11. 0.20094
12. 1.7202 and 1.3993
13. 0.10754
14. 103,973.04
15. 1800.355
16.

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1473.1458	1411.765	1200
107	1185.185	1066.667	800
108	768	640	400
109	304.762	228.571	100

17. 99,798.4
18. 12.11066
19.
    i. 0.82764
    ii. 0.20950
    iii. 0.16236