Chapter 6

1. A whole life policy for 50,000 is issued to (65). The death benefit is payable at the end of the year of death. The level premiums are payable for the life of the insured.

   You are given:
   a. Mortality follows the Standard Ultimate Life Table.
   b. $i = 5\%$.
   c. Deaths are uniformly distributed between integer ages.
   d. The equivalence principle applies.

   For this life insurance:

   a. Calculate the level annual net premium payable at the beginning of each year.
   Solution:

$$PVP = PVB$$
$$P\ddot{a}_{65} = 50,000A_{65}$$
$$P = \frac{50,000(0.35477)}{13.5498} = 1309.13$$

   b. Write an expression for the loss at issue random variable L_0^n
   Solution:

$$L_0^n = 50,000v^{(K_x+1)} - 1309.13\ddot{a}_{\overline{K_x+1|}}$$

   c. Calculate the $Var[L_0^n]$.
   Solution:

$$Var[L_0^n] = \left(S + \frac{P}{d}\right)^2 ({}^2A_x - (A_x)^2)$$
$$= \left(50,000 + \frac{1309.13}{(0.05/1.05)}\right)^2 \left(0.15420 - (0.35477)^2\right) = 170,170,273$$

d. Calculate the monthly net premium payable at the beginning of each month.

Solution:

$$PVP = PVB ==> P\ddot{a}_{65}^{(12)} = 50,000A_{65}$$

$$P = \frac{50,000(0.35477)}{12\left(\alpha(12)\ddot{a}_{65} - \beta(12)\right)} = \frac{50,000(0.35477)}{12\left((1.00020)(13.5498) - 0.46651\right)} = 112.96$$

2. A whole life policy for 50,000 is issued to (75). The death benefit is payable at the moment of death. The premiums are payable continuously for the life of the insured.

You are given:

a. Mortality follows the Standard Ultimate Life Table.
b. $i = 5\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

For this life insurance:

a. Calculate the net level premium payable continuously.

Solution:

$$PVP = PVB ==> P\bar{a}_{75} = 50,000\bar{A}_{75}$$

$$P = \frac{50,000\bar{A}_{75}}{\bar{a}_{75}} = \frac{50,000\bar{A}_{75}}{\left(\frac{1-\bar{A}_{75}}{\delta}\right)} = \frac{50,000\left(\frac{i}{\delta}\right)A_{75}(\delta)}{1-\left(\frac{i}{\delta}\right)A_{75}} = \frac{50,000(0.05)(0.50868)}{1-1.02480(0.50868)} = 2656.54$$

b. Write an expression for the loss at issue random variable L_0^n

Solution:

$$L_0^n = 50,000v^{(T_x)} - 2656.54\bar{a}_{\overline{T_x}|}$$

c. Calculate the $Var[L_0^n]$.

Solution:

$$Var[L_0^n] = \left(S + \frac{P}{\delta}\right)^2 \left({}^2\bar{A}_{75} - \left(\bar{A}_{75}\right)^2\right)$$

$$= \left(50,000 + \frac{2656.54}{\ln(1.05)}\right)^2 \left(\left(\frac{1.05^2 - 1}{2\ln(1.05)}\right)(0.29079) - \left(1.02480\right)^2 \left(0.50868\right)^2\right) = 367,668,895$$

3. A 20 year endowment policy for 25,000 is issued to (40). The death benefit is payable at the end of the year of death. The level premiums are payable for the life of the insured during the term of the policy.

   You are given:

   a. Mortality follows the Standard Ultimate Life Table.
   b. $i = 5\%$.
   c. Deaths are uniformly distributed between integer ages.
   d. The equivalence principle applies.

   For this endowment insurance:

   a. Calculate the level annual net premium payable at the beginning of each year.

   Solution:

$$PVP = PVB ==> P\ddot{a}_{40:\overline{20}|} = 25,000A_{40:\overline{20}|}$$

$$P = \frac{25,000A_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}} = \frac{25,000(0.38126)}{12.9935} = 733.56$$

   b. Write an expression for the loss at issue random variable L_0^n

   Solution:

$$L_0^n = 25,000v^{\min(K_x+1,20)} - 733.56\ddot{a}_{\overline{\min(K_x+1,20)}|}$$

c. Calculate the $Var[L_0^n]$.

Solution:

$$Var = \left(S + \frac{P}{d}\right)^2 \left({}^2A_{40:\overline{20|}} - \left(A_{40:\overline{20|}}\right)^2 \right)$$

$$= \left(25,000 + \frac{733.56}{(0.05/1.05)}\right)^2 \bullet \left({}^2A_{40} - v^{20}{}_{20}E_{40}\ {}^2A_{60} + v^{20}{}_{20}E_{40} - \left(A_{40:\overline{20|}}\right)^2 \right)$$

$$= 1,632,544,631\left(0.14668 - \left(0.38126\right)^2\right) = 2,156,285$$

d. Calculate the monthly net premium payable at the beginning of each month.
Solution:

$$PVP = PVB ==> 12P\ddot{a}^{(12)}_{40:\overline{20|}} = 25,000A_{40:\overline{20|}}$$

$$P = \frac{25,000(0.38126)}{12\left(\ddot{a}^{(12)}_{40} - {}_{20}E_{40} \cdot \ddot{a}^{(12)}_{60}\right)} = \frac{25,000(0.38126)}{12\left(\alpha(12)\ddot{a}_{40} - \beta(12) - {}_{20}E_{40} \cdot \left[\alpha(12)\ddot{a}_{60} - \beta(12)\right]\right)}$$

$$P = \frac{25,000(0.38126)}{12\left(1.00020(18.4578) - 0.46651 - 0.36663\left[1.00020(14.9041) - 0.46651\right]\right)} = 62.54$$

4. Tiannan buys a Term to Age 65. Tiannan is age 35. The term policy pays a death benefit of 500,000 immediately upon Tiannan's death. Level premiums are payable for 15 years.

You are given:

a. Mortality follows the Standard Ultimate Life Table.
b. $i = 5\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

Calculate the monthly net premium payable at the beginning of each month.

Solution:

$$PVP = PVB ==> 12P\ddot{a}^{(12)}_{35:\overline{15}|} = 500,000\bar{A}^{\,1}_{35:\overline{30}|}$$

$$P = \frac{500,000(i/\delta)(A_{35} - {}_{30}E_{35}A_{65})}{12(\alpha(12)\ddot{a}_{35} - \beta(12) - {}_{15}E_{35}[\alpha(12)\ddot{a}_{50} - \beta(12)])} =$$

$$\frac{500,000(1.02480)(0.09653 - (0.37041)(0.59342)(0.35477))}{12((1.00020)(18.9728) - 0.46651) - (0.61069)(0.77991)[(1.00020)(17.0245) - 0.46651])}$$

$$= \frac{9504.234}{127.466} = 74.56$$

5. Brittany age 25 purchases an annuity due that a monthly benefit of 1000 for as long she lives with the first payment made today.

   You are given:

   a. Mortality follows the Standard Ultimate Life Table.
   b. $i = 5\%$.
   c. Deaths are uniformly distributed between integer ages.
   d. The equivalence principle applies.

   Calculate the net single premium that Brittany would pay to purchase this annuity.

   Solution:

$$PVP = PVB ==> P = 1000(12)(\ddot{a}_{25}^{(12)}) = 1000(12)(\alpha(12)\ddot{a}_{25} - \beta(12))$$

$$(12,000)\left[(1.00020)(19.7090) - 0.46651\right] = 230,957$$

6. Alex, age 20, purchases a deferred life annuity. The life annuity will pay an annual benefit of 100,000 beginning at age 65. Alex will pay a level annual net premium of P for the next 10 years to pay for this annuity.

You are given:

a. Mortality follows the Standard Ultimate Life Table.
b. $i = 5\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

Calculate P.

Solution:

$$PVP = PVB ==> P\ddot{a}_{20:\overline{10}|} = 100,000\,{}_{45|}\ddot{a}_{20}$$

$$P = \frac{100,000\,{}_{45}E_{20}\ddot{a}_{65}}{\ddot{a}_{20} - {}_{10}E_{20}\ddot{a}_{30}} = \frac{100,000v^{45}\left(\frac{l_{65}}{l_{20}}\right)(13.5498)}{19.9664 - (0.61224)(19.3834)} = 17,610.64$$

7. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

Assume that deaths are uniformly distributed between integral ages and that the equivalence principle applies. Calculate at $i = 4\%$:

a. The level annual premium for a whole life of 5000 to (90). The death benefit is payable at the end of the year of death and the premium is payable for life.
Solution:

$$PVP = PVB$$
$$P(1000 + 900v + 720v^2 + 432v^3 + 216v^4)$$
$$= 5000(100v + 180v^2 + 288v^3 + 216v^4 + 216v^5)$$
$$P = \frac{4,403,894.381}{3099.749221} = 1420.73$$

b. The variance of the loss at issue random variable for the insurance in a.
Solution:

$$\left(5000 + \frac{1420.73}{d}\right)^2 \left({}^2A_{90} - (A_{90})^2\right)$$
$$A_{90} = 0.8807788762$$
$${}^2A_{90} = \frac{100v^2 + 180v^4 + 288v^6 + 216v^8 + 216v^{10}}{1000}$$
$${}^2A_{90} = 0.777681904$$
$$Var = 3,360,293$$

c. The monthly premium for a whole life of 5000 to (90). The death benefit is payable at the moment of death and the premium is payable for two years during the insured's lifetime.
Solution:

$$PVP = PVB$$

$$12P\ddot{a}^{(12)}_{90:\overline{2}|} = 5000\bar{A}_{90}$$

$$P = \frac{5000\left(\frac{.04}{\ln(1.04)}\right)(.8807788762)}{12\left(\frac{1-\frac{0.04}{12[1.04^{1/12}-1]}\left(\frac{100v+180v^2}{1000}\right)-\frac{720}{1000}v^2}{12[1-(1.04)^{-1/12}]}\right)}$$

$$P = \frac{4491.396537}{12(1.710200869)} = 218.85$$

8. *Matthew and Lingxiao each purchase a fully discrete 3-year term insurance of 100,000. Matthew and Lingxiao are each 21 years old at the time of purchase.

You are given:

i. The symbol μ_{21+t} is the force of mortality consistent with the Standard Ultimate Life Table for $t \geq 0$.

ii. Lingxiao is a standard life and her mortality follows the Standard Ultimate Life Table.

iii. Matthew is a substandard life and has a force of mortality equal to μ^*_{21+t} where $\mu^*_{21+t} = \mu_{21+t} + 0.05$.

iv. $i = 5\%$

Calculate the difference between the annual benefit premium for Matthew and the annual benefit premium for Lingxiao.

Solution:

For both Matthew and Lingxiao (with different p's and q's):

$$PVP = PVB$$

$$P\left(1 + vp_{21} + v^2\,{}_2p_{21}\right) = 100{,}000\left(vq_{21} + v^2 p_{21}q_{22} + v^3\,{}_2p_{21}q_{23}\right)$$

$$P = \frac{100{,}000\left(vq_{21} + v^2 p_{21}q_{22} + v^3\,{}_2p_{21}q_{23}\right)}{1 + vp_{21} + v^2\,{}_2p_{21}}$$

For Lingxiao, we use values straight out of our table:

$$P = \frac{100{,}000\left[\left(\frac{1}{1.05}\right)(0.000253) + \left(\frac{1}{1.05}\right)^2(1-0.000253)(0.000257) + \left(\frac{1}{1.05}\right)^3(1-0.000253)(1-0.000257)(0.000262)\right]}{1 + \left(\frac{1}{1.05}\right)(1-0.000253) + \left(\frac{1}{1.05}\right)^2(1-0.000253)(1-0.000257)}$$

$$= \frac{70.021}{2.85871} = 24.49$$

For Matthew, the mortality is different:

$$ {}_1p_{21} = (p_{21})(e^{-0.05}) = (1-0.000253)e^{-0.05} = 0.95099 $$
$$ {}_2p_{21} = ({}_2p_{21})(e^{-0.05(2)}) = (1-0.000253)(1-0.000257)e^{-0.10} = 0.90438 $$
$$ {}_3p_{21} = ({}_3p_{21})(e^{-0.05(3)}) = (1-0.000253)(1-0.000257)(1-0.000262)e^{-0.15} = 0.86004 $$
$$ q_{21} = 1 - p_{21} = 0.04901 $$
$$ {}_2p_{21} = (p_{21})(p_{22}) = (p_{21})(1-q_{22}) $$
$$ \therefore q_{22} = 1 - \frac{{}_2p_{21}}{p_{21}} = 1 - \frac{0.90438}{0.95099} = 0.04901 $$
$$ {}_3p_{21} = ({}_2p_{21})(p_{23}) = ({}_2p_{21})(1-q_{23}) $$
$$ \therefore q_{23} = 1 - \frac{{}_3p_{21}}{{}_2p_{21}} = 1 - \frac{0.86004}{0.90438} = 0.04903 $$
$$ P = \frac{100{,}000\left[v(0.04901) + v^2(0.95099)(0.04901) + v^3(0.90438)(.04903)\right]}{\left(1 + v(0.95099) + v^2(0.90438)\right)} = \frac{12{,}725.51105}{2.72600} = 4668.19 $$

$$ P_{Matthew} - P_{Lingxiao} = 4668.19 - 24.49 = 4643.70 $$

9. *Amy who is 25 years old purchases a 3-year term insurance with a death benefit of 25,000.

You are given that mortality follows the select and ultimate mortality table below

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
25	1100	1060	1000	27
26	1020	970	900	28
27	940	880	800	29

You are also given:

i. The death benefit is payable at the end of the year of death.
ii. Level premiums are payable at the beginning of each quarter.
iii. Deaths are uniformly distributed over each year of age.
iv. $i = 6\%$

Calculate the amount of each quarterly benefit premium.

Solution:

$$PVP = PVB$$

$$PVP = P\ddot{a}^{(4)}_{[25]:\overline{3}|}$$

$$PVB = 25,000 A^{\;1}_{[25]:\overline{3}|}$$

$$l_{[25]} A^{\;1}_{[25]:\overline{3}|} = vd_{[25]} + v^2 d_{[25]+1} + v^3 d_{[25]+2}$$

$$1100 A^{\;1}_{[25]:\overline{3}|} = \left(\frac{1}{1.06}\right)(40) + \left(\frac{1}{1.06}\right)^2 (60) + \left(\frac{1}{1.06}\right)^3 (100)$$

$$A^{\;1}_{[25]:\overline{3}|} = \frac{175.09756}{1100} = 0.1591796$$

$$\ddot{a}^{(4)}_{[25]:\overline{3}|} = \ddot{a}_{[25]:\overline{3}|}\alpha(4) - \beta(4)(1 - {}_3E_{[25]})$$

$${}_3E_{[25]} = v^3 {}_3p_{[25]} = \left(\frac{1}{1.06}\right)^3 \left(\frac{900}{1100}\right) = 0.68696$$

$$\ddot{a}_{[25]:\overline{3}|} = 1 + vp_{[25]} + v^2 {}_2p_{[25]}$$

$$= 1 + \frac{1}{1.06}\left(\frac{1060}{1100}\right) + \left(\frac{1}{1.06}\right)^2 \left(\frac{1000}{1100}\right)$$

$$= 2.71818$$

$$\ddot{a}^{(4)}_{[25]:\overline{3}|} = (2.71818)(1.00027) - 0.38424(1 - 0.68696)$$

$$= 2.59863$$

$$P = \frac{25,000(0.1591796)}{2.59863} = 1531.38$$

$$Quarterly = \frac{1531.38}{4} = 382.84$$

10. Emily, (40), purchases a whole life policy. The policy pays a death benefit of 50,000 at the end of the year of death if Emily dies prior to age 65. It pays a death benefit of 25,000 at the end of the year of death if Emily dies after age 65.

Additionally, the policy pays a pure endowment of 25,000 if Emily survives to age 65.

Emily will pay annual benefit premiums for this policy. The annual benefit premium during the first 10 years is P. The annual benefit premium thereafter is $2P$.

You are given that mortality follows the Standard Ultimate Life Table with $i = 5\%$.

Calculate P.

Solution:

$$PVB = PVP$$

$$PVB = 50,000 A_{40} - 25,000 \, {}_{25}E_{40} A_{65} + 25,000 \, {}_{25}E_{40}$$
$$= 50,000(0.12106) - 25,000(0.36663)(0.76687)(0.35477) + 25,000(0.36663)(0.76687)$$
$$= 10,588.28$$

$$PVP = P\left(\ddot{a}_{40}\right) + P\left({}_{10}E_{40}\ddot{a}_{50}\right)$$
$$= P(18.4578 + 0.60920(17.0245)) = P(28.8291)$$

$$P = \frac{10,588.28}{28.8291} = 367.28$$

11. A whole life policy on (60) pays a death benefit of 40,000 at the moment of death. Premiums are paid annually for as long as the insured lives.

    You are given:

    a. Mortality follows the Standard Ultimate Life Table.
    b. $i = 0.05$
    c. Commissions are 80% of premiums in the first year and 5% of premiums thereafter.
    d. The issue expenses at time zero are 300 per policy.
    e. The renewal expense at the beginning of each year beginning with the second year is 25.

    i. Calculate the gross premium for this policy using the equivalence principle.

    Solution:

$$PVP = PVB + PVE ==> P\ddot{a}_{60} = 40,000\bar{A}_{60} + .75P + .05P\ddot{a}_{60} + 275 + 25\ddot{a}_{60}$$

$$P = \frac{40,000\left(\frac{i}{\delta}\right)A_{60} + 275 + 25\ddot{a}_{60}}{0.95\ddot{a}_{60} - 0.75} = \frac{40,000(1.02480)(0.29028) + 275 + (25)(14.9041)}{(0.95)(14.9041) - 0.75}$$

$$= \frac{12,546.76026}{13.40890} = 935.70$$

    ii. Write an expression for L_0^g for this policy

    Solution:

$$L_0^g = 40,000v^{T_x} + 0.75(935.70) + 0.05(935.70)\ddot{a}_{\overline{K_x+1}|} + 275 + 25\ddot{a}_{\overline{K_x+1}|} - 935.70\ddot{a}_{\overline{K_x+1}|}$$

$$= 40,000v^{T_x} + 701.775 + 46.785\ddot{a}_{\overline{K_x+1}|} + 275 + 25\ddot{a}_{\overline{K_x+1}|} - 935.70\ddot{a}_{\overline{K_x+1}|}$$

$$= 40,000v^{T_x} + 976.775 - 863.915\ddot{a}_{\overline{K_x+1}|}$$

12. A whole life policy on (80) pays a death benefit of 10,000 at the end of the year of death. Premiums are paid annually for as long as the insured lives.

You are given:

a. Mortality follows the Standard Ultimate Life Table.
b. $i = 0.05$
c. Commissions are c% of premiums in the first year and 5% of premiums thereafter.
d. The issue expenses at time zero are 300 per policy.
e. The renewal expense at the beginning of each year beginning with the second year is 25.
f. The gross premium for this policy using the equivalence principle is 1279.21.

Calculate c.

Solution:

$$PVP = PVB + PVE ==> P\ddot{a}_{80} = 10,000A_{80} + 0.05P\ddot{a}_{80} + (c - 0.05)P + 275 + 25\ddot{a}_{80}$$

$$(797.67)(8.5484) = 10,000(0.59293) + 0.05(797.67)(8.5484)$$
$$+(c - 0.05)(797.67) + 275 + 25(8.5484)$$

$$c = \frac{99.736}{797.67} = 12.5\%$$

13. Lalani Life Insurance Company sells a 2 year term policy to Jackson who is (x). The death benefit in the first year is 10,000 and in the second year is 6000. The death benefit is paid at the end of the year of death.

Annual net premiums are paid for two years.

Let L_0^n be the loss at issue random variable.

You are given that:

- $v = 0.9$
- $q_{x+t} = 0.08 + 0.04t \qquad \text{for } t = 0,1,2,3,4,5$

a. Calculate the net premium based on the equivalence principle.

$PVB = PVP$

set $l_x = 1{,}000$

$l_{x+1} = 1{,}000(1-0.08) = 920$

$l_{x+2} = 920(1-0.12) = 809.60$

$(1{,}000-920)(10{,}000v) + (920-809.60)(6{,}000v^2) = P(1{,}000+920v)$

$1{,}256{,}544 = 1{,}828P$

$P = 687.39$

b. Calculate $E[L_0^n]$.

net premium based on the equivalence principle, therefore $E[L_0^n] = 0$

c. Calculate $Var[L_0^n]$.

Case	L_0^n	Probability
Die in Year 1	$10{,}000v - 687.39 = 8{,}312.61$	0.08
Die in Year 2	$6{,}000v^2 - 687.39(1+v) = 3{,}553.96$	$(0.92)(0.12) = 0.1104$
Live 2 Years	$-687.39(1+v) = -1{,}306.041$	$(0.92)(0.88) = 0.8096$

$Var(L_0^n) = E[(L_0^n)^2] - [E(L_0^n)]^2$

$E(L_0^n) = 0$

$E[(L_0^n)^2] = (.08)(8{,}312.61)^2 + (.1104)(3{,}553.96)^2 + (.8096)(-1{,}306.041)^2 = 8{,}303{,}348$

$Var(L_0^n) = 8{,}303{,}348 - (0)^2 = 8{,}303{,}348$

14. Cong Actuarial Consulting provides a life insurance benefit to Candace who is an consultant age 40. If Candace dies after age 60, a death benefit of 100,000 will be paid at the end of the year of death.

Cong will pay level gross premiums for 20 years during the deferral period. No premiums are payable after 20 years.

You are given:

i. Mortality follows the Standard Ultimate Life Table.
ii. $i = 0.05$.
iii. The gross premium is 125% of the annual benefit premium. The annual benefit premium is the net premium calculated using benefits only and the equivalence principle.
iv. Commissions are 25% in the first year and 5% thereafter. No commissions are paid after the premiums stop.
v. There is a per policy expense of 110 in the first year and 50 each year thereafter. This expense does not stop when the premiums stop.
vi. L_0 is the present value of future losses at issue random variable.

Calculate $E[L_0]$.

Solution:

First, find the annual benefit premium

$$PVP = PVB ==> P(\ddot{a}_{40} - {}_{20}E_{40}\ddot{a}_{60}) = 100,000\,{}_{20}E_{40}A_{60}$$

$$P = \frac{100,000(0.36663)(0.29028)}{18.4578 - (0.36663)(14.9041)} = 819.07$$

$$Gross = 1.25P = 1.25(819.07) = 1023.83$$

$$E\left[L_0\right] = PVB + PVE - PV(GrossP)$$

$$= PVB + PVE - PV(1.25BenefitP)$$

$$= PVE - 0.25PV(BenefitP)$$

$$E\left[L_0\right] = 0.2(1023.83) + 0.05(1023.83)(18.4578 - (0.36663)(14.9041))$$

$$+60 + 50(18.4578) - 0.25(819.07)(18.4578 - (0.36663)(14.9041))$$

$$= -807.84$$

15. A 20 year term insurance policy is issued to (70) with a death benefit of 1,000,000 payable at the end of the year of death. Premiums are paid annually during the term of the policy.

You are given:

a. Mortality follows the Standard Ultimate Life Table.
b. $i = 0.05$
c. Commissions are 50% of premiums in the first year and 7% of premiums thereafter.
d. The issue expenses at time zero are 1000 per policy plus 1 per 1000 of death benefit.
e. The renewal expense at the beginning of each year including the first year is 40.
f. A termination expense of 500 is incurred at the end of the year of death.

Calculate the gross premium for this policy using the equivalence principle.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{70:\overline{20|}} = 1,000,000 A^{\;1}_{70:\overline{20|}} + 0.07 P\ddot{a}_{70:\overline{20|}} + .43P + 1000 + (1)(1000) + 40\ddot{a}_{70:\overline{20|}} + 500 A^{\;1}_{70:\overline{20|}}$$

$$P = \frac{1,000,500 A^{\;1}_{70:\overline{20|}} + 2000 + 40\ddot{a}_{70:\overline{20|}}}{0.93\ddot{a}_{70:\overline{20|}} - 0.43}$$

$$A^{\;1}_{70:\overline{20|}} = A_{70:\overline{20|}} - {}_{20}E_{70} = 0.47091 - 0.17313 = 0.29778$$

$$P = \frac{1,000,500(0.29778) + 2000 + 40(11.1109)}{0.93(11.1109) - 0.43} = 30,331.13$$

16. A special 30 year term policy on (35) provides a death benefit that is paid at the end of the year of death. The death benefit is 300,000 for death during the first 10 years of the policy. The death benefit is 200,000 if the insured dies after 10 years, but before 20 years. The death benefit is 100,000 if the insured dies during the last 10 years of the policy.

    Gross premiums are payable annually for the term of the policy. The annual gross premium is $3G$ during the first 10 years, $2G$ during the second 10 years, and G during the last 10 years.

    You are given:

    i. Mortality follows the Standard Ultimate Life Table.
    ii. $i = 0.05$
    iii. Commissions are 50% of the premium in the first year and 5% thereafter.
    iv. Maintenance expenses are 50 per year payable at the start of every year.
    v. The issue expense is 400 payable at issue.
    vi. The gross premium is determined using the equivalence principle.

    Determine G.

    Solution:

$$PVB + PVE = PVP$$

$$\begin{aligned} &300,000A_{35} - 100,000\,{}_{10}E_{35}A_{45} - 100,000\,{}_{20}E_{35}A_{55} - 100,000\,{}_{30}E_{35}A_{65} \\ &+0.45(3G) + 50(\ddot{a}_{35} - {}_{30}E_{35}\ddot{a}_{65}) + 400 \\ &= (0.95)(3G\ddot{a}_{35} - G\,{}_{10}E_{35}\ddot{a}_{45} - G\,{}_{20}E_{35}\ddot{a}_{55} - G\,{}_{30}E_{35}\ddot{a}_{65}) \end{aligned}$$

$$\begin{aligned} &100,000\big[3(0.09653) - (0.61069)(0.15161) - (0.37041)(0.23524) - (0.37041)(0.59342)(0.35477)\big] \\ &+50\big[18.9728 - (0.37041)(0.59342)(13.5498)\big] + 400 \\ &= G\left[0.95\begin{bmatrix} 3(18.9728) - (0.61069)(17.8162) - (0.37041)(16.0599) \\ -(0.37041)(0.59342)(13.5498) \end{bmatrix} - 0.45(3)\right] \end{aligned}$$

$$G = \frac{4388.3725}{33.905558} = 129.43$$

17. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a whole life to (91) with a death benefit of 10,000 payable at the end of the year of death and level annual premiums, the expenses are 200 per policy at issue and 40 per policy at the beginning of each year including the first year.

You are given that $i = 4\%$.

a. Calculate the level gross premium using the equivalence principle.
Solution:

$$PVP = PVB + PVE$$

$$P(900 + 720v + 432v^2 + 216v^3)$$

$$= 10,000(180v + 288v^2 + 216v^3 + 216v^4) + 900(200) + 40(900 + 720v + 432v^2 + 216v^3)$$

$$P = \frac{10,000(816.0100312) + 180,000 + 40(2183.73919)}{2183.73919}$$

$$P = 3859.18333$$

b. Complete the following table:
Solution:

$$L_0^g = 10,000v^{K_x+1} + 200 + 40\ddot{a}_{\overline{K_x+1|}} - 3859.18\ddot{a}_{\overline{K_x+1|}}$$

	L_0^g	**Prob**
K_x =0	5996.20	180/900
K_x =1	1954.09	288/900
K_x =2	-1932.55	216/900
K_x =3	-5669.71	216/900

c. The variance of the loss at issue random variable.
Solution:

$$Var(L) = E(L^2) - (E(L))^2$$
$$= E(L^2) - (0)^2 = E(L^2)$$
$$E(L)^2 = (5996.20)^2\left(\frac{180}{900}\right) + (1954.09)^2\left(\frac{288}{900}\right) + (-1932.55)^2\left(\frac{216}{900}\right) + (-5669.71)^2\left(\frac{216}{900}\right)$$
$$= 17,024,079.2$$

d. Calculate the expected value and the variance of the loss at issue random variable if the gross premium was 4000.

Solution:

$$L_0^g = 10,000v^{K_x+1} + 200 + 40\ddot{a}_{\overline{K_x+1}|} - 4000\ddot{a}_{\overline{K_x+1}|}$$

	L_0^g	**Prob**
K_x=1	5855.38	180/900
K_x=2	1677.87	288/900
K_x=3	-2338.97	216/900
K_x=4	-6201.32	216/900

$$E\left[L_0^g\right] = -341.68$$
$$E\left[\left(L_0^g\right)^2\right] = 18,300,490 - (341.68)^2 = 18,183,745$$