Chapter 7

1. You are given that Mortality follows the Standard Ultimate Life Table with $i = 5\%$. Assume that mortality is uniformly distributed between integral ages. Calculate:

   a. Calculate ${}_{10}V^n$ for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for the life of the insured.
   Solution:

$$P = \frac{100,000A_{60}}{\ddot{a}_{60}} = \frac{100,000(0.29028)}{14.9041} = 1947.65$$

$${}_{10}V = PVFB - PBFP$$

$$= 100,000A_{70} - 1947.65\ddot{a}_{70}$$

$$= 42,818 - 23,387.97 = 19,430.03$$

   b. Calculate ${}_{10}V^n$ for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for 20 years during the life of the insured.
   Solution:

$$P = \frac{250,000A^{\,1}_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}} = \frac{250,000\left(A_{40:\overline{20}|} - {}_{20}E_{40}\right)}{\ddot{a}_{40:\overline{20}|}} = \frac{250,000(0.38126 - 0.36663)}{12.9935} = 281.49$$

$${}_{10}V = PVFB - PVFP = 250,000A^{\,1}_{50:\overline{10}|} - 281.49\ddot{a}_{50:\overline{10}|}$$

$$= 250,000\left(0.61643 - 0.60182\right) - 281.49(8.0550) = 1385.10$$

c. Calculate ${}_5V^n$ and ${}_{10}V^n$ for an Endowment to 65 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual premiums payable for 10 years during the life of the insured.

Solution:

$$PVP = PVB ==> P\ddot{a}_{35:\overline{10}|} = 10,000A_{35:\overline{30}|}$$

$$P = \frac{10,000\left(A_{35} - {}_{30}E_{35} \cdot A_{65} + {}_{30}E_{35}\right)}{\ddot{a}_{35:\overline{10}|}}$$

$$= \frac{10,000\big(0.09653 - (0.37041)(0.59342)(0.35477 - 1)\big)}{8.0926} = 294.54$$

$${}_5V = 10,000A_{40:\overline{25}|} - 294.54\ddot{a}_{40:\overline{5}|}$$

$$= 10,000\big(0.12106 - (0.36663)(0.76687)(0.35477 - 1)\big)$$
$$- 294.54\big(18.4578 - (0.78113)(17.8162)\big) = 1687.20$$

$${}_{10}V = 10,000A_{45:\overline{20}|} - 0 = 10,000(0.38385) = 3838.50$$

*Note, for ${}_{10}V$ there are no more premiums, so only the PVFB is calculated

d. Calculate ${}_{10}V^n$ for an annuity issued to (65). The annuity has level annual payments during the life of the annuitant of 1000. The annuity was purchased for a single premium. Calculate ${}_{10}V^n$ both immediately before the annuity payment at age 75 and immediately after the annuity payment at 75.

Solution:

${}_{10}V = 1000\ddot{a}_{75} = 10,317.8$ before the payment

$10,317.80 - 1000 = 9317.80$ after the payment

e. Calculate ${}_{10}V^{n}$ for a whole life policy issued to (60). The death benefit is 50,000 and is payable at the moment of death. The insurance has level annual premiums payable for the life of the insured.
Solution:

$$PVP = PVB ==> P\ddot{a}_{60} = 50,000\bar{A}_{60}$$

$$P = \frac{50,000(1.02480)(0.29028)}{14.9041} = 997.98$$

$${}_{10}V = PVFB - PVFP = 50,000\bar{A}_{70} - 997.98\ddot{a}_{70}$$

$$= 50,000(1.02480)(0.42818) - 997.98(12.0083) = 9955.90$$

f. Calculate ${}_{10}V^{n}$ for a whole life policy issued to (60). The death benefit is 50,000 and is payable at the moment of death. The insurance has level monthly premiums payable for the life of the insured.
Solution:

First find net premium. Let P be the monthly premium

$$12P\ddot{a}_{60}^{(12)} = 50,000\bar{A}_{60}$$

$$12P = \frac{50,000(1.02480)(0.29028)}{1.00020(14.9041) - 0.46651} = 1030.01$$

$${}_{10}V = 50,000\bar{A}_{70} - 12P\ddot{a}_{70}^{(12)}$$

$$= 50,000(1.02480)(0.42818) - (1030.01)(1.00020(12.0083) - 0.46651) = 10,049.31$$

g. Calculate ${}_{10}V^{FPT}$ (the modified premium reserve using the Full Preliminary Term method) for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for the life of the insured.

Solution:

$$
\begin{aligned}
{}_{10}V^{FPT} &= PVB - PVP_{x+1} \\
&= 100,000\left(A_{70} - \frac{A_{61}}{\ddot{a}_{61}}\ddot{a}_{70} \right) =
\end{aligned}
$$

$$
100,000(0.42818) - \frac{0.30243(100,000)}{14.6491}(12.0083) = 18,026.92
$$

h. Calculate ${}_{10}V^{FPT}$ (the modified premium reserve using the Full Preliminary Term method) for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for 20 years during the life of the insured.

Solution:

$$
\begin{aligned}
{}_{10}V^{FPT} &= PVB - PVP_{X+1} \\
&= 250,000\left(A_{50:\overline{10}|} - {}_{10}E_{50} \right) - 250,000\left(\frac{A_{41} - {}_{19}E_{41}\cdot A_{60}}{\ddot{a}_{41} - {}_{19}E_{41}\cdot \ddot{a}_{60}} \right)\left(\ddot{a}_{50:\overline{10}|} \right) \\
&= 250,000(0.61643 - 0.60182) \\
&\quad - 250,000\left(\frac{0.12665 - 1.05^{-19}\left(\frac{96,634.1}{99,285.9} \right)(0.29028)}{18.3403 - 1.05^{-19}\left(\frac{96,634.1}{99,285.9} \right)(14.9041)} \right)(8.0550)
\end{aligned}
$$

$$
= 3652.5 - (294.54)(8.0550) = 1279.98
$$

i. Calculate ${}_{5}V^{FPT}$ and ${}_{10}V^{FPT}$ (the modified premium reserves using the Full Preliminary Term method) for an Endowment to 65 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual premiums payable for 10 years during the life of the insured.

Solution:

$$ {}_{5}V^{FPT} = PVB - PVP_{x+1} $$

$$ P_{x+1} = (10,000)\left[\frac{A_{36} - {}_{29}E_{36} \cdot A_{65} + {}_{29}E_{36}}{\ddot{a}_{36} - {}_{9}E_{36} \cdot \ddot{a}_{45}}\right] $$

$$ = 10,000\left[\frac{0.10101-(1.05)^{-29}\left(\dfrac{94,579.7}{99,517.8}\right)(0.35477-1)}{18.8788-(1.05)^{-9}\left(\dfrac{99,033.9}{99,517.8}\right)(17.8162)}\right] = 335.55 $$

$$ {}_{5}V^{FPT} = 10,000\left[A_{40} - {}_{25}E_{40}\left(A_{65}-1\right)\right] - 335.55\left(\ddot{a}_{40} - {}_{5}E_{40} \cdot \ddot{a}_{45}\right) $$
$$ = 10,000\left[0.12106-(0.36663)(0.76687)(0.35477-1)\right] $$
$$ -335.55\left[18.4578-(0.78113)(17.8162)\right] $$
$$ = 1500.97 $$

$$ {}_{10}V^{FPT} = PVB $$
$$ = 10,000A_{45:\overline{20|}} = (10,000)(0.38385) = 3838.50 $$

*Note, only calculate PVB because there are no more premiums

2. You are given that Mortality follows the Standard Ultimate Life Table with $i = 5\%$. Assume that mortality is uniformly distributed between integral ages.

   The expenses associated with the following policies is:

   a. Commissions of 50% of premium in the first year and 8% of premium thereafter
   b. 100 per policy at the beginning of the first year
   c. 25 per policy at the beginning of every year including the first
   d. 250 at the time that a claim payment is made or upon the payment of an endowment

   Calculate:

   a. Calculate ${}_{10}V^g$ for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual gross premiums payable for the life of the insured. The gross premium is calculated using the equivalence principle.
   Solution:

$$PVP = PVB + PVE ==> P\ddot{a}_{60} = 100,000A_{60} + 0.42P + 0.08P\ddot{a}_{60} + 100 + 25\ddot{a}_{60} + 250A_{60}$$

$$P = \frac{100,250(0.29028) + 100 + 25(14.9041)}{0.92(14.9041) - 0.42} = 2224.93$$

$${}_{10}V = PVFB + PVFE - PVFP$$
$$= 100,250(A_{70}) + (0.08)(2224.93)\ddot{a}_{70} + 25\ddot{a}_{70} - 2224.93\ddot{a}_{70}$$
$$= 100,250(0.42818) - [(0.92)(2224.93) - 25](12.0083) = 18,645.04$$

   b. Calculate ${}_{0}V^g$ and ${}_{10}V^g$ for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual gross premiums of 400 payable for 20 years during the life of the insured.
   Solution:

$${}_{0}V = PVFB + PVFE - PVFP$$

$$= 250,000A^{1}_{40:\overline{20|}} + (0.42)(400) + (0.08)(400)\ddot{a}_{40:\overline{20|}} + 100 + 25\ddot{a}_{40:\overline{20|}} + 250A^{1}_{40:\overline{20|}} - 400\ddot{a}_{40:\overline{20|}}$$

$$= 250,250(0.38126 - 0.36663) + 168 + 100 - [.92(400) - 25](12.9935) = -527.61$$

$$ {}_{10}V = 250{,}000 A^{\,1}_{50:\overline{10}|} + (0.08)(400)\,\ddot{a}_{50:\overline{10}|} + 25\ddot{a}_{50:\overline{10}|} + 250 A^{\,1}_{50:\overline{10}|} - 400\ddot{a}_{50:\overline{10}|} $$

$$ = 250{,}250(0.61643 - 0.60182) - [.92(400) - 25](8.0550) = 893.29 $$

c. Calculate ${}_5V^g$ and ${}_{10}V^g$ for an Endowment to 55 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual gross premiums payable for 10 years during the life of the insured. The gross premium was determined as $P+50$ where P is determined using the equivalence principle. (Note: The 25 per policy expense continues for all 20 years.)

Solution:

$$ P\ddot{a}_{35:\overline{10}|} = 10{,}000 A_{35:\overline{20}|} + 0.42P + 0.08P\ddot{a}_{35:\overline{10}|} + 100 + 25\ddot{a}_{35:\overline{20}|} + 250 A_{35:\overline{20}|} $$

$$ P = \frac{10{,}250(0.37981) + 100 + 25(13.0240)}{(0.92)(8.0926) - 0.42} = 614.74 $$

$$ G = 614.74 + 50 = 664.74 $$

$$ {}_5V = 10{,}250 A_{40:\overline{15}|} + 25\ddot{a}_{40:\overline{15}|} + 0.08(664.74)\left(\ddot{a}_{40:\overline{5}|}\right) - 664.74\ddot{a}_{40:\overline{5}|} $$

$$ \begin{aligned} &= 10{,}250\big(0.12106 - (0.60920)(0.77772)(0.23524 - 1)\big) \\ &+25\big(18.4578 - (0.60920)(0.77772)(16.0599)\big) \\ &-(0.92)(664.74)\big[\big(18.4578 - (0.78113)(17.8162)\big)\big] \\ &= 2448.89 \end{aligned} $$

$$ {}_{10}V = 10{,}250 A_{45:\overline{10}|} + 25\ddot{a}_{45:\overline{10}|} = 10{,}250(0.61547) + 25(8.0751) = 6510.45 $$

3. You are given:

   i. $1000\bar{A}_x = 400$

   ii. $1000\bar{A}_{x+t} = 500$

Calculate ${}_tV^n$ for a whole life insurance issued to (x) with a death benefit of 1000 payable at the moment of death. Premiums are payable continuously during (x)'s lifetime.

Solution:

$$_tV_n = 1000\bar{A}_{x+t} - P\bar{a}_{x+t}$$
$$P = \frac{1000\bar{A}_x}{\bar{a}_x}$$
$$\bar{a}_{x+t} = \frac{1-\bar{A}_{x+t}}{d}; \bar{a}_x = \frac{1-\bar{A}_x}{d}$$
$$_tV^n = 1000\bar{A}_{x+t} - \frac{1000\bar{A}_x}{\bar{a}_x}\left(\bar{a}_{x+t}\right)$$
$$= 500 - 400\left(\frac{1-\bar{A}_{x+t}}{\delta}\right) / \left(\frac{1-\bar{A}_x}{\delta}\right)$$
$$= 500 - 400\left(\frac{1-\bar{A}_{x+t}}{1-\bar{A}_x}\right)$$
$$= 500 - 400\left(\frac{1-.5}{1-.4}\right)$$
$$= 500 - 400\left(5/6\right) = 166.67$$

4. You are given:

   i. $\bar{a}_x = 12$

   ii. $\bar{a}_{x+t} = 8.4$

   Calculate $_tV^n$ for a whole life insurance issued to (x) with a death benefit of 1000 payable at the moment of death. Premiums are payable continuously during (x)'s lifetime.

   Solution:

$$
{}_tV^n = 1000\bar{A}_{x+t} - P\bar{a}_{x+t}
$$

$$
P = \frac{1000\bar{A}_x}{\bar{a}_x}; \bar{A}_x = 1 - \delta\bar{a}_x; \bar{A}_{x+t} = 1 - \delta\bar{a}_{x+t}
$$

$$
1000\left(1 - \delta\bar{a}_{x+t}\right) - \frac{1000\left(1 - \delta\bar{a}_x\right)}{\bar{a}_x}\bar{a}_{x+t}
$$

$$
= 1000 - \delta(1000)(8.4) - \frac{1000\left(1 - \delta(12)\right)}{12}(8.4)
$$

$$
= 1000 - 1000(\delta)(8.4) - \frac{1000(8.4)}{12} + 1000(\delta)(8.4)
$$

$$
= 1000 - \frac{1000(8.4)}{12} = 300
$$

5. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a whole life to (91) with a death benefit of 10,000 payable at the end of the year, the net premium is 3736.756. The interest rate is 4%.

Calculate ${}_tV$ for $t = 1, 2, 3, \text{and } 4$. The easiest way to find these reserves is using the recursive formula.

Solution:

$$ {}_0V = 0 $$

$$ {}_{t+1}V = \frac{\left({}_tV + P_t\right)\left(1+i\right) - q_{x+t}\left(S_{t+1}\right)}{p_{x+t}} $$

$$ {}_1V = \frac{\left(0 + 3736.756\right)\left(1.04\right) - \left(.2\right)\left(10,000\right)}{0.8} = 2357.78 $$

$$ {}_2V = \frac{\left(2357.78 + 3736.756\right)\left(1.04\right) - \left(.4\right)\left(10,000\right)}{0.6} = 3897.20 $$

$$ {}_3V = \frac{\left(3897.20 + 3736.756\right)\left(1.04\right) - \left(0.5\right)\left(10000\right)}{0.5} = 5878.63 $$

$$ {}_4V = 0 $$

6. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a special 4 year term issued to (91) which pays death benefits at the end of the year, you are given:

i. $i = 4\%$
ii. The death benefit during the first two years is 1000
iii. The death benefit during the second two years is 500
iv. The net annual premium for the first two years is twice the net annual premium for the last two years.

Calculate ${}_2V$, the net premium reserve at the end of the second year for this special term insurance.

Solution:

$$PVP = PVB$$

$$2P(900) + 2P(720)v + P(432)v^2 + P(216)v^3$$

$$= 1000(180)v + 1000(288)v^2 + 500(216)v^3 + 500(216)v^4$$

$$P = \frac{627,679.5718}{3776.04688} = 166.226636$$

$${}_2V = PVFB - PVFP$$

$$= 500v\left(\frac{216}{432}\right) + 500v^2\left(\frac{216}{432}\right) - 166.226636\left(1 + \frac{216}{432}v\right)$$

$$= 225.38$$

7. You are given the following mortality table:

x	l_x	q_x	p_x	${}_{x-90}p_{90}$
90	1000	0.10	0.90	1.000
91	900	0.20	0.80	0.900
92	720	0.40	0.60	0.720
93	432	0.50	0.50	0.432
94	216	1.00	0.00	0.216
95	0			

For a special 3 year endowment insurance issued to (90), you are given:

i. $i = 4\%$

ii. The death benefit payable at the end of the year of death and the endowment amount are 2000

iii. The net annual premium for the first year is twice the net annual premium for the second year which is twice the net annual premium for the third year.

If the premium at the start of the third year is 282.235, calculate the net premium reserves using the recursive formula.

Solution:

$$P_2 = 282.235$$

$$P_1 = 2(P_2) = 564.470$$

$$P_0 = 2(P_1) = 1128.940$$

$${}_0V = 0$$

$${}_1V = \frac{({}_0V + P_0)(1+i) - s_1 q_x}{1 - q_x}$$

$$= \frac{(0 + 1128.940)(1.04) - 2000(.1)}{.9} = 1,082.33$$

$${}_2V = \frac{(1082.33 + 564.47)(1.04) - 2000(.2)}{.8} = 1,640.84$$

$${}_3V = (1640.84 + 282.235)(1.04) - 2000(1) = 0$$

8. For a whole life policy (50) with annual premiums and death benefit of 1000 payable at the end of the year of death, you are given:
   a. ${}_{9}V^{n} = 500$
   b. ${}_{10}V^{n} = 560$
   c. $i = 10\%$
   d. The net premium is 60.

Calculate q_{59}

Solution:

$$
{}_{10}V = \frac{\left({}_{9}V + P\right)(1+i) - 1000q_{59}}{1 - q_{59}}
$$

$$
560 = \frac{(500+60)(1.1) - 1000\left(q_{59}\right)}{1 - q_{59}}
$$

$$
560 - 560q_{59} = 616 - 1000q_{59}
$$

$$
q_{59} = \frac{56}{440} = \frac{7}{55}
$$

9. For a whole life policy on (x) with a death benefit of 10,000 payable at the end of the year of death, you are given:
   a. The annual gross premium is calculated using the equivalence principle.
   b. The gross premium reserve at the end of the first year is -79.56. (Note: This is a negative reserve.)
   c. $q_x = 0.0020$
   d. $q_{x+1} = 0.0021$
   e. $i = 8\%$
   f. Our commissions are 50% of premium in the first year and 10% thereafter.
   g. Our per policy expenses are 100 in the first year and 20 thereafter.

Calculate the annual gross premium.

Solution:

${}_0V = 0$ since premium is calculated using the equivalence principle

$${}_1V = \frac{\left({}_0V + P_0 - e_0 - X_0^{BOY}\right)(1+i) - \left(S_0 + E_0\right)q_x}{1-q_x}$$

$$-79.56 = \frac{(0 + P - 0.5P - 100)(1.08) - 10,000(0.002)}{1-0.002}$$

$$(-79.56)(.998) = .54P - 108 - 20$$

$$P = \frac{128 - 79.56(.998)}{54} = .90$$

10. A whole life insurance of 1000 is issued on (65). Death benefits are paid at the end of the year of death. Mortality follows the Standard Ultimate Life Table with interest at 5%. Net premiums are paid annually.

Determine ${}_{10.7}V^n$.

Solution:

$$P = \frac{1000A_{65}}{\ddot{a}_{65}} = \frac{354.77}{13.5498} = 26.18267$$

$${}_{10}V = 1000A_{75} - 26.18267\ddot{a}_{75} = 238.53$$

$${}_{11}V = 1000A_{76} - 26.18267\ddot{a}_{76} = 264.39$$

$${}_{10.7}V = (1-s)\left({}_{10}V + P\right) + (s)\left({}_{11}V\right) = (0.3)(238.53 + 26.18267) + (0.7)(264.39) = 264.49$$

11. A fully discrete whole life on (70) provides a death benefit of 25,000. The annual gross premium is determined using the equivalence principle using the following assumptions:

    a. Mortality follows the Standard Ultimate Life Table.
    b. $i = 0.05$
    c. Commissions as a percent of premium which are 60% in the first year and 4% in renewal years.
    d. The issue expense at the start of the first year is 100.
    e. The annual maintenance expense is 20 at the start of each year including the first year.

    The actual experience in the 6th year is:

    a. Mortality is 85% of the Standard Ultimate Life Table
    b. $i = 0.048$
    c. The annual maintenance expense is 30.
    d. Commissions and issue expenses are equal to expected.

    Calculate:

    a. The annual gross premium
    Solution:

$$PVB + PVE = PVP \implies P\ddot{a}_{70} = 25000A_{70} + 0.56P + 0.04P\ddot{a}_{70} + 100 + 20\ddot{a}_{70}$$

$$P = \frac{25,000A_{70} + 100 + 20\ddot{a}_{70}}{\left(0.96\ddot{a}_{70} - 0.56\right)} = \frac{25,000\left(0.42818\right) + 100 + 20\left(12.0083\right)}{\left(0.96\right)\left(12.0083\right) - 0.56} = 1006.9929$$

    b. The gross premium reserve at the end of 5 years
    Solution:

$$PVB + PVE - PVP = 25,000A_{75} + 0.04\left(1006.9929\right)\ddot{a}_{75} + 20\ddot{a}_{75} - \left(1006.9929\right)\ddot{a}_{75}$$

$$= 25,000\left(0.50868\right) - \left[\left(0.96\right)\left(1006.9929\right) - 20\right]\left(10.3178\right) = 2949.00$$

c. The gross premium reserve at the end of 6 years
Solution:

$$
\begin{aligned}
{}_{6}V &= \frac{\left({}_{5}V + P - E\right)(1.06) - 25{,}000 q_{x+5}}{1 - q_{x+5}} \\
&= \frac{\left(2949.00 + (0.96)(1006.9929) - 20\right)(1.05) - 25{,}000(0.018433)}{1 - 0.018433} = 3697.84
\end{aligned}
$$

d. The total gain in the 6th year
Solution:

$$
\begin{aligned}
G_6^{Total} = {}& \left(2949.00 + 0.96(1006.9929) - 30\right)(1.048) \\
& - 25{,}000(0.85)(0.018433) - (3697.84)\left[1 - (0.85)(0.018433)\right]
\end{aligned}
$$

$$= 40.62$$

e. Allocate the gain to mortality, interest rate, and expenses in that order.
Solution:

$$
\begin{aligned}
G_6^{Mort} = {}& \left(2949.00 + 0.96(1006.9929) - 20\right)(1.05) \\
& - 25{,}000(0.85)(0.018433) - (3697.84)\left[1 - (0.85)(0.018433)\right]
\end{aligned}
$$

$$= 58.90$$

$$
\begin{aligned}
G_6^{Mort\&Int} = {}& \left(2949.00 + 0.96(1006.9929) - 20\right)(1.048) \\
& - 25{,}000(0.85)(0.018433) - (3697.84)\left[1 - (0.85)(0.018433)\right]
\end{aligned}
$$

$$= 51.10$$

$$G_6^{Int} = G_6^{Mort\&Int} - G_6^{Mort} = 51.10 - 58.90 = -7.80$$

Gain from Expense:

$$= G_6^{Total} - G_6^{Mort\&Int} = 40.62 - 51.10 = -10.48$$

12. *Your company issues fully discrete whole policies to a group of lives age 40. For each policy, you are given:

    a. The death benefit is 50,000.
    b. Assumed mortality and interest are the Standard Ultimate Life Table at 5%.
    c. Assumed gross premium is 125% of the net premium.
    d. Assumed expenses are 5% of gross premium, payable at the beginning of each year, and 300 to process each death claim, payable at the end of the year of death.
    e. Profits are based on gross premium reserves.

    During year 11, actual experience is as follows:

    a. There are 1000 lives in force at the beginning of the year.
    b. There is one death.
    c. Interest earned equals 5%.
    d. Expenses equal 6% of gross premiums and 100 to process each death claim.

    For year 11, you calculate the gain due to mortality and then the gain due to expenses.

    Calculate the gain due to expenses during year 11.

Solution:

$G^M =$ Gain from Mortality

$G^E =$ Gain from Expenses

$$P = \frac{50,000A_{40}}{\ddot{a}_{40}} = \frac{50,000(0.12106)}{18.4578} = 327.93724$$

Therefore Gross $= 327.93724(1.25) = 409.92$

$$_{10}V = (50,000 + 300)A_{50} - (1 - 0.050)(409.92)\ddot{a}_{50}$$
$$= 50,300(0.18931) - 0.95(409.92)(17.0245) = 2892.54$$

$$_{11}V = (50,000 + 300)A_{51} - (1 - 0.050)(409.92)\ddot{a}_{51}$$
$$= 50,300(0.19780) - 0.95(409.92)(16.8461) = 3389.06$$

$$G^M = \left(2892.54 + (409.92)(0.95)\right)(1.05) - 0.001(50,300) - (1 - 0.001)(3389.06) = 10.09$$

$$G^{M\&E} = \left(2892.54 + (409.92)(0.94)\right)(1.05) - 0.001(50,100) - (1 - 0.001)(3389.06) = 5.99$$

$$G^E = G^{M\&E} - G^M = 5.99 - 10.09 == -4.10$$

But this is per policy and we had 1000 policies:

$-4.10(1000) = -4100$

13. * For a whole life insurance on (40), you are given:

    i. The premium is payable continuously at a level annual premium rate of 66, payable for the first 20 years.
    ii. The death benefit payable at the moment of death is 2000 for the first 20 years and 1000 thereafter.
    iii. $\delta = 0.06$
    iv. $1000\bar{A}_{50} = 333.33$
    v. $1000\bar{A}^{1}_{50:\overline{10}|} = 197.81$
    vi. $1000\,{}_{10}E_{50} = 406.57$

Calculate ${}_{10}V$, the net premium reserve for this insurance at time 10.

Solution:

$$
\begin{aligned}
{}_{10}V &= PVFB - PFVP \\
&= 1000\bar{A}_{50} + 1000\bar{A}^{\;1}_{50:\overline{10}|} - 66\bar{a}_{50:\overline{10}|} \\
&= 333.33 + 197.81 - 66\left(\frac{1-\bar{A}_{50:\overline{10}|}}{\delta}\right) \\
&= 531.14 - 66\left(\frac{1-0.19781-0.40657}{.06}\right) \\
&= 95.96
\end{aligned}
$$

14. * For a 3-year endowment insurance of 1000 on (x):

i. The annual premium is paid at the beginning of the year and the death benefit is paid at the end of the year of death.

ii. $q_x = q_{x+1} = 0.20$

iii. $i = 0.06$

iv. The net benefit premium is 373.63

Calculate ${}_2V - {}_1V$.

Solution:

Use the recursive formula:

$${}_0V = 0$$

$${}_1V = \frac{\left({}_0V + P\right)\left(1+i\right) - \left(s\right)\left(q_x\right)}{p_x}$$

$$= \frac{\left(0+373.63\right)\left(1.06\right)-1000\left(.2\right)}{1-.2} = 245.06$$

$${}_2V = \frac{\left({}_1V + P\right)\left(1+i\right) - 1000q_{x+1}}{1-q_{x+1}}$$

$$= \frac{\left(245.06+373.63\right)\left(1.06\right)-1000\left(.2\right)}{.8} = 569.76$$

$${}_2V - {}_1V = 569.76 - 245.06 = 324.70$$

15. * For a 5-payment 10-year decreasing term insurance on (60), you are given:

i. The death benefit payable at the end of year $k+1 = 1000(10-k), k = 0,1,2...,9$

ii. Level annual net premiums are payable for five years and equal 218.15 each.

iii. $q_{60+k} = 0.02 + 0.001k, k = 0,1,2,...,9$

iv. $i = 0.06$

Calculate ${}_2V$, the net premium reserve at the end of year 2.

Solution:

Use recursive formula:

$${}_0V = 0$$

$${}_1V = \frac{\left({}_0V + P\right)\left(1+i\right) - sq_x}{p_x}$$

$$= \frac{\left(0+218.15\right)\left(1.06\right) - \left(10,000\right)\left(.02\right)}{.98} = 31.8765$$

$${}_2V = \frac{\left(31.8765+218.15\right)\left(1.06\right) - \left(9,000\right)\left(.021\right)}{1-.021} = 77.66$$

16. Norris Life Insurance Company sells a whole life policy with a benefit of 1 million to (65). The death benefit is payable at the end of the year of death. The policy has level annual premiums. You are given the following assumptions:

   i. Mortality follows the mortality in the Standard Ultimate Life Table.
   ii. Interest is 5% per annum.
   iii. Expenses at the beginning of each year are as follows:
      1. Per policy expense is \$100 in the first year and \$40 per policy for each year thereafter;
      2. Percent of premium expenses are 50% in the first year and 3% thereafter;
      3. Per 1000 expense of \$1.00 in the first year and \$0.10 thereafter; and
      4. \$200 per policy per claim.

a. Calculate the annual net premium.

Solution:

$$P^n = \frac{1{,}000{,}000 A_{65}}{\ddot{a}_{65}} = \frac{1{,}000{,}000(0.35477)}{13.5498} = 26{,}182.67$$

b. Calculate the annual gross premium using the equivalence principle.

Solution:

$$PVP = PVB + PVE ==>$$

$$P^g \ddot{a}_{65} = 1{,}000{,}000 A_{65} + 60 + 40\ddot{a}_{65} + 0.47P^g$$
$$+0.03P^g \ddot{a}_{65} + 1000(0.9) + 1000(0.1)\ddot{a}_{65} + 200A_{65}$$

$$P^g = \frac{1{,}000{,}200 A_{65} + 960 + 140\ddot{a}_{65}}{0.97\ddot{a}_{65} - 0.47} = \frac{1{,}000{,}200(0.35477) + 960 + 140(13.5498)}{0.97(13.5498) - 0.47}$$

$$= 28{,}224.52$$

c. Calculate the level annual expense premium.

Solution:

$$P^e = P^g - P^n = 28{,}224.52 - 26{,}182.67 = 2041.84$$

d. Calculate the net premium reserve at the end of the 10th year.

Solution:

$$ {}_{10}V^n = 1,000,000\left(1-\frac{\ddot{a}_{75}}{\ddot{a}_{65}}\right) = 1,000,000\left(1-\frac{10.3178}{13.5498}\right) = 238,527.51 $$

e. Calculate the expense reserve at the end of the 10th year.
Solution:

$$ {}_{10}V^e = PVFE - PVFP^e $$
$$ = 40\ddot{a}_{75} + (0.03)(28,224.52)\ddot{a}_{75} + 1000(0.1)\ddot{a}_{75} + 200A_{75} - 2041.84\ddot{a}_{75} $$

$$ = 200A_{75} - (1055.10)\ddot{a}_{75} = 200(0.50868) - (1055.10)(10.3178) = -10,784.57 $$

f. Calculate the gross premium reserve at the end of the 10th year.
Solution:

$$ {}_tV^g = {}_tV^n + {}_tV^e = 238,527.51 - 10,784.57 = 227,742.93 $$

17. Norris Life also sells a three year term insurance policy with a benefit of 10,000 to (x). Annual premiums are payable for three years. Death benefits are assumed to be paid at the end of the year. You are given the following:

i. All expenses occur at the beginning of the year.
ii. Interest is 8%.
iii.

Year	Mortality	Per Policy Expense	Percent of Premium Expense
1	0.010	130	20%
2	0.015	30	8%
3	0.020	30	8%

Calculate the gross premium using the equivalence principle.

Complete the following table:

Solution:

t	${}_tV^n$	${}_tV^e$
0	0	0
1	47.50	-88.72
2	49.05	-46.40
3	0	0

$$PVP = PVB + PVE$$

$$P\left(1+0.99v+0.99(0.985)v^2\right)$$

$$=10,000\left(0.01v+.099(0.015)v^2+(0.99)(0.985)(0.02)v^3\right)+0.12P+0.08P\ddot{a}_{x:\overline{3}|}+100+30\ddot{a}_{x:\overline{3}|}$$

$$P=\frac{10,000\left[0.01v+0.99(0.015)v^2+0.99(0.985)(0.02)v^3\right]+100+30\left(1+0.99v+0.99(0.985)v^2\right)}{(0.92)\left(1+0.99v+0.99(0.985)v^2\right)-0.12}$$

$$=231.01$$

Use recursive formula for Gross Premium Reserve:

${}_0V^g = 0$ Since the premium was determined using the equivalence principle

$${}_1V^g = \frac{\left(0+(231.01)(0.8)-130\right)(1.08)-10,000(0.01)}{0.99} = -41.22$$

$${}_2V^g = \frac{\left(-41.22-(231.01)(0.92)-30\right)(1.08)-10,000(0.015)}{0.985} = 2.65$$

$${}_3V^g = 0$$

We will find the benefit premium and then use the recursive formula to find the net benefit reserves:

$$Net = \frac{10,000\left(0.01v+0.99(0.015)v^2+0.99(0.985)(0.02)v^3\right)}{1+0.99v+(0.99)(0.985)v^2} = 136.13$$

$${}_0V^n = 0$$

$${}_1V^n = \frac{(0+136.13)(1.08)-10,000(0.01)}{0.99} = 47.50$$

$${}_2V^n = \frac{(47.50+136.13)(1.08)-10,000(0.015)}{0.985} = 49.05$$

${}_3V^n = 0$ (by definition)

$${}_tV^n + {}_tV^e = {}_tV^g$$

$${}_0V^e = 0$$

$${}_1V^e = -41.22-47.50 = -88.72$$

$${}_2V^e = 2.65-49.05 = -46.40$$

$${}_3V^e = 0$$

18. A whole life policy of 100,000 on (60) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate:

a. The first year net premium under Full Preliminary Term.
Solution:

$$ {}_1P^{FPT} = 100,000(v)(q_{60}) = 100,000\left(\frac{1}{1.05}\right)(0.003398) = 323.62 $$

b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
Solution:

$$ P_{x+1}^{FPT} = \frac{100,000A_{61}}{\ddot{a}_{61}} = \frac{100,000(0.30243)}{14.6491} = 2064.4954 $$

c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.
Solution:

$$ {}_{10}V^{FPT} = PVFB - PVFP^{PFT} = 100,000A_{70} - 2064.4954\ddot{a}_{70} $$

$$ = 100,000(0.42818) - 2064.4954(12.0083) = 18,026.92 $$

OR

$$ (100,000)\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{61}}\right) = (100,000)\left(1 - \frac{12.0083}{14.6491}\right) = 18,027.05 $$

d. Calculate the ${}_{0.7}V^{FPT}$, the modified net premium reserve at time 0.7.
Solution:

$$ {}_{0.7}V^{FPT} = (1-0.7)\left({}_0V^{FPT} + {}_1P^{FPT}\right) + (0.7)\left({}_1V^{FPT}\right) $$

$$ = (0.3)(0 + 323.62) + (.7)(0) = 97.09 $$

e. Calculate the ${}_{10.7}V^{FPT}$, the modified net premium reserve at the end of 10 years.

Solution:

$$ {}_{10.7}V^{FPT} = (0.3)\left({}_{10}V^{FPT} + P_{x+1}^{FPT}\right) + (0.7)\left({}_{11}V^{FPT}\right) $$

$$ {}_{11}V^{FPT} = 100,000\left(1 - \frac{\ddot{a}_{71}}{\ddot{a}_{61}}\right) = (100,000)\left(1 - \frac{11.6803}{14.6491}\right) = 20,266.09 $$

$$ {}_{10.7}V^{FPT} = (0.3)(18,026.92 + 2064.4954) + (0.7)(20,266.09) = 20,213.69 $$

19. A 20 Year Term policy of 500,000 on (40) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate:

a. The first year net premium under Full Preliminary Term.
Solution:

$$ {}_{1}P^{FPT} = S\left(v\right)\left(q_x\right) = 500,000\left(\frac{1}{1.05}\right)\left(0.000527\right) = 250.95 $$

b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
Solution:

$$ P_{x+1}^{FPT} = \frac{500,000 A_{\overset{1}{41}:\overline{19}|}}{\ddot{a}_{41:\overline{19}|}} = \frac{500,000\left(0.12665 - v^{19}\left(\frac{l_{60}}{l_{41}}\right)\left(0.29028\right)\right)}{18.3403 - v^{19}\left(\frac{l_{60}}{l_{41}}\right)\left(14.9041\right)} $$

$$ v^{19}\left(\frac{l_{60}}{l_{41}}\right) = 1.05^{-19}\left(\frac{96,634.1}{99,285.9}\right) = 0.3851644 $$

$$ P_{x+1}^{FPT} = \frac{7422.238}{12.59977} = 589.08 $$

c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.
Solution:

$$ {}_{10}V^{FPT} = PVFB - PVFP^{FPT} = 500,000 A_{\overset{1}{50}:\overline{10}|} - 589.08\ddot{a}_{50:\overline{10}|} $$

$$ = 500,000\left(0.61643 - 0.60182\right) - 589.08(8.0550) = 2559.96 $$

20. A 20 Endowment policy of 20,000 on (65) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate:

a. The first year net premium under Full Preliminary Term.
Solution:

$$ {}_1P^{FPT} = S(v)(q_x) = 20,000(1.05)^{-1}(0.005915) = 112.67 $$

b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
Solution:

$$ P_{x+1}^{FPT} = \frac{20,000A_{66:\overline{19}|}}{\ddot{a}_{66:\overline{19}|}} = \frac{20,000\left(A_{66} - v^{19}\left(\frac{l_{85}}{l_{66}}\right)A_{85} + v^{19}\left(\frac{l_{85}}{l_{66}}\right)\right)}{\ddot{a}_{66} - v^{19}\left(\frac{l_{85}}{l_{66}}\right)\ddot{a}_{85}} $$

$$ v^{19}\left(\frac{l_{85}}{l_{66}}\right) = (1.05)^{-19}\left(\frac{61,184.9}{94,020.3}\right) = 0.25752888 $$

$$ P_{x+1}^{FPT} = \frac{20,000\left(0.36878 - 0.2575288(0.67622 - 1)\right)}{13.2557 - 0.2575288(6.7993)} = 786.0498 $$

c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.
Solution:

$$ {}_{10}V^{FPT} = PVFB - PVFP^{FPT} = 20,000A_{75:\overline{10}|} - 786.0498\ddot{a}_{75:\overline{10}|} $$
$$ = 20,000(0.65142) - 786.0498(7.3203) = 7274.28 $$

21. *For a fully discrete whole life insurance of 1000 on (80):

a. $i = 0.06$
b. $\ddot{a}_{80} = 5.89$
c. $\ddot{a}_{90} = 3.65$
d. $q_{80} = 0.077$

Calculate ${}_{10}V^{FPT}$, the full preliminary term reserve for this policy at the end of year 10.

Solution:

$$PVB - PVP_{x+1}$$
$$P_{81} = \frac{1000A_{81}}{\ddot{a}_{81}} = \frac{1000\left(1 - d\ddot{a}_{81}\right)}{\ddot{a}_{81}}$$
$$\ddot{a}_{81} = \left(\ddot{a}_{80} - 1\right)\left(1 + i\right) / \left(p_{80}\right)$$
$$= \left(5.89 - 1\right)\left(1.06\right) / \left(1 - .077\right)$$
$$= 5.616$$
$$P_{81} = \frac{1000\left(1 - \left(.06/1.06\right)\left(5.616\right)\right)}{5.616} = 121.46$$
$$PVB - PVP_{x+1}$$
$$= 1000A_{90} - 121.46\ddot{a}_{90}$$
$$= 1000\left(1 - d\ddot{a}_{90}\right) - 121.46\ddot{a}_{90}$$
$$= 1000\left(1 - \left(.06/1.06\right)\left(3.65\right)\right) - 121.46\left(3.65\right)$$
$$= 350.07$$

22. *A special fully discrete 3-year endowment insurance on (x) pays a death benefit of 25,000 if death occurs during the first year, 50,000 if death occurs during the 2nd year and 75,000 if death occurs during the 3rd year.

You are given:

a. The maturity benefit is 75,000.
b. Annual benefit premiums increase 20% per year, compounded annually.
c. $i = 8\%$
d. $q_x = 0.08$
e. $q_{x+1} = 0.10$

Calculate ${}_2V$, the benefit reserve at the end of year 2.

Solution:

First, we need the premium,

$$PVP = PVB$$

$$P\left[1+(1.2)(v)\,p_x+(1.2)^2\left(v^2\right){}_2p_x\right]$$

$$=25,000(v)\,q_x+50,000\left(v^2\right)(p_x)\,q_{x+1}+75,000\left(v^3\right){}_2p_x$$

$$P=\frac{25,000(1.08)^{-1}(.08)+50,000(1.08)^{-2}(.92)(.1)+75,000(1.08)^{-3}(.92)(.9)}{1+(1.2)(1.08)^{-1}(.92)+(1.2)^2(1.08)^{-2}(.92)(.9)}$$

$$=18,096.11$$

$${}_3V=\left({}_2V+P\right)(1+i)-S$$

$$0=\left({}_2V+(18,096.11)(1.44)\right)(1.08)-75,000$$

$${}_2V=43,386.05$$