Chapter 4

1. You are given that the interest rate is 6% and that mortality follows Makeham's law with A=0.0003, B=0.000004 and C=1.1.

   Write an integral expression for $\bar{A}_{40}$.

2. You are given that mortality follows Gompertz law and $i = 8\%$. Further, you are given that

   $X \bullet \bar{A}_0 = \int_0^\infty (1.08)^{-t} \bullet \exp\{-0.002049593(1.05^t - 1)\} \bullet 1.05^t \, dt$.

   Determine X .

3. You are given:

   a. $\delta = 0.04$
   b. ${}_t p_x = 1 - 0.02t - 0.0008t^2 \text{ for } 0 \le t \le 25$

   Calculate $1000\bar{A}_x$.

4. You are given the following mortality table:

x	l_x	q_x	p_x	n	${}_n p_{90}$
90	1000	0.10	0.90	0	1.000
91	900	0.20	0.80	1	0.900
92	720	0.40	0.60	2	0.720
93	432	0.50	0.50	3	0.432
94	216	1.00	0.00	4	0.216
95	0			5	0.000

Assume that deaths are uniformly distributed between integral ages. Calculate at $i = 4\%$:

a. A_{91}
b. $A^1_{91:\overline{3}|}$
c. ${}_3E_{91}$
d. $1000A_{91:\overline{3}|}$
e. ${}_{2|}A_{91}$
f. $Var[Z]$ if Z is the present value random variable for a whole life on (91).
g. $A^{(4)}_{91}$
h. $1000A^{(12)}_{93}$
i. $(IA)_{91}$
j. $(IA)^1_{91:\overline{3}|}$

5. Use the Standard Ultimate Life Table with interest at 5% to calculate the following:

    a. A_{50}
    b. $A^{1}_{50:\overline{20|}}$
    c. $A_{50:\overline{25|}}$
    d. ${}_{20|}A_{50}$
    e. $A^{1}_{50:\overline{35|}}$
    f. $Var[Z]$ where Z is the present value random variable for A_{50}
    g. $Var[Z]$ where Z is the present value random variable for $A^{1}_{50:\overline{20|}}$
    h. ${}_{13}E_{40}$

6. You are given:

    i. $A_x = 0.500$
    ii. $A_{x+1} = 0.50617$
    iii. ${}_{1|}A_x = 0.410$

    Calculate q_x and i.

7. You are given:

    i. $i = 8\%$
    ii. $p_{90} = 0.92$
    iii. Z is the present value random variable for A_{91}
    iv. $Var[Z] = 0.00706$
    v. ${}^{2}A_{91} = 0.54696$

    Calculate A_{90}.

8. You are given:

    i. Mortality follows the Standard Ultimate Life Table except for age 70 where $q_{70} =$ twice the mortality rate listed in the Standard Ultimate Life Table.
    ii. $i = 5\%$

    Calculate $100,000A_{70}$.

9. Assume that deaths are uniformly distributed between integral ages. Use the Standard Ultimate Life Table with interest at 5% to calculate the following:

   a. $1000\bar{A}_{50}$
   b. $1000\bar{A}^{1}_{50:\overline{20|}}$
   c. $1000\bar{A}_{50:\overline{20|}}$
   d. $1000\,{}_{20|}\bar{A}_{50}$
   e. $1000A^{(4)}_{50}$

10. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

If $i = 0.06$, calculate:

   a. $A^{1}_{[54]:\overline{3|}}$
   b. $A_{[54]:\overline{3|}}$

11. You are given:

    i. $i = 10\%$

    ii. $q_x = 0.20$

    iii. $q_{x+1} = 0.40$

    Calculate $A_{x:\overline{3}|}$

12. A special whole life insurance on (30) provides a death benefit of $(1.06)^t$ where t is measured from the issue date of the policy. The death benefit is payable at the moment of death.

    Calculate the Expected Present Value at $i = 6\%$.

13. A special decreasing whole life insurance is issued to (60). The special whole life pays the following death benefits at the end of the year of death:

k	b_{k+1}
0-9	4000
10-19	2000
20+	1000

    You are also given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

    Calculate the actuarial present value of this special whole life.

14. A special decreasing whole life insurance is issued to (45). The special whole life pays a death benefit of 5000 until age 65 and a death benefit of 2000 after age 65. The death benefit is payable at the end of the year of death.

    You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

    Z is the present value random variable for this insurance coverage.

    Calculate $E[Z]$ and $Var[Z]$.

15. A special whole life insurance policy on (80) pays a death benefit of $(1.04)^{K+1}$ where K is the number of complete years lived by (80). For example, if (80) dies between 80 and 81, then $K=0$. If (80) dies between 81 and 82, then $K=1$.

You are given:

i. Mortality follows the Standard Ultimate Life Table
ii. $i=9.2\%$

Calculate the EPV of this special whole life.

16. * For a group of individuals all age x, you are given:

i. 25% are smokers and 75% are nonsmokers
ii. $i=0.02$
iii. Mortality as follows:

k	q_{x+k} for smokers	q_{x+k} for nonsmokers
0	0.10	0.05
1	0.20	0.10
2	0.30	0.15

Calculate $10,000A^{1}_{x:\overline{2}|}$ for an individual chosen at random from this group.

17. * A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs. The light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.

You are given:

i. For new light bulbs,

$$q_0=0.10$$
$$q_1=0.30$$
$$q_2=0.50$$

ii. Each light bulb costs 1.
iii. $i=0.05$

Calculate the actuarial present value of this contract.

18. * A decreasing term life insurance on (80) pays $(20-k)$ at the end of the year of death if (80) dies in year $k+1$, for $k=0,1,2,...,19$.

You are given:

i. $i=0.06$
ii. For a certain mortality table with $q_{80}=0.2$, the actuarial present value for this insurance is 13.
iii. For the same mortality table, except that $q_{80}=0.1$, the actuarial present value for this insurance is APV.

Calculate APV.

19. * For an increasing 10 year term insurance, you are given:

i. $b_{k+1}=100,000(1+k), k=0,1,2,...,9$
ii. Benefits are payable at the end of the year of death.
iii. Mortality follows the Standard Ultimate Life Table
iv. $i=0.05$
v. The actuarial present value of this insurance on (41) is 3608 .

Calculate the actuarial present value for this insurance on (40).

20. A whole insurance on (25) pays a death benefit immediately upon death. The death benefit at time t is $(1.05)^{-t}$.

You are given:

a. Mortality follows Gompertz Law with B = 0.0002 and c = 1.1025.
b. $i=0.05$
c. $\mathring{e}_{25}=53.06$

Calculate the Expected Present Value of this insurance benefit.

21. * For a special 3 year term insurance on (x), you are given:

i. Z is the present-value random variable for this insurance.

ii. $q_{x+k} = 0.02(k+1), k = 0,1,2$

iii. The following benefits are payable at the end of the year of death:

k	b_{k+1}
0	300
1	350
2	400

iv. $i = 0.06$

Calculate $Var[Z]$.

22. Knapp Insurance Company issues 100 whole life policies to independent lives each age 65. The death benefit for each life is 10,000 payable at the end of the year of death.

    Mortality follows the Standard Ultimate Life Table and $i = 5\%$.

    Assuming the normal distribution, calculate the amount that Knapp must have on hand at time 0 to be 90% certain that the company can cover the future death benefits.

23. Li Life Insurance Company issues 400 20 year term policies to independent lives each age 70. The death benefit is 100,000 payable at the moment of death.

    Mortality follows the Standard Ultimate Life Table and $i = 5\%$. Death are uniformly distributed between integral ages.

    At the time these policies are issued, Li sets aside 13 million.

    Assuming the normal distribution, calculate the probability that Li will have sufficient funds to pay all the death benefits.

24. Z is the present value random variable for a whole life of 1000 to (55) where the death benefit is paid at the end of the year of death.

    Mortality follows the Standard Ultimate Life Table and $i = 5\%$.

    Calculate $\Pr(Z < E[Z])$.

25. Exercise 4.7 from the book – (a) and (c) only.

26. Exercise 4.12 from the book

27. Exercise 4.13 from the book

1. $\int_0^{\infty}(1.06)^{-t}\exp\left(-0.0003t-\frac{0.000004}{\ln(1.1)}(1.1^{40})(1.1^{t}-1)\right)\left(0.0003+(0.000004)(1.1^{40+t})\right)$
2. 10,000
3. 580.30
4.
    a. 0.90668
    b. 0.70152
    c. 0.21336
    d. 914.88
    e. 0.41851
    f. 0.00142
    g. 0.92017
    h. 960.21411
    i. 2.24471
    j. 1.42410

5.
    a. 0.18931
    b. 0.04020
    c. 0.31471
    d. 0.14911
    e. 0.11322
    f. 0.01524
    g. 0.02128
    h. 0.52415
6. $i=11.11111\%$ and $q_x=0.10$
7. 0.70000
8. 43,369
9.
    a. 194.00
    b. 41.20
    c. 389.44
    d. 152.81
    e. 192.82
10.
    a. 0.14262
    b. 0.84643
11. 0.80691
12. 1
13. 490.63
14. 374.96 and 285,870

15. 0.59293
16. 1730.10
17. 6688.26
18. 12.27
19. 3307
20. 0.121692
21. 10,417
22. 376,351
23. 90.49%
24. 65.93%
25.
    c. 5.07307
26. 0.59704
27. 0.01