Chapter 8

1. For a multiple state model where there are two states:

   i. State 0 is a person is alive
   ii. State 1 is a person is dead

   Further you are given that a person can transition from State 0 to State 1 but not back again.

   Using the Standard Ultimate Life Table, determine the following:

   a. ${}_{10}p_{80}^{00}$

      Solution:

      $${}_{10}p_{80}^{00} = \frac{l_{90}}{l_{80}} = \frac{41,841.1}{75,657.2} = 0.5530$$

   b. ${}_{10}p_{80}^{01}$

      Solution:

      $${}_{10}p_{80}^{01} = \frac{l_{80} - l_{90}}{l_{80}} = 1 - 0.5530 = 0.4470$$

   c. ${}_{10}p_{80}^{10}$

      Solution:

      0 because a life cannot move from state 1 to state 0

2. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is permanently disabled
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 2, but not to State 0. Finally, a person in State 2 cannot transition. You are given the following transitional intensities:

i. $\mu_x^{01} = 0.05$
ii. $\mu_x^{02} = 0.01$
iii. $\mu_x^{12} = 0.02$

Calculate the following:

a. ${}_{10}p_x^{00}$

Solution:

$${}_{10}p_x^{00} = e^{-\int_0^{10}\left(\mu_x^{01}+\mu_x^{02}\right)ds} = e^{-\int_0^{10} 0.06 ds} = e^{-10(.06)} = e^{-.60} = 0.54881$$

b. ${}_{10}p_x^{01}$

Solution:

$${}_{10}p_x^{01} = \int_0^{10} {}_tp_x^{00}\left(\mu_{x+t}^{01}\right)\left({}_{10-t}p_{x+t}\right)dt = \int_0^{10} e^{-.06t}\left(0.05\right)e^{-0.02(10-t)}dt$$

$$= 0.05e^{-0.2}\int_0^{10} e^{-.06t}e^{0.02t}dt$$

$$= 0.05e^{-0.2}\int_0^{10} e^{-.04t}dt$$

$$= 0.05e^{-0.2}\left[\frac{e^{-0.04t}\,|_0^{10}}{-0.04}\right]$$

$$= \frac{0.05}{0.04}e^{-0.2}\left(1-e^{-0.4}\right) = 0.33740$$

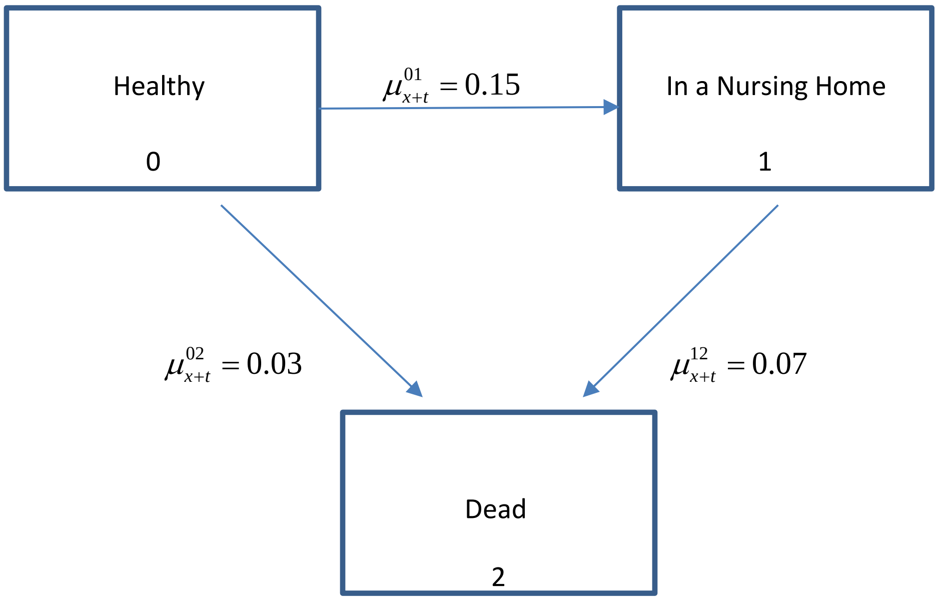
c. ${}_{10}p_x^{02}$

Solution:

$${}_{10}p_x^{02} = 1 - {}_{10}p_x^{00} - {}_{10}p_x^{01}$$

$$= 1 - 0.54881 - 0.33740 = 0.11379$$

3.



The above mutli-state model is used for a long term care policy which has a term of 20 years. Jeff who is age x purchases this policy.

The policy pays four benefits:

- Benefit 1 is a lump sum benefit of 50,000 at the moment of transition from State 0 to State 1.
- Benefit 2 is a lump sum benefit of 100,000 at the moment of transition from State 0 to State 2.
- Benefit 3 is a lump sum benefit of 25,000 at the moment of transition from State 1 to State 2.
- Benefit 4 is a continuous annuity at an annual rate of 40,000 per year while a person is in State 1.

The premium for this policy is payable continuously while the insured is in State 0. The premium was determined using the equivalence principle.

You are given that $\delta = 0.02$.

a. Determine ${}_t p_x^{00}$.

Solution:

$${}_t p_x^{00} = e^{-\int_0^t (\mu_{x+s}^{01} + \mu_{x+s}^{02}) ds} = e^{-\int_0^t (0.15+0.03) ds} = e^{-0.18t}$$

b. Calculate the probability that Jeff will receive no benefits.

Solution:

$$Probability\ of\ \ no\ benefits = {}_{20}\ p_x^{00} = e^{-\int_0^{20} \left(\mu_{x+t}^{01} + \mu_{x+t}^{02}\right) dt} = e^{-\int_0^{20} (0.15+0.03) dt} = e^{-(0.18)(20)} = 0.0273237$$

c. Determine ${}_t p_x^{01}$.

Solution:

$${}_t p_x^{01} = \int_0^t {}_s p_x^{00} \cdot \mu_{x+s}^{01} \cdot {}_{t-s}\ p_{x+s}^{11} \cdot ds = \int_0^t e^{-0.18s} \cdot 0.15 \cdot e^{-0.07(t-s)} \cdot ds =$$

$$0.15 \int_0^t e^{-0.11s} \cdot e^{-0.07t} \cdot ds = 0.15 \cdot e^{-0.07t} \int_0^t e^{-0.11s} \cdot ds = 0.15 \cdot e^{-0.07t} \left(\frac{1 - e^{-0.11t}}{0.11} \right)$$

$$= (15) \left(\frac{e^{-0.07t} - e^{-0.18t}}{11} \right)$$

d. Calculate the actuarial present value of Benefit 1.

Solution:

$$PV = 50,000 \int_0^{20} v^{t} \cdot {}_t\ p_x^{00} \cdot \mu_{x+t}^{01} \cdot dt = 50,000 \int_0^{20} e^{-0.02t} \cdot e^{-0.18t} \cdot 0.15 \cdot dt$$

$$= (50,000)(0.15) \int_0^{20} e^{-0.2t} \cdot dt = 7500 \left[\frac{1 - e^{-0.2(20)}}{0.2} \right] = 36,813.16$$

e. Calculate the actuarial present value of Benefit 2

Solution:

$$PV = 100,000\int_0^{20} v^{t} \cdot {}_t p_x^{00} \cdot \mu_{x+t}^{02} \cdot dt = 50,000\int_0^{20} e^{-0.02t} \cdot e^{-0.18t} \cdot 0.03 \cdot dt$$

$$= (100,000)(0.03)\int_0^{20} e^{-0.2t} \cdot dt = 3000\left[\frac{1-e^{-0.2(20)}}{0.2}\right] = 14,725.27$$

f. Calculate the actuarial present value of Benefit 3

Solution:

$$PV = 25,000\int_0^{20} v^{t} \cdot {}_t p_x^{01} \cdot \mu_{x+t}^{12} \cdot dt = 25,000\int_0^{20} e^{-0.02t} \cdot (15)\left(\frac{e^{-0.07t} - e^{-0.18t}}{11}\right) \cdot 0.07 \cdot dt$$

$$= \frac{(25,000)(15)(0.07)}{11}\int_0^{20}\left(e^{-0.09t} - e^{-0.20t}\right) \cdot dt = 2386.363636\left[\frac{1-e^{-0.09(20)}}{0.09} - \frac{1-e^{-0.20(20)}}{0.20}\right]$$

$$= 10,418.95$$

g. Calculate the actuarial present value of Benefit 4.

Solution:

$$PV = 40,000\int_0^{20} v^{t} \cdot {}_t p_x^{01} \cdot dt = 40,000\int_0^{20} e^{-0.02t} \cdot (15)\left(\frac{e^{-0.07t} - e^{-0.18t}}{11}\right) \cdot dt$$

$$= \frac{(40,000)(15)}{11}\int_0^{20}\left(e^{-0.09t} - e^{-0.20t}\right) \cdot dt = 54,545.45455\left[\frac{1-e^{-0.09(20)}}{0.09} - \frac{1-e^{-0.20(20)}}{0.20}\right]$$

$$= 238,147.36$$

h. Calculate the premium for this policy.

Solution:

$$PVP = P\int_0^{20} v^{t} \cdot {}_t p_x^{00} \cdot dt = P\int_0^{20} e^{-0.02t} \cdot e^{-0.18t} \cdot dt$$

$$= P\int_0^{20} e^{-0.2t} \cdot dt = P\left[\frac{1-e^{-0.2(20)}}{0.2}\right] = 4.908421806P$$

$$PVB = 36,813.16 + 14,725.27 + 10,418.95 + 238,147.36 = 300,104.74$$

$$P = \frac{300,104.74}{4.908421806} = 61,140.78$$

4. For a multiple state model, there are three states:

   i. State 0 is a person is healthy
   ii. State 1 is a person is sick
   iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

   i. $\mu_x^{01} = 0.05$
   ii. $\mu_x^{10} = 0.03$
   iii. $\mu_x^{02} = 0.01$
   iv. $\mu_x^{12} = 0.02$

Calculate the following:

   a. ${}_{10}p_x^{\overline{00}}$

   Solution:

   $${}_{10}p_{80}^{00} = e^{-\int_0^{10}\left(\mu_{x+s}^{01}+\mu_{x+s}^{02}\right)ds} = e^{-\int_0^{10}(0.05+0.01)ds} = e^{-0.6} = 0.54881$$

   b. Assuming that only one transition can occur in any monthly period, use the Euler method to calculate:

      i. ${}_0p_x^{00}$

      Solution:

      ${}_0p_x^{00} = 1$ (Everyone is in state 0 at time 0)

      ii. ${}_0p_x^{01}$

      Solution:

      ${}_0p_x^{01} = 0$ (Everyone is in state 0 at time 0)

iii. ${}_{1/12}p_x^{00}$

Solution:

$${}_{1/12}p_x^{00} = {}_0p_x^{00} - {}_0p_x^{00}\left(\frac{1}{12}\right)\left(\mu_x^{01} + \mu_x^{02}\right) + {}_0p_x^{01}\left(\frac{1}{12}\right)\mu_x^{10}$$

$$= 1 - 1\left(\frac{1}{12}\right)(0.06) + (0)\left(\frac{1}{12}\right)(0.03)$$

$$= 0.995$$

iv. ${}_{1/12}p_x^{01}$

Solution:

$${}_{1/12}p_x^{01} = {}_0p_x^{01} - {}_0p_x^{01}\left(\frac{1}{12}\right)\left(\mu_x^{10} + \mu_x^{12}\right) + {}_0p_x^{00}\left(\frac{1}{12}\right)\mu_x^{01}$$

$$= 0 - 0 + (1)\left(\frac{1}{12}\right)(0.05)$$

$$= 0.004167$$

v. ${}_{2/12}p_x^{00}$

Solution:

$${}_{2/12}p_x^{00} = {}_{1/12}p_x^{00} - {}_{1/12}p_x^{00}\left(\frac{1}{12}\right)\left(\mu_x^{01} + \mu_x^{02}\right) + {}_{1/12}p_x^{01}\left(\frac{1}{12}\right)\mu_x^{10}$$

$$= 0.995 - 0.995\left(\frac{1}{12}\right)(0.06) + (0.004167)\left(\frac{1}{12}\right)(0.03)$$

$$= 0.990035$$

vi. ${}_{2/12}p_x^{01}$

Solution:

$${}_{2/12}p_x^{01} = {}_{1/12}p_x^{01} - {}_{1/12}p_x^{01}\left(\frac{1}{12}\right)\left(\mu_x^{10} + \mu_x^{12}\right) + {}_{1/12}p_x^{00}\left(\frac{1}{12}\right)\mu_x^{01}$$

$$= 0.004167 - 0.004167\left(\frac{1}{12}\right)(0.05) + (0.995)\left(\frac{1}{12}\right)(0.05)$$

$$= 0.008295$$

5. For a multiple state model, there are three states:

   i. State 0 is a person is healthy
   ii. State 1 is a person is sick
   iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

   i. $\mu_{x+t}^{01} = 0.05 + .001t$
   ii. $\mu_x^{10} = 0.03 - .0005t$
   iii. $\mu_x^{02} = 0.01$
   iv. $\mu_x^{12} = 0.02$

Assume that only one transition can occur in any monthly period.

If ${}_{10}p_x^{00} = 0.90$ and ${}_{10}p_x^{01} = 0.07$, use the Euler method to calculate ${}_{121/12}p_x^{01}$.

Solution:

$${}_{121/12}p_x^{00} = {}_{10}p_x^{00} - {}_{10}p_x^{00}\left(\frac{1}{12}\right)\left(\mu_x^{01} + \mu_x^{02}\right) + {}_{10}p_x^{01}\left(\frac{1}{12}\right)\mu_x^{10}$$

$$= 0.9 - 0.9\left(\frac{1}{12}\right)(0.06 + 0.01) + 0.07\left(\frac{1}{12}\right)(0.025)$$

$$= 0.895041667$$

$${}_{121/12}p_x^{01} = {}_{10}p_x^{01} - {}_{10}p_x^{01}\left(\frac{1}{12}\right)\left(\mu_x^{10} + \mu_x^{12}\right) + {}_{10}p_x^{00}\left(\frac{1}{12}\right)\mu_x^{01}$$

$$= 0.07 - 0.07\left(\frac{1}{12}\right)(0.025 + 0.02) + 0.9\left(\frac{1}{12}\right)(0.06)$$

$$= 0.074238$$

6. *For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

v. $\mu_{x+t}^{01} = 0.06$
vi. $\mu_x^{10} = 0.03$
vii. $\mu_x^{02} = 0.01$
viii. $\mu_x^{12} = 0.04$

Calculate the probability that a disabled life on July 1, 2012 will become healthy at some time before July 1, 2017 but will not remain continuously healthy until July 1, 2017.

Solution:

Pr(Transition from 1 to 0 at time t but then not remaining continually in 0 until 5)

$$= \int_0^5 {}_t p_x^{\overline{11}} \mu_{x+t}^{10} \left(1 - {}_{5-t} p_{x+t}^{\overline{00}}\right) dt$$

$$= \int_0^5 \exp\left[-\int_0^t \left(\mu_x^{10} + \mu_x^{12}\right) ds\right] \bullet \mu^{10} \bullet \left[1 - \exp\left[-\int_t^5 \left(\mu_x^{01} + \mu_x^{02}\right) ds\right]\right] dt$$

$$= 0.03 \int_0^5 \exp\left[-\int_0^t 0.07 ds\right]\left[1 - \exp\left[-\int_t^5 0.07 ds\right]\right] dt$$

$$= 0.03 \int_0^5 e^{-.07t}\left[1 - e^{-0.07(5-t)}\right] ds$$

$$= 0.03 \int_0^5 \left(e^{-0.07t} - e^{-0.35}\right) dt$$

$$= 0.03 \left[\frac{1 - e^{-0.35}}{0.07} - 5e^{-0.35}\right]$$

$$= 0.03(4.2187 - 3.5234) = 0.0209$$

7. * Employees in Purdue Life Insurance Company (PLIC) can be in:

i. State 0: Non-Executive employee
ii. State 1: Executive employee
iii. State 2: Terminated from employment

Emily joins PLIC as a non-executive employee at age 25.

You are given:

i. $\mu^{01} = 0.008$
ii. $\mu^{02} = 0.02$
iii. $\mu^{12} = 0.01$
iv. Executive employees never return to non-employee executive state.
v. Employees terminated from employment are never rehired.
vi. The probability that Emily lives for 30 years is 0.92, regardless of state.

Calculate the probability that Emily will be an executive employee of PLIC at age 55.

Solution:

The probability that Emily will be an executive employee at age 55 is ${}_{30}p_{25}^{01}$• Pr(surviving 30 years)

Pr(surviving 30 years)=0.92

$${}_{30}p_{25}^{01} = \int_0^{30} {}_tp_{25}^{00}\left(\mu_{25+t}^{01}\right)\left({}_{30-t}p_{25+t}\right)dt$$

$$= \int_0^{30} e^{-\int_0^t \left(\mu^{01}+\mu^{02}\right)ds}\left(\mu_{25+t}^{01}\right)e^{-\int_t^{30}\mu^{12}ds}\,dt$$

$$= \int_0^{30} e^{-\int_0^t 0.028ds}\left(0.008\right)e^{-\int_t^{30}0.01ds}\,dt$$

$$= \int_0^{30} e^{-0.028t}\left(0.008\right)e^{-.01(30-t)}dt$$

$$= \left(0.008\right)e^{-.30}\int_0^{30} e^{-0.018t}dt$$

$$= \frac{0.008}{0.018}e^{-.30}\left(1-e^{-0.54}\right)$$

$$= 0.13738$$

$$0.13738(.92) = 0.12639$$

8. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

a. In Good Standing
b. Out of Favor
c. Dead

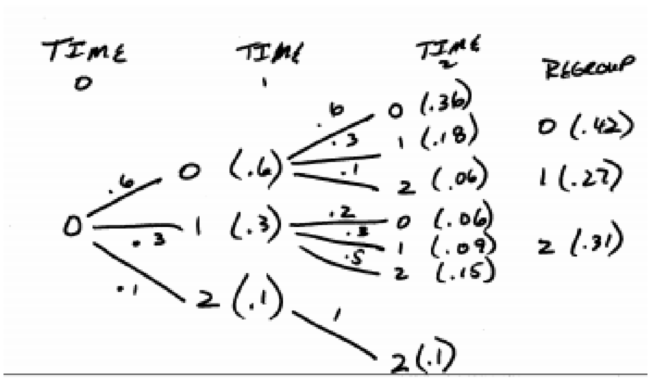
The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

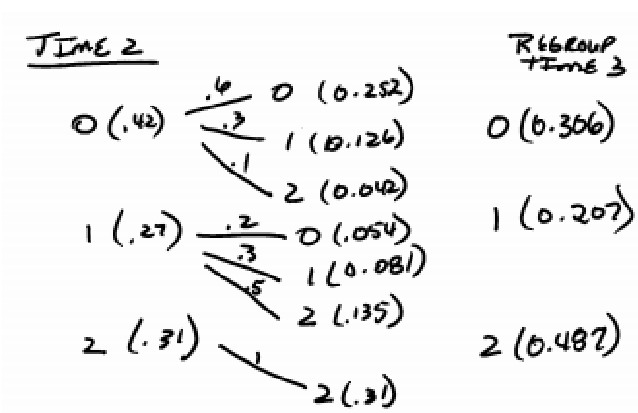
	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

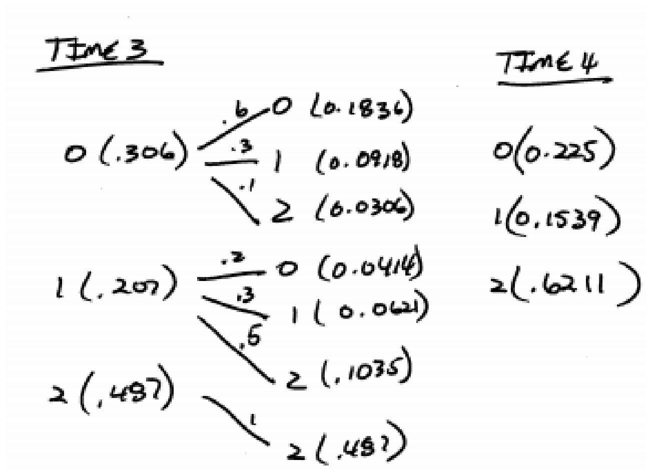
Calculate the probability that a person In Good Standing now will be Out of Favor at the end of the fourth year.

Solution:

State 0 → In good standing
State 1 → Out of favor
State 2 → Dead
Use a tree approach:







Answer: 15.39%

9. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
    a. In Good Standing
    b. Out of Favor
    c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	**Out of Favor**	**Dead**
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

At the beginning of the year, there are 1000 employees In Good Standing. All future states are assumed to be independent.

i. Calculate the expected number of deaths over the next four years.

Solution:

See the work above:

$$1000(0.6211) = 621.1$$

ii. Calculate the variance of the number of the original 1000 employees who die within four years.

Solution:

Because it is binomial

$$N(p)(q) = 1000(0.6211)(1-0.6211) = 235.33479$$

10. Animals species have three possible states: Healthy (row 1 and column 1 in the matrices), Endangered (row 2 and column 2 in the matrices), and Extinct (row 3 and column 3 in the matrices). Transitions between states vary by year where the subscript indicates the beginning of the year.

$$Q_0 = \begin{pmatrix} 0.80 & 0.20 & 0 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0.90 & 0.10 & 0 \\ 0.20 & 0.70 & 0.10 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.25 & 0.70 & 0.05 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.3 & 0.70 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
for $i > 2$

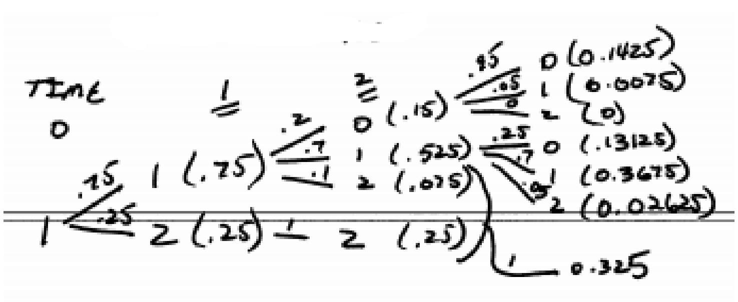
Calculate the probability that a species endangered at time 0 will become extinct.

Solution:

State 0 → Healthy

State 1 → Endangered

State 2 → Extinct



At the end of 3 years:

State 0= 0.27375

State 1= 0.375

State 2= 0.35125

Note under Q_i nobody becomes extinct, so answer=0.35125

11. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death and Decrement 2 is lapse. The benefit upon death is 1000. No benefit is payable upon lapse.

You are given:

a. $\mu_x^{(1)} = 0.015$

b. $\mu_x^{(2)} = 0.100$

c. $\delta = 0.045$

Calculate the P, the premium rate payable annually.

Solution:

$$A_x^{01} = \int_0^\infty v^t \, {}_tp_x^{00} \mu_{x+t}^{01} dt$$

$$= \int_0^\infty e^{-.045t} e^{-0.115t} (0.15) dt$$

$$= \frac{0.015}{0.16} = \frac{15}{160} = \frac{3}{32}$$

$$\ddot{a}_x^{00} = \int_0^\infty v^t \, {}_tp^{00} dt$$

$$= \int_0^\infty e^{-0.04t} e^{-0.115t} dt$$

$$= \frac{1}{0.16}$$

$$P = \frac{1000(0.015/.16)}{(1/.16)} = 1000(0.015) = 15$$

12. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death by accident and Decrement 2 is death by any other cause. The benefit upon death by accident is 2000. The death benefit upon death by any other cause is 1000.

You are given:

a. $\mu_x^{(1)} = 0.015$

b. $\mu_x^{(2)} = 0.025$

c. $\delta = 0.06$

Calculate the P, the premium rate payable annually.

Solution:

$$A_x^{01} = \int_0^\infty v^t \, {}_t p_x^{00} \mu_{x+t}^{01} dt$$

$$= \int_0^\infty e^{-0.06t} e^{-0.04t} (0.15) dt$$

$$= \frac{0.015}{0.10}$$

$$A_x^{02} = \int_0^\infty v^t \, {}_t p_x^{00} \mu_{x+t}^{02} dt$$

$$= \int_0^\infty e^{-0.06t} e^{-0.04t} (0.025) dt$$

$$= \frac{0.025}{0.10}$$

$$\ddot{a}_x^{00} = \int_0^\infty v^t \, {}_t p_x^{00} dt$$

$$= \int_0^\infty e^{-0.06t} e^{-0.04t} dt$$

$$= \frac{1}{0.10}$$

$$P = \frac{2000(0.015/.10) + 1000(0.025/.10)}{(1/.10)}$$

$$= 2000(.015) + 1000(.025) = 30 + 25 = 55$$

13. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following matrix of transitional probabilities:

$$\begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.60 & 0.25 & 0.15 \\ 0 & 0 & 1 \end{bmatrix}$$

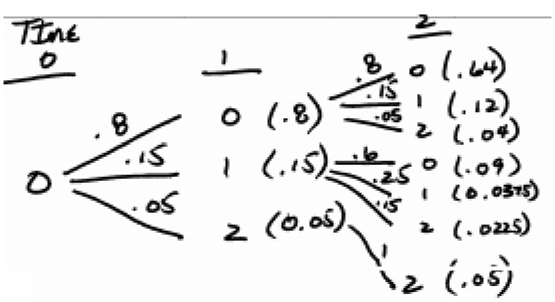
A special 3-year term policy pays 500,000 at the end of the year of death. It also pays 100,000 at the end of the year if the insured is disabled.

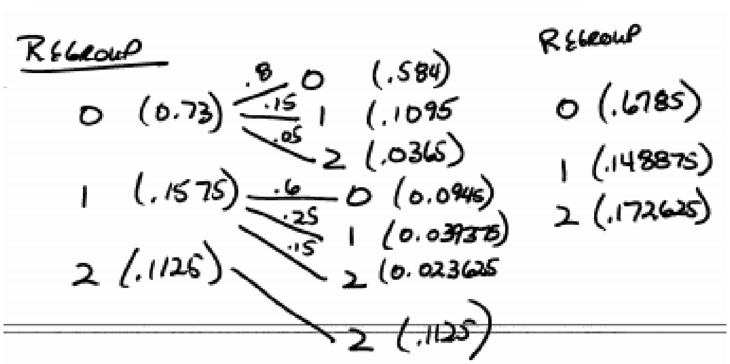
Premiums are payable annually if the insured is healthy (state 0).

You are given $i = 10\%$.

Solution:

First, note that everyone is healthy at time zero. We can use the tree approach:





i. Calculate the present value of the death benefits to be paid.

Solution:

$$500,000\left(0.05v+(.1125-.05)v^2+(.172625-.1125)v^3\right)$$
$$=71,140.12$$

ii. Calculate the present value of the disability benefits to be paid.

Solution:

$$100,000\left(0.15v+0.1575v^2+0.148875v^3\right)$$
$$=37,838.09$$

iii. Calculate the annual benefit premium.

Solution:

$$PVP=PVB$$
$$PVP=P\left(1+.8v+.73v^2\right)=2.33058P$$
$$PVB=71,140.12+37,838.09$$
$$P=\frac{71,140.12+37,838.09}{2.33058}=46,760.15$$

iv. Calculate the total reserve that would be held at the end of the first year.

Solution:

$${}_1V=\frac{\left({}_0V+P_0\right)(1+i)-PaidBenefits}{\Pr(alive)}$$
$$=\frac{(0+46,760.15)(1.1)-100,000(.15)-500,000(.05)}{(1-.05)}$$
$$=12,038.07$$

v. Calculate the reserve associated with each person in state 0 at the end of the first year.

Solution:

$${}_1V=PVFB-PVFP$$

	Time		
State	**1**	**2**	**3**
0	1.00	0.8	0.73
1		0.15	0.1575
2		0.05	0.1125

Note: Since these are Homogenous Markov chains, we can use values from part 1

$$500,000\left[0.05v+(.1125-.05)v^2\right]+100,000\left[0.15v+0.1575v^2\right]-46,760.15[1+.8v]$$
$$=-5560.92$$

vi. Calculate the reserve associated with each person in state 1 at the end of the first year.

Solution:

	Time		
State	**1**	**2**	**3**
0		0.60	0.63
1	1.00	0.25	0.1525
2		0.15	0.2175

Note: These can be developed from tree

$$PVFB-PVFP$$
$$=500,000\left[.15v+(.2175-.15)v^2\right]+100,000\left(.25v+0.1575v^2\right)-46,760.15[0+.6v]$$
$$=105,899.42$$

Note relationship between parts iv, v, and vi:

$$\frac{(.8)(-5560.92)+(.15)(105,899.42)}{.95}$$
$$=12,083.08$$

14. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
    a. In Good Standing
    b. Out of Favor
    c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

The Italian Life Insurance Company issues a special 4 year term insurance policy covering employees of OCI. The policy pays a death benefit of 10,000 at the end of the year of death.

Assume that the interest rate is 25% (remember who we are dealing with).

Solution:

	Time				
State	**0**	**1**	**2**	**3**	**4**
0	1.00	0.6	0.42	.306	.225
1		0.3	0.27	.207	.1539
2		0.1	0.31	.487	.6211

(From Problem 9)

i. Calculate the actuarial present value of the death benefit for an employee who is In Good Standing at the issue of the policy.
Solution:
$$APV = 10,000\left[.1v + .21v^2 + .177v^3 + .1341v^4\right]$$
$$= 3599.5136$$

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.
Solution:
$$PVP = PVB$$
$$P\left(1 + 0.9v + 0.69v^2 + 0.513v^3\right) = 3599.5136$$
$$P = 1484.79$$

iii. Calculate the total reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.

Solution:

Use recursive formula:

$$
\begin{aligned}
{}_1V &= \frac{\left({}_0V+P\right)(1+i)-BenefitsPaid}{1-\Pr(dead)} \\
&= \frac{(0+1484.79)(1.25)-(.1)(10,000)}{.9} \\
&= 951.10 \\
{}_2V &= \frac{(951.10+1484.79)(1.25)-(.21/.9)(10,000)}{(1-.21/.9)} \\
&= 928.08
\end{aligned}
$$

iv. Calculate the actuarial present value of the death benefit for an employee who is Out of Favor at the issue of the policy.

Solution:

	Time				
State	**0**	**1**	**2**	**3**	**4**
0		0.2	0.18	0.138	0.1026
1	1.00	0.3	0.15	0.099	0.0711
2		0.5	0.67	0.763	0.8263

$$
\begin{aligned}
PVB &= 10,00\left[.5v+.17v^2+0.093v^3+0.0633v^4\right] \\
&= 5823.44
\end{aligned}
$$

v. For an employee who is Out of Favor when the policy is issued, the annual contract premium payable at the beginning of each year that the employee is not Dead is 3500. Calculate the actuarial present value of the expected profit for Italian Life. The actuarial present value of the expected profit is the actuarial present value of the contract premiums less the actuarial present value of the death benefits.

Solution:

$$
\begin{aligned}
PVP-PVB &= 3500\left[1+.5v+.33v^2+.237v^3\right]-5823.44 \\
&= 240.46
\end{aligned}
$$

15. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

    d. In Good Standing
    e. Out of Favor
    f. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

The Italian Life Insurance Company issues a special four year annuity covering employees of OCI. The annuity pays a benefit of 100,000 at the end of a year if the employee is In Good Standing at the end of the year. It pays a benefit of 50,000 if an employee is Out of Favor at the end of a year. No benefit is paid if the employee is Dead at the end of a year.

Assume that the interest rate is 25% (remember who we are dealing with).

Solution:

	Time				
State	**0**	**1**	**2**	**3**	**4**
0	1.00	0.6	0.42	0.306	0.225
1		0.3	0.27	0.207	0.1539
2		0.1	0.31	0.487	0.6211

(From problem 118)

i. Calculate the actuarial present value of the annuity benefit for an employee who is In Good Standing at the issue of the policy.

   Solution:

   $$PVB = 100,000\left[.6v + .42v^2 + .306v^3 + .225v^4\right] +$$
   $$500,000\left[.3v + .27v^2 + .207v^3 + .1539v^4\right]$$
   $$= 128,854.27$$

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.

Solution:

$$PVP = PVB$$

$$P\left[1+.9v+.69v^2+.513v^3\right]=128,854.27$$

$$P=\frac{128,854.27}{2.424256}=53,152.09$$

iii. Calculate the reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.

Solution:

Using recursive formula:

$$_1V=\frac{(0+53,152.09)(1.25)-100,000(0.6)-50,000(.3)}{.9}=-9510.99$$

$$_2V=\frac{(-9510.99+53,152.09)(1.25)-100,000(0.42/.9)-50,000(.27/.9)}{(.69/.90)}=-9280.82$$