Chapter 5

1. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

Assume that deaths are uniformly distributed between integral ages. Calculate at $i = 4\%$:

a. $\ddot{a}_{91}$
b. a_{91}
c. $\ddot{a}_{91:\overline{3}|}$
d. $Var[Y]$ if Y is the present value random variable for an annual whole life annuity due on (91).
e. $\ddot{a}_{\overline{91:\overline{3}|}}$
f. $Var[Y]$ if Y is the present value random variable for an annual 3 year temporary life annuity due on (91).
g. ${}_{2|}\ddot{a}_{91}$
h. $\bar{a}_{91}$
i. $Var[Y]$ if Y is the present value random variable for a continuous whole life annuity on (91).
j. $(I\ddot{a})_{91}$
k. $(I\ddot{a})_{91:\overline{3}|}$

2. Using the Standard Ultimate Life Table with $i = 0.05$, calculate:

   a. $\ddot{a}_{60}$
   b. a_{60}
   c. ${}_{10|}\ddot{a}_{60}$
   d. $\ddot{a}_{60:\overline{20|}}$
   e. $\bar{a}_{80}$ *assuming UDD between integer ages*
   f. $\bar{a}_{50:\overline{20|}}$ *assuming UDD between integer ages*
   g. $\ddot{a}_{\overline{60:\overline{10|}}}$
   h. $\ddot{a}_{\overline{60:\overline{13|}}}$
   i. $\ddot{a}_{60}^{(12)}$ assuming UDD between integral ages.
   j. $\ddot{a}_{60}^{(12)}$ estimated using α and β formula.
   k. $\ddot{a}_{60}^{(12)}$ estimated using Woolhouse formula to three terms and estimating μ_x
   l. $Var[Y]$ where Y is the present value random variable for $\ddot{a}_{60}$.
   m. $Var[Y]$ where Y is the present value random variable for $\ddot{a}_{60:\overline{20|}}$.

3. You are given that a continuous whole life annuity to (50) pays at a rate of 100 per year for the first 20 years and 500 per year thereafter. You are given that deaths are uniformly distributed between integral ages. Calculate the actuarial present value if mortality follows the Standard Ultimate Life Table with $i = 0.05$.

4. Peter retires from Purdue at age 60. He receives a monthly pension of 800 per month. The payments are guaranteed for 5 years.

   You are given:
   a. Mortality follows the Standard Ultimate Life Table.
   b. Deaths are uniformly distributed between integral ages.
   c. $i = 5\%$

   Calculate the present value of Peter's retirement benefit.

5. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

If $i = 0.06$, calculate:

a. $\ddot{a}_{[54]:\overline{3}|}$

b. $a_{[54]:\overline{3}|}$

6. Mortality follows the Standard Ultimate Life Table except for age 90. For age 90, $q_{90} = 0.15$. Calculate $\ddot{a}_{90}$ at $i = 0.05$.

7. Problem 5.3 in the book.

8. Problem 5.4 in the book.

9. Problem 5.8 in the book.

10. You are given:

    i. $a_x = 9$

    ii. $A_x = 0.6$

    iii. Deaths are uniformly distributed between integral ages.

    Calculate $1000\bar{A}_x$.

11. You are given:

    i. $\ddot{a}_{\overline{x:\overline{n}|}} = 22.9$
    ii. $\ddot{a}_{x:\overline{n}|} = 8$
    iii. $\ddot{a}_x = 20$
    iv. $i = 0.05$

    Calculate n.

12. Your boss has asked you to use the Standard Ultimate Life Table with interest at 5% to estimate $100{,}000\bar{A}_{85}$.
    d. Determine an estimate assuming UDD.
    e. Determine an estimate using Woolhouse's formula with three terms. Use the values of p_x to estimate μ_x.

13. * For a special 3-year temporary life annuity-due on (x), you are given:

    i. $i = 0.06$
    ii.

t	Annuity Payment	p_{x+t}
0	15	0.95
1	20	0.90
2	25	0.85

    Calculate the variance of the present value random variable for this annuity.

14. A life annuity due payable to (60) pays annual payments of 1000.

    You are given:
    i. Mortality follows the Standard Ultimate Life Table.
    ii. $i = 5\%$
    iii. Y is the present value random variable for this annuity.

    Calculate the probability that Y is greater than the expected value of Y plus one half the standard deviation of Y.

15. A life annuity due payable to (40) pays monthly payments of 100.

You are given:

i. Mortality follows the Standard Ultimate Life Table.
ii. $i = 5\%$
iii. Deaths are uniformly distributed between integral ages.
iv. Y is the present value random variable for this annuity.

Calculate the probability that Y is greater than 12,000.

Answers

1.
    a. 2.42638
    b. 1.42638
    c. 2.21302
    d. 0.96334
    e. 3.09945
    f. 0.53197
    g. 0.65715
    h. 1.9200
    i. 1.03247
    j. 4.72326
    k. 3.86982
2.
    a. 14.9041
    b. 13.9041
    c. 6.94848
    d. 12.38164
    e. 8.04192
    f. 12.51409
    g. 15.05630
    h. 15.1745
    i. 14.4409
    j. 14.4406
    k. 14.4415
    l. 10.6182
    m. 3.84192
3. 3254.20
4. 139,022.11
5.
    a. 2.71307
    b. 2.41688
6. 4.9552
7. 4.0014%
8. 0.98220

9.
   a. 3.66643
   b. 3.45057
   c. 8.37502
   d. 3.16305
   e. 119.14
   f. 0.00421
10. 612.415
11. 15
12.
   a. 69,299.03
   b. 69,308.25
13. 114.42
14. 0.3893
15. 0.9873