1)

a.

$$900\ddot{a}_{91} = 900 + 720\left(\frac{1}{1.04}\right) + 432\left(\frac{1}{1.04}\right)^2 + 216\left(\frac{1}{1.04}\right)^3$$

$$\ddot{a}_{91} = 2.42638$$

b.

$$a_{91} = \ddot{a}_{91} - 1 = 1.42638$$

c.

$$900\ddot{a}_{91:\overline{3}|} = 900 + 720\left(\frac{1}{1.04}\right) + 432\left(\frac{1}{1.04}\right)^2$$

$$\ddot{a}_{91:\overline{3}|} = 1 + 0.8\left(\frac{1}{1.04}\right) + 0.48\left(\frac{1}{1.04}\right)^2 = 2.21302$$

d.

$$Var[Y] = \frac{{}^2A_{91} - \left(A_{91}\right)^2}{d^2}$$

$${}^2A_{91} = \frac{180v^2 + 288v^4 + 216v^6 + 216v^8}{900} = 0.82349$$

$$A_{91} = \frac{180v + 288v^2 + 216v^3 + 216v^4}{900} = 0.90668$$

$$Var = \frac{0.82349 - (0.90668)^2}{\left(\frac{0.04}{1.04}\right)^2} = 0.96334$$

e.

$$\ddot{a}_{\overline{91:3}|} = 1 + v + v^2 + v^3\left(\frac{216}{900}\right) = 3.09945$$

f.

$$Var[Y] = \frac{{}^{2}A_{91:\overline{3}|} - \left(A_{91:\overline{3}|}\right)^2}{d^2}$$

$${}^{2}A_{91:\overline{3}|} = \frac{180v + 288v^4 + 432v^6}{900} = 0.83780$$

$$A_{91:\overline{3}|} = \frac{180v + 288v^2 + 432v^3}{900} = 0.91488$$

$$Var = \frac{0.83780 - (0.91488)^2}{\left(\frac{0.04}{1.04}\right)^2} = 0.53197$$

g.

$${}_{2|}\ddot{a}_{91} = {}_{2}E_{91}\ddot{a}_{93} = v^2\left(\frac{432}{900}\right)\left(1 + \frac{216}{432}v\right) = 0.65715$$

h.

$$\bar{a}_{91} = \frac{1 - \bar{A}_{91}}{\delta} = \frac{1 - \frac{i}{\delta}A_{91}}{\delta} = \frac{1 - \left(\frac{0.04}{\ln(1.04)}\right)(0.90668)}{\ln(1.04)} = 1.9200$$

i.

$$Var[Y] = \frac{{}^{2}\bar{A}_{91} - \left(\bar{A}_{91}\right)^2}{\delta^2} = \frac{\frac{\left[(1+i)^2 - 1\right]}{2\delta}{}^{2}A_{91} - \left(\frac{i}{\delta}A_{91}\right)^2}{\delta^2}$$

$$Var = \frac{\frac{\left[(1.04)^2 - 1\right]}{2\ln(1.04)}(0.82349) - \left[\frac{0.04}{\ln(1.04)}(0.90668)\right]}{\left[\ln(1.04)\right]^2}$$

$$Var = 1.03247$$

j.

$$(I\ddot{a})_{91} = \frac{900(1) + 720(2)v + 432(3)v^2 + 216(4)v^3}{900} = 4.72326$$

k.

$$(I\ddot{a})_{91:\overline{3}|} = \frac{900(1)+720(2)v+432(3)v^2}{900} = 3.86982$$

2)

a.

$$\ddot{a}_{60} = 14.9041 \text{ (straight from Table)}$$

b.

$$a_{60} = \ddot{a}_{60} - 1 = 13.9041$$

c.

$${}_{10|}\ddot{a}_{60} = {}_{10}E_{60}(\ddot{a}_{70}) = (0.57864)(12.0083) = 6.94848$$

d.

$$\ddot{a}_{60:\overline{20}|} = \ddot{a}_{60} - {}_{20}E_{60}(\ddot{a}_{80}) = 14.9041 - (0.29508)(8.5484) = 12.38164$$

e.

$$\bar{a}_{80} = \frac{1-\bar{A}_{80}}{\delta} = \frac{1-\left(\frac{i}{\delta}\right)A_{80}}{\delta} = \frac{1-(1.02480)(0.59293)}{0.04879} = 8.04192$$

f.

$$\bar{a}_{50:\overline{20}|} = \frac{1-\bar{A}_{50:\overline{20}|}}{\delta} = \frac{1-\left[\bar{A}_{50} - {}_{20}E_{50}(\bar{A}_{70}) + {}_{20}E_{50}\right]}{\delta}$$

$$\bar{a}_{50:\overline{20}|} = \frac{1-(\frac{i}{\delta})A_{50} + {}_{20}E_{50}(\frac{i}{\delta})A_{70} - {}_{20}E_{50}}{\delta}$$

$$\bar{a}_{50:\overline{20}|} = \frac{1-(1.02480)(0.18931)+(0.34824)(1.02480)(0.42818)-0.34824}{0.04879}$$

$$\bar{a}_{50:\overline{20}|} = 12.51409$$

g.

$$\ddot{a}_{\overline{60:10}|} = \ddot{a}_{\overline{10}|} + {}_{10}E_{60}(\ddot{a}_{70})$$

$$\ddot{a}_{\overline{60:10}|} = \frac{1-(1.05)^{-10}}{0.05}(1.05) + (0.57864)(12.0083) = 15.05630$$

h.

$$\ddot{a}_{\overline{60:\overline{13}|}} = \ddot{a}_{\overline{13}|} + {}_{13}E_{60}(\ddot{a}_{73})$$

$$\ddot{a}_{\overline{60:\overline{13}|}} = \frac{1-(1.05)^{-13}}{0.05}(1.05) + (1.05)^{-13}\left(\frac{l_{73}}{l_{60}}\right)\ddot{a}_{73}$$

$$\ddot{a}_{\overline{60:\overline{13}|}} = 9.8633 + (1.05)^{-13}\left(\frac{87{,}916.8}{96{,}634.1}\right)(11.0081) = 15.1745$$

i.

$$\ddot{a}_{60}^{(12)} = \frac{1-A_{60}^{(12)}}{d^{(12)}} = \frac{1-\left(\frac{i}{i^{(12)}}\right)A_{60}}{d^{(12)}}$$

$$\ddot{a}_{60}^{(12)} = \frac{1-(1.02271)(0.29028)}{0.04869} = 14.4409$$

j.

$$\ddot{a}_{60}^{(12)} = \alpha(12)\ddot{a}_{60} - \beta(12) = 1.00020(14.9041) - 0.46651 = 14.4406$$

k.

$$\mu_x \approx -\tfrac{1}{2}\left[\ln(p_{59}) + \ln(p_{60})\right] = -\tfrac{1}{2}\left[\ln(1-0.003048) + \ln(1-0.003398)\right] = 0.00322822$$

$$\ddot{a}_{60}^{(12)} = \ddot{a}_{60} - \frac{12-1}{2(12)} - \frac{12^2-1}{12^3}(\delta + \mu_x)$$

$$\ddot{a}_{60}^{(12)} = 14.9041 - \frac{11}{24} - \frac{143}{1728}(0.04879 + 0.00322822) = 14.4415$$

l.

$$\frac{{}^2A_{60} - (A_{60})^2}{d^2} = \frac{0.10834 - (0.29028)^2}{(\frac{0.05}{1.05})^2} = 10.61819$$

m.

$$\frac{{}^2A_{60:\overline{20}|} - (A_{60:\overline{20}|})^2}{d^2}$$

$${}^2A_{60:\overline{20}|} = {}^2A_{60} - v^{20}\,{}_{20}E_{60}({}^2A_{80}) + v^{20}\,{}_{20}E_{60}$$

$${}^2A_{60:\overline{20}|} = 0.10834 - (\tfrac{1}{1.05})^{20}(0.29508)(0.38134) + (\tfrac{1}{1.05})^{20}(0.29508) = 0.17714$$

$$A_{60:\overline{20}|} = 0.41040$$

$$Var = \frac{0.17714 - (0.41040)^2}{(\frac{0.05}{1.05})^2} = 3.84192$$

3)

$$100\left(\frac{1-\bar{A}_{50}}{\delta}\right) + 400 \cdot {}_{20}E_{50}\left(\frac{1-\bar{A}_{70}}{\delta}\right)$$

$$= 100\left(\frac{1-(1.02480)(0.18931)}{0.04879}\right) + (400)(0.34824)\left(\frac{1-(1.02480)(0.42818)}{0.04879}\right) = 3254.20$$

4)

$$(12)(800)\ddot{a}^{(12)}_{\overline{60:5}|} = (12)(800)\left[\frac{1-v^5}{d^{(12)}} + {}_5E_{60}\left(\frac{1-\frac{i}{i^{(12)}}A_{65}}{d^{(12)}}\right)\right]$$

$$= 9600\left[4.44596 + (0.76687)\left(\frac{1-(1.02271)(0.35477)}{0.04869}\right)\right] = 139{,}022.11$$

5)

a.

$$\ddot{a}_{[54]:\overline{3}|} = 1 + v^1\,{}_1p_{[54]} + v^2\,{}_2p_{[54]} = 1 + \tfrac{1}{1.06}\left[1 - q_{[54]}\right] + (\tfrac{1}{1.06})^2\left[1 - q_{[54]}\right]\left[1 - q_{[54]+1}\right]$$

$$= 1 + \frac{0.96}{1.06} + \frac{(0.96)(0.945)}{(1.06)^2} = 2.71307$$

b.

$$\ddot{a}_{[54]:\overline{3}|} = v\,{}_1p_{[54]} + v^2\,{}_2p_{[54]} + v^3\,{}_3p_{[54]}$$

$$= \frac{0.96}{1.06} + \frac{(0.96)(0.945)}{(1.06)^2} + \frac{(0.96)(0.945)(0.924)}{(1.06)^3} = 2.41688$$

6)

$$\ddot{a}_{90} = 1 + vp_{90}\ddot{a}_{91} = 1 + \left(\frac{0.85}{1.05}\right)(4.8858) = 4.9552$$

7)

$$\ddot{a}_{50:\overline{10}|} - 1 + {}_{10}E_{50} = a_{50:\overline{10}|}$$

$$8.2066 - 1 + v^{10}(0.9195) = 7.8277 \Longrightarrow v^{10} = 0.675476$$

$$i = \left(\frac{1}{0.675476}\right)^{\frac{1}{10}} - 1 = 0.040014\%$$

8)

$$\ddot{a}_{60} = a_{60} + 1 = 11.996$$

$$\ddot{a}_{61} = a_{61} + 1 = 11.756$$

$$\ddot{a}_{62} = a_{62} + 1 = 11.509$$

$$\ddot{a}_{60} = 1 + vp_{60}\ddot{a}_{61} \Longrightarrow 11.996 = 1 + \left(\frac{1}{1.06}\right)p_{60}(11.756) \Longrightarrow p_{60} = 0.991473$$

$$\ddot{a}_{61} = 1 + vp_{61}\ddot{a}_{62} \Longrightarrow 11.756 = 1 + \left(\frac{1}{1.06}\right)p_{61}(11.509) \Longrightarrow p_{61} = 0.990647$$

$${}_2p_{60} = (p_{60})(p_{61}) = (0.991473)(0.990647) = 0.98220$$

9)

a.

$$\ddot{a}_{[40]:\overline{4}|} = 1 + v\,{}_1p_x + v^2\,{}_2p_x + v^3\,{}_3p_x$$

$$\ddot{a}_{[40]:\overline{4}|} = 1 + \left(\frac{1}{1.06}\right)\left(\frac{33,485}{33,519}\right) + \left(\frac{1}{1.06}\right)^2\left(\frac{33,440}{33,519}\right) + \left(\frac{1}{1.06}\right)^3\left(\frac{33,328}{33,519}\right) = 3.66643$$

b.

$$a_{[40]+1:\overline{4}|} = v\,{}_1p_{[40]+1} + v^2\,{}_2p_{[40]+1} + v^3\,{}_3p_{[40]+1} + v^4\,{}_4p_{[40]+1}$$

$$a_{[40]+1:\overline{4}|} = \left(\frac{1}{1.06}\right)\left(\frac{33,440}{33,485}\right) + \left(\frac{1}{1.06}\right)^2\left(\frac{33,378}{33,485}\right) + \left(\frac{1}{1.06}\right)^3\left(\frac{33,309}{33,485}\right) + \left(\frac{1}{1.06}\right)^4\left(\frac{33,231}{33,485}\right)$$

$$a_{[40]+1:\overline{4}|} = 3.45057$$

c.

$$(Ia)_{[40]:\overline{4}|} = v\,{}_1p_{[40]} + 2v^2\,{}_2p_{[40]} + 3v^3\,{}_3p_{[40]} + 4v^4\,{}_4p_{[40]}$$

$$(Ia)_{[40]:\overline{4}|} = \left(\frac{1}{1.06}\right)\left(\frac{33,485}{33,519}\right) + 2\left(\frac{1}{1.06}\right)^2\left(\frac{33,440}{33,519}\right) + 3\left(\frac{1}{1.06}\right)^3\left(\frac{33,328}{33,519}\right) + 4\left(\frac{1}{1.06}\right)^4\left(\frac{33,309}{33,519}\right)$$

$$(Ia)_{[40]:\overline{4}|} = 8.37502$$

d.

$$(IA)_{[40]:\overline{4}|} = v\left(\frac{d_{[40]}}{l_{[40]}}\right) + 2v^2\left(\frac{d_{[40]+1}}{l_{[40]}}\right) + 3v^3\left(\frac{d_{[40]+2}}{l_{[40]}}\right) + 4v^4\left(\frac{l_{43}}{l_{[40]}}\right)$$

$$= \left(\frac{1}{1.06}\right)\left(\frac{34}{33,519}\right) + 2\left(\frac{1}{1.06}\right)^2\left(\frac{45}{33,519}\right) + 3\left(\frac{1}{1.06}\right)^3\left(\frac{62}{33,519}\right) + 4\left(\frac{1}{1.06}\right)^4\left(\frac{33,378}{33,519}\right)$$

$$= 3.16305$$

e.

$$\left[(1000)^2\left(\frac{{}^2A_{[41]:\overline{4}|} - (A_{[41]:\overline{4}|})^2}{d^2}\right)\right]^{\frac{1}{2}}$$

$${}^2A_{[41]:\overline{4}|} = v^2\left(\frac{39}{33,467}\right) + v^4\left(\frac{50}{33,467}\right) + v^6\left(\frac{69}{33,467}\right) + v^8\left(\frac{33,309}{33,467}\right) = 0.62812$$

$$A_{[41]:\overline{4}|} = v\left(\frac{39}{33,467}\right) + v^2\left(\frac{50}{33,467}\right) + v^3\left(\frac{69}{33,467}\right) + v^4\left(\frac{33,309}{33,467}\right) = 0.79251$$

$$SD = \frac{1000\sqrt{0.62812 - (0.79251)^2}}{\left(\frac{0.06}{1.06}\right)} = 119.1387$$

f.

$$\Pr(Y<3)=\Pr\left(\frac{1-v^{K+1}}{d}<3\right)\Longrightarrow \Pr(Y<3)=\Pr\left[1-v^{K+1}<3\left(\frac{0.06}{1.06}\right)=0.16981\right]$$

$$\Pr(Y<3)=\Pr(v^{K+1}>0.83019)$$

$$\Pr(Y<3)=\Pr\left[K+1<\frac{\ln(0.83019)}{\ln(\frac{1}{1.06})}\right]\Longrightarrow \Pr(Y<3)=\Pr(K+1<3.19)$$

$$\Pr(Y<3)=\Pr(K<2.19)=1-{}_3p_x=1-\frac{l_{43}}{l_{[40]}}=1-\frac{33{,}378}{33{,}519}=0.00421$$

10)

$$\ddot{a}_x=a_x+1=10$$

$$A_x=1-d\ddot{a}_x \Longrightarrow 0.6=1-d(10)\Longrightarrow d=0.04$$

$$i=\frac{0.04}{1-0.04} \text{ and } \delta=\ln(1+i)$$

$$1000\bar{A}_x=1000(\tfrac{i}{\delta})A_x \Longrightarrow 1000\bar{A}_x=1000\left[\left(\frac{\frac{0.04}{0.96}}{\ln(1+\frac{0.04}{0.96})}\right)(0.6)\right]=612.415$$

11)

$$\ddot{a}_{\overline{x:n}|} = 22.9 = \ddot{a}_{\overline{n}|} + {}_{n|}\ddot{a}_x$$

$$\ddot{a}_{\overline{x:n}|} = 8 = \ddot{a}_x - {}_{n|}\ddot{a}_x = 20 - {}_{n|}\ddot{a}_x$$

$$\therefore {}_{n|}\ddot{a}_x = 12$$

$$\therefore \ddot{a}_{\overline{n}|} = 10.90$$

Now using BA-II Plus
SET BGN

$$I/Y = 5$$

$$PV = 10.90$$

$$PMT = -1$$

$$CPT \rightarrow N = 15$$