1.

$$\bar{A}_{40} = \int_0^\infty v^t \bullet {}_t p_{40} \bullet \mu_{40+t} =$$

$$\int_0^\infty (1.06)^{-t} \exp\left(-0.0003t - \frac{0.000004}{\ln(1.1)}(1.1^{40})(1.1^t - 1)\right)\left(0.0003 + (0.000004)(1.1^{40+t})\right)$$

2.

$$\bar{A}_0 = \int_0^\infty v^t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = \int_0^\infty v^t \cdot \exp\{-\frac{B}{\ln c} \cdot c^0 \cdot (c^t - 1)\} \cdot Bc^{0+t} \cdot dt$$

$$X \cdot \bar{A}_0 = X \int_0^\infty v^t \cdot \exp\{-\frac{B}{\ln c} \cdot (c^t - 1)\} \cdot Bc^t \cdot dt = \int_0^\infty (1.08)^{-t} \bullet \exp\{-0.002049593(1.05^t - 1)\} \bullet 1.05^t \, dt$$

By inspection, c=1.05 then $\frac{B}{\ln(1.05)} = 0.002049593 \Rightarrow B = 0.0001$

Since i=0.08, $v^t = (1.08)^{-t}$, *Therefore*

$$X \int_0^\infty (1.08)^{-t} \cdot \exp\{-\frac{0.0001}{\ln(1.05)} \cdot (1.05^t - 1)\} \cdot B \cdot 1.05^t \cdot dt = \int_0^\infty (1.08)^{-t} \bullet \exp\{-0.002049593(1.05^t - 1)\} \bullet 1.05^t \, dt$$

$$BX \int_0^\infty (1.08)^{-t} \cdot \exp\{-\frac{0.0001}{\ln(1.05)} \cdot (1.05^t - 1)\} \cdot 1.05^t \cdot dt = \int_0^\infty (1.08)^{-t} \bullet \exp\{-0.002049593(1.05^t - 1)\} \bullet 1.05^t \, dt$$

$$\therefore BX = 1 \Longrightarrow X = \frac{1}{B} = \frac{1}{0.0001} = 10{,}000$$

3.

$$1000\bar{A}_x = 1000\int_0^{25} v^t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt$$

$$\mu_{x+t} = -\frac{\frac{d}{dt}({}_t p_x)}{{}_t p_x} = \frac{0.02 + 0.0016t}{{}_t p_x} \Longrightarrow {}_t p_x \cdot \mu_{x+t} = 0.02 + 0.0016t$$

$$1000\int_0^{25} v^t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt = 1000\int_0^{25} v^t \cdot (0.02 + 0.0016t) \cdot dt = 1000\int_0^{25} v^t \cdot (0.02)dt + 1000\int_0^{25} v^t \cdot (0.0016t) \cdot dt =$$

$$1000(0.02)\int_0^{25} v^t \cdot dt + 1000(0.0016)\int_0^{25} v^t \cdot t \cdot dt = 20\bar{a}_{\overline{25|}} + 1.6(\bar{I}\bar{a})_{\overline{25|}} =$$

$$20\frac{1 - e^{-(0.04)(25)}}{\delta} + 1.6\frac{\bar{a}_{\overline{25|}} - 25e^{-(0.04)(25)}}{\delta} = 580.30$$

If you do not remember these formulas from interest theory, you can just do the integration. The second one requires integration by parts.

4.

a.

$$900(A_{91}) = (900-720)v + (720-432)\,v^2 + (432-216)v^3 + (216-0)v^4 = 816.01003$$

$$A_{91} = \frac{816.01003}{900} = 0.90668$$

b.

$$900(A^{1}_{91:\overline{3}|}) = (900-720)v + (720-432)\,v^2 + (432-216)v^3 = 631.37233$$

$$A^{1}_{91:\overline{3}|} = \frac{631.37233}{900} = 0.70152$$

c.

$${}_3E_{91} = v^3\,{}_3p_{91} = \left(\frac{1}{1.04}\right)^3 \frac{l_{94}}{l_{91}} = \left(\frac{1}{1.04}\right)^3 \frac{216}{900} = 0.21336$$

d.

$$1000A_{91:\overline{3}|} = 1000(A^{1}_{91:\overline{3}|} + {}_3E_{91}) = 1000(0.70152 + 0.21336) = 914.88 \text{ using parts b. and c.}$$

or

$$900A_{91:\overline{3}|} = (900-720)v + (720-432)\,v^2 + (432-216)v^3 + 216v^3 = 823.39554$$

$$A_{91:\overline{3}|} = \frac{823.39554}{900} = 0.91488 \Longrightarrow 1000A_{91:\overline{3}|} = (1000)(0.91488) = 914.88$$

e.

$${}_{2|}A_{91} = {}_2E_{91} \cdot A_{93} = v^2\,{}_2p_{91}A_{93} = \left(\frac{1}{1.04}\right)^2 \frac{432}{900}(0.94305) = 0.41851$$

$$\text{since } A_{93} = \frac{216v + 216v^2}{432} = 0.94305$$

f.

$$Var[Z] = \left({}^{2}A_{91}\right) - \left(A_{91}\right)^2$$

From Part a, $A_{91} = 0.90668$

$$900({}^{2}A_{91}) = (900-720)v^2 + (720-432)\,v^4 + (432-216)v^6 + (216-0)v^8 = 741.14075$$

$${}^{2}A_{91} = \frac{741.14075}{900} = 0.82349$$

$$Var[Z] = 0.82349 - \left(0.90668\right)^2 = 0.0014251$$

g.

There are 180 deaths (900 - 720) between age 91 and 92 which means there are 45 ($180 \div 4$) each quarter. There are 288 deaths (720 - 432) between age 92 and 93 which means there are 72 ($288 \div 4$) deaths each quarter. Finally, there are 216 deaths between ages 93 and 94 and another 216 deaths between age 94 and 95. This means that there are 54 ($216 \div 4$) deaths each quarter for these two years. Now using l_x and d_x :

$$900A_{91}^{(4)} = 45v^{0.25} + 45v^{0.5} + 45v^{0.75} + 45v^{1} + 72v^{1.25} + \ldots + 72v^{2} + 54v^{2.25} + \ldots + 54v^{4} =$$

$$\frac{45(v^{0.25} - v^{1.25})}{1-v^{0.25}} + \frac{72(v^{1.25} - v^{2.25})}{1-v^{0.25}} + \frac{54(v^{2.25} - v^{4.25})}{1-v^{0.25}} = 828.15017$$

$$A_{91}^{(4)} = \frac{828.15017}{900} = 0.92017$$

h.

We want $1000A_{93}^{(12)}$ so first we must find $A_{93}^{(12)}$. We will do that using l_x and d_x.

We know that 216 people die each year from ages 93 to 94 and from 94 to 95. This means that 18 $(216 \div 12)$ die each month. Therefore:

$$432(A_{93}^{(12)}) = 18v^{\frac{1}{12}} + 18v^{\frac{2}{12}} + 18v^{\frac{3}{12}} + \ldots + 18v^{\frac{24}{12}} = \frac{18\left(v^{\frac{1}{12}} - v^{\frac{25}{12}}\right)}{1 - v^{\frac{1}{12}}} = 414.81249$$

$$A_{93}^{(12)} = \frac{414.81249}{432} = 0.96021 ==> 1000A_{93}^{(12)} = (1000)(0.96021) = 960.21$$

i.

Doing it with $l's$

$$900(IA)_{91} = (1)(180)v + (2)(288)v^2 + (3)(216)v^3 + (4)(216)v^4$$

$$(IA)_{91} = \frac{(1)(180)v + (2)(288)v^2 + (3)(216)v^3 + (4)(216)v^4}{900} = 2.24471$$

j.

$$900(IA)_{91:\overline{3}|}^{1} = (1)(180)v + (2)(288)v^2 + (3)(216)v^3$$

$$(IA)_{91} = \frac{(1)(180)v + (2)(288)v^2 + (3)(216)v^3}{900} = 1.42410$$

5.

a.

We can just look this up in the SULT.

$$A_{50} = 0.18931$$

b.

$$A^{1}_{50:\overline{20|}} = A^{1}_{50:\overline{20|}} - {}_{20}E_{50} = 0.38844 - 0.34824 = 0.0402$$

or

$$A^{1}_{50:\overline{20|}} = A_{50} - {}_{20}E_{50} \cdot A_{70} = 0.18931 - (0.34824)(0.42818) = 0.0402$$

c.

$$A_{50:\overline{25|}} = A^{1}_{50:\overline{25|}} + {}_{25}E_{50} = A_{50} - {}_{25}E_{50} \cdot A_{70} + {}_{25}E_{50} =$$

$$0.18931 - (0.77772)(0.32819)(0.50868) + (0.77772)(0.32819) = 0.31471$$

d.

$${}_{20|}A_{50} = {}_{20}E_{50} \cdot A_{70} = (0.34824)(0.42818) = 0.14911$$

e.

$$A^{1}_{50:\overline{35|}} = A_{50} - {}_{35}E_{50} \cdot A_{85} = A_{50} - {}_{20}E_{50} \cdot {}_{10}E_{70} \cdot {}_{5}E_{80} \cdot A_{85} =$$

$$0.18931 - (0.34824)(0.50994)(0.63365)(0.67622) = 0.11322$$

f.

$$Var[Z] = \left({}^{2}A_{50}\right) - \left(A_{50}\right)^{2} \text{ ==> Look these up in SULT}$$

$$Var[Z] = \left(0.05108\right) - \left(0.18931\right)^{2} = 0.01524$$

g.

$${}^{2}A^{1}_{50:\overline{20|}} = {}^{2}A_{50} - v^{20}{}_{20}E_{50} \cdot \left({}^{2}A_{70}\right) = 0.05108 - v^{20}(0.34824)(0.21467) = 0.02290$$

$$A^{1}_{50:\overline{20|}} = 0.0402 \text{ from part b.}$$

$$Var[Z] = \left({}^{2}A^{1}_{50:\overline{20|}}\right) - \left(A^{1}_{50:\overline{20|}}\right)^{2} = 0.02290 - (0.0402)^{2} = 0.02128$$

h.

$${}_{13}E_{40} = v^{13}{}_{13}p_{40} = \left(\frac{1}{1.05}\right)^{13}\left(\frac{l_{53}}{l_{40}}\right) = \left(\frac{1}{1.05}\right)^{13}\left(\frac{98{,}181.8}{99{,}338.3}\right) = 0.52415$$

6.

$$ {}_{1|}A_x = vp_x A_{x+1} \text{ ==> } 0.410 = vp_x\left(0.50617\right) \text{ ==> } vp_x = \frac{0.410}{0.50617} $$

$$ A_x = vq_x + vp_x A_{x+1} = v(1 - p_x) + vp_x A_{x+1} = v - vp_x + vp_x A_{x+1} $$

$$ \text{==> } 0.500 = v - \frac{0.410}{0.50617} + \frac{0.410}{0.50617}(0.50617) $$

$$ \text{==> } v = 0.500 + \frac{0.410}{0.50617} - \frac{0.410}{0.50617}(0.50617) = 0.90000 \text{ ==> } i = 0.1111111 $$

$$ vp_x = \frac{0.410}{0.50617} \text{ ==> } p_x = \frac{0.410}{0.50617}(1+i) $$

$$ \text{==> } q_x = 1 - p_x = 1 - \frac{0.410}{0.50617}(1.1111111) = 0.10 $$

7.

This problem screams recursive formula.

$$ Var[Z] = \left({}^2A_{91}\right) - \left(A_{91}\right)^2 \text{ ==> } 0.00706 = 0.54696 - \left(A_{91}\right)^2 \text{ ==> } A_{91} = 0.73478 $$

$$ A_{90} = v \cdot q_{90} + v \cdot p_{90} \cdot A_{91} = \left(\frac{1}{1.08}\right)\left(1 - 0.92\right) + \left(\frac{1}{1.08}\right)\left(0.92\right)(0.73478) = 0.70000 $$

8.

Use the recursive formula.

$$A_{70} = v \cdot q_{70} + v \cdot p_{70} \cdot A_{71}$$

We can use A_{71} straight from the SULT since the mortality at ages 71 and over is not effected. Therefore, $A_{71} = 0.44379$.

$v = \dfrac{1}{1.05}$ and q_{70} is twice that in the SULT ==> $q_{70} = (2)(0.010413) = 0.020826$

==> $A_{70} = v \cdot q_{70} + v \cdot p_{70} \cdot A_{71} =$

$$\left(\frac{1}{1.05}\right)(0.020826) + \left(\frac{1}{1.05}\right)(1-0.020826)(0.44379) = 0.43369$$

==> $100,000 A_{70} = (100,000)(0.43369) = 43,369$

9.

a. $1000\bar{A}_{50} = 1000\left(\dfrac{i}{\delta}\right)A_{50} = 1000(1.02480)(0.18931) = 194.00$

b.

$$1000\bar{A}^{1}_{50:\overline{20|}} = 1000\left(\frac{i}{\delta}\right)A^{1}_{50:\overline{20|}} = 1000(1.0248)(A_{50:\overline{20|}} - {}_{20}E_{50})$$

$$= 1000(1.0248)[0.38844 - 0.34824] = 41.20$$

c. $1000\bar{A}_{50:\overline{20|}} = 1000\left[\bar{A}^{1}_{50:\overline{20|}} + {}_{20}E_{50}\right] = 41.20 + 1000(0.34824) = 389.44$

d.

$$1000\, {}_{20|}\bar{A}_{50} = 1000\left(\frac{i}{\delta}\right){}_{20|}A_{50} = 1000\left(\frac{i}{\delta}\right){}_{20}E_{50} \cdot A_{70}$$

$$= 1000(1.0248)(0.34824)(0.42818) = 152.81$$

e. $1000A_{50}^{(4)} = 1000\left(\frac{i}{i^{(4)}}\right)A_{50} = 1000(1.01856)(0.18931) = 192.82$

10.

a.

$$l_{[54]} = 1000$$
$$l_{[54]+1} = l_{[54]}\left(1-q_{[54]}\right) = 1000(1-0.04) = 960$$
$$l_{[54]+2} = l_{[54]+1}\left(1-q_{[54]+1}\right) = 960(1-0.055) = 907.2$$
$$l_{[54]+3} = l_{[54]+2}\left(1-q_{[54]+2}\right) = 907.2(1-0.076) = 838.2528$$

$$1000A^{1}_{[54]:\overline{3}|} = \frac{1000-960}{1.06} + \frac{960-907.2}{(1.06)^2} + \frac{907.2-838.2528}{(1.06)^3} = 142.61706$$

$$A^{1}_{[54]:\overline{3}|} = \frac{142.61706}{1000} = 0.14262$$

b.

$$l_{[54]} = 1000$$
$$l_{[54]+1} = l_{[54]}\left(1-q_{[54]}\right) = 1000(1-0.04) = 960$$
$$l_{[54]+2} = l_{[54]+1}\left(1-q_{[54]+1}\right) = 960(1-0.055) = 907.2$$
$$l_{[54]+3} = l_{[54]+2}\left(1-q_{[54]+2}\right) = 907.2(1-0.076) = 838.2528$$

$$1000A_{[54]:\overline{3}|} = \frac{1000-960}{1.06} + \frac{960-907.2}{(1.06)^2} + \frac{907.2-838.2528}{(1.06)^3} + \frac{838.2528}{(1.06)^3} = 846.43027$$

$$A_{[54]:\overline{3}|} = \frac{846.43027}{1000} = 0.84643$$

11.

$$l_x = 1000$$
$$l_{x+1} = l_x(1-q_x) = (1000)(1-0.2) = 800$$
$$l_{x+2} = l_{x+1}(1-q_{x+1}) = (800)(1-0.4) = 480$$

$$1000A_{x:\overline{3}|} = \frac{1000-800}{1.1} + \frac{800-480}{(1.1)^2} + \frac{480}{(1.1)^3} = 806.91210$$

$$A_{x:\overline{3}|} = \frac{806.91210}{1000} = 0.80691$$

Note that the 480 people alive after 2 years will get paid at the end of the 3rd year. Those that die are paid a death benefit and those that live get the endowment.

12. $PV = \int_0^{\infty} b_t \cdot v^t \cdot {}_t p_x \cdot \mu_{x+t} dt = \int_0^{\infty} (1.06)^t \cdot (1.06)^{-t} \cdot {}_t p_x \cdot \mu_{x+t} dt = \int_0^{\infty} {}_t p_x \cdot \mu_{x+t} dt = {}_{\infty} q_x = 1$

13.

$$PV = 4000A_{60} - 2000{}_{10}E_{60} \cdot A_{70} - 1000{}_{20}E_{60} \cdot A_{80} =$$

$$(4000)(0.29028) - (2000)(0.57864)(0.42818) - (1000)(0.29508)(0.59293) = 490.63$$

14.

$$PV = 5000A_{45} - 3000\,{}_{20}E_{45}\cdot A_{65} = (5000)(0.15161) - (3000)(0.35994)(0.35477) = 374.96$$

$$Var[Z] = E[Z^2] - \left(E[Z]\right)^2$$

$$E[Z^2] = E\left[\left(5000v^{K_x+1} - 3000v^{K_x+1}(I\{K_x \geq 20\})\right)^2\right]$$

$$= (1000)^2 E\left[25v^{2(K_x+1)} - (2)(5)(3)v^{K_x+1}v^{K_x+1}(I\{K_x \geq 20\}) + 9v^{2(K_x+1)}(I\{K_x \geq 20\})\right]$$

$$= (1000)^2 E\left[25v^{2(K_x+1)} - (21)v^{2(K_x+1)}(I\{K_x \geq 20\})\right]$$

$$= (1000)^2\left[25\cdot {}^2A_{45} - 21\cdot {}_{20}E_{45}\cdot v^{20}\cdot {}^2A_{65}\right] = (1000)^2\left[25(0.03463) - 21(0.35994)(1.05)^{-20}(0.15420)\right]$$

$$= 426{,}463.56$$

$$Var[Z] = 426{,}463.56 - (374.96)^2 = 285{,}869$$

15.

$$PV = \sum_{k=0}^{\infty} b_{k+1}\cdot v^{k+1}\cdot {}_k p_x\cdot q_{x+k} = \sum_{k=0}^{\infty}(1.04)^{k+1}\cdot\left(\frac{1}{1.092}\right)^{k+1}\cdot {}_k p_x\cdot q_{x+k} = \sum_{k=0}^{\infty}\left(\frac{1}{1.05}\right)^{k+1}\cdot {}_k p_x\cdot q_{x+k} = A_x$$

$$A_{80} = 0.59293$$

16.

We calculate the value for smokers and nonsmokers and then we wait the present values.

Smokers ==> $A^1_{x:\overline{2}|} = vq_x + v^2 p_x q_{x+1} = \frac{0.1}{1.02} + \frac{(1-0.1)(0.2)}{(1.02)^2} = 0.2710496$

Nonsmokers ==> $A^1_{x:\overline{2}|} = vq_x + v^2 p_x q_{x+1} = \frac{0.05}{1.02} + \frac{(1-0.05)(0.1)}{(1.02)^2} = 0.1403306$

$$10{,}000A^1_{x:\overline{2}|} = (10{,}000)(0.25\cdot 0.2710496 + 0.75\cdot 0.1403306) = 1730.10$$

17.

This problem is different because we replace the light bulbs which we then cover with the insurance. The easiest way to do this is still to determine how many bulbs will be replaced at the ends of 1, 2, and 3 years.

End of first year ==> Start year with 10000 new bulbs
==> Bulbs burning out are $(10000)(0.1) = 1000$

End of second year ==> Begin year with 9000 one year old bulbs and 1000 new bulbs that were replaced because they were burned out in the first year
==> Bulbs burning out are $(9000)(0.3)+(1000)(0.1) = 2800$

End of third year ==> Begin year with 6300 bulbs that are two years old, 900 bulbs that are one year old and 2800 new bulbs ==> Bulbs burning out are $(6300)(0.5)+(900)(0.3)+(2800)(0.1) = 3700$

$$APV = 1000v + 2800v^2 + 3700v^3 = 6688.26$$

18.

$$APV_{80}^{q=0.2} = 13 = 20vq_{80} + vp_{80}(APV_{81}) = \frac{(20)(0.2)}{1.06} + \frac{0.8}{1.06}(APV_{81})$$

$$\therefore APV_{81} = \frac{13 - \frac{(20)(0.2)}{1.06}}{\frac{0.8}{1.06}} = 12.225$$

$$APV_{80}^{q=0.1} = 20vq_{80} + vp_{80}(APV_{81}) = \frac{(20)(0.1)}{1.06} + \frac{0.9}{1.06}(12.225) = 12.27$$

19.

$$APV_{40} = vp_{40} \cdot APV_{41} + 100,000 A^{1}_{40:\overline{10}|} - 1,000,000\, {}_{10}E_{40} \cdot vq_{50}$$

$$\left(\frac{1-0.000527}{1.05}\right)(3608) + (100,000)(0.61494 - 0.60920) - (1,000,000)(0.60920)\left[\frac{0.001209}{1.05}\right]$$

$$= 3307$$

20.

$$PV = \int_0^\infty b_t \cdot v^t \cdot {}_t p_{25} \cdot \mu_{25+t} \cdot dt = \int_0^\infty (1.05)^{-t} \cdot (1.05)^{-t} \cdot {}_t p_{25} \cdot (0.0002)(1.1025)^{25+t} \cdot dt =$$

$$\int_0^\infty (1.1025)^{-t} \cdot {}_t p_{25} \cdot (0.0002)(1.1025)^{25}(1.1025)^{t} \cdot dt =$$

$$\int_0^\infty {}_t p_{25} \cdot (0.0002)(1.1025)^{25} dt = (0.0002)(1.1025)^{25} \int_0^\infty {}_t p_{25} \cdot dt =$$

$$(0.0002)(1.1025)^{25}[\mathring{e}_{25}] = (0.0002)(1.1025)^{25}(53.06) = 0.121692$$

21.

$$l_x = 1000$$
$$l_{x+1} = (1000)(1-0.02) = 980$$
$$l_{x+2} = (980)(1-0.04) = 940.80$$
$$l_{x+3} = (940.80)(1-0.06) = 884.352$$

$$E[Z] = \frac{(300v)(1000-980) + (350v^2)(980-940.80) + (400v^3)(940.80-884.352)}{1000}$$

$$= 36.829$$

$$E[Z] = \frac{(300v)^2(1000-980) + (350v^2)^2(980-940.80) + (400v^3)^2(940.80-884.352)}{1000}$$

$$= 11,772.61$$

$$Var[Z] = 11,772.61 - (36.829)^2 = 10,416.22$$

22.

$$E[Z] = (10,000)A_{65} = 3547.70$$
$$E[Port] = (100)(E[Z]) = (100)(3547.70) = 354,770$$

$$Var[Z] = (10,000)^2\left[{}^2A_{65} - (A_{65})^2\right] = (10,000)^2\left[0.15420 - (0.35477)^2\right] = 2,833,825$$
$$Var[Port] = 100Var[Z] = 283,382,500$$

$$Amount = E[Port] + 1.282\sqrt{Var[Port]} = 354,770 + (1.282)\sqrt{283,382,500} = 376,351$$

23.

$$E[Z]=100,000[\bar{A}_{70}-{}_{20}E_{70}\cdot\bar{A}_{90}]=(100,000)(1.02480)[0.42818-(0.17313)(0.75317)]=30,516.87$$
$$E[Port]=(400)E[Z]=(400)(30,516.87)=12,206,748$$

$$Var[Z]=(100,000)^2\left[{}^2\bar{A}^{1}_{70:\overline{20}|}-(\bar{A}^{1}_{70:\overline{20}|})^2\right]=$$

$$(100,000)^2\left[{}^2\bar{A}_{70}-{}_{20}E_{70}v^{20}\cdot{}^2\bar{A}_{90}-(0.3051687)^2\right]=$$

$$(100,000)^2\left[\frac{(1+i)^2-1}{2\delta}\left\{{}^2A_{70}-{}_{20}E_{70}v^{20}\cdot{}^2A_{90}\right\}-(0.3051687)^2\right]=$$

$$(100,000)^2\left[\frac{(1.05)^2-1}{2\ln(1.05)}\left\{0.21467-(0.17313)(1.05)^{-20}(0.58528)\right\}-(0.3051687)^2\right]=$$

$$(100,000)^2(0.092249558)$$

$$Var[Port]=(400)(100,000)^2(0.092249558)$$

$$==>SD[Port]=\sqrt{(400)(100,000)^2(0.092249558)}=607,452$$

$$E[Port]+\gamma\cdot SD[Port]=13,000,000$$

$$12,206,748+\gamma(607,452)=13,000,000 ==> \gamma=1.306$$

Look up 1.31 in the table. We see that the probability is 0.9049

24.

$$E[Z]=235.24$$

$$\Pr[Z<235.24]=\Pr[1000v^{K_x+1}<235.24]=\Pr[v^{K_x+1}<0.23524]=$$

$$\Pr\left[K_x+1>\frac{\ln(0.23524)}{\ln[(1.05)^{-1}]}\right]=\Pr[K_x+1>29.66]=\Pr[K_x>28.66]={}_{29}p_{55}=\frac{l_{84}}{l_{55}}=0.65926$$