Mortality Improvement

1. You are given that mortality follows the Standard Ultimate Life Table based on mortality today. However, mortality is assumed to improve at a rate of 1% per year into perpetuity. Using the mortality reflecting mortality improvement, calculate:

   i. ${}_5p_{80}$

Solution:

$${}_5p_{80} = {}_1p_{80} \cdot {}_1p_{81} \cdot {}_1p_{82} \cdot {}_1p_{83} \cdot {}_1p_{84} =$$

$$[1-0.032658][1-(0.99)(0.036607)][1-(0.99)^2(0.041025)]$$
$$[1-(0.99)^3(0.045968)][1-(0.99)^4(0.051493)]$$

$$= 0.81260$$

   ii. $e_{80:\overline{5|}}$

Solution:

$$e_{80:\overline{5|}} = {}_1p_{80} + {}_2p_{80} + {}_3p_{80} + {}_4p_{80} + {}_5p_{80} =$$

$$[1-0.032658]+[1-0.032658][1-(0.99)(0.036607)]$$

$$+[1-0.032658][1-(0.99)(0.036607)][1-(0.99)^2(0.041025)]$$

$$+[1-0.032658][1-(0.99)(0.036607)][1-(0.99)^2(0.041025)][1-(0.99)^3(0.045968)]$$

$$+0.81260$$

$$= 4.4619$$

2. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.2	0.20	0.09
1	0.4	0.15	0.07
2	0.6	0.10	0.05

Calculate the probability that a robot placed into service today is still functioning at the end of three years.

$${}_3p_{0,0} = [1-q(0,0)][1-q(1,1)][1-q(2,2)]$$

$$=[1-q(0,0)][1-\{1-\varphi(1,1)\}q(1,0)][1-\{1-\varphi(2,1)\}\{1-\varphi(2,2)\}q(2,0)]$$

$$=[1-0.2][1-\{1-0.15\}(0.4)][1-\{1-0.10\}\{1-0.05\}(0.6)]$$

$$(0.8)(1-0.85\times 0.4)(1-0.90\times 0.95\times 0.6) = 0.257136$$

3. The actuaries at Purdue have set short term improvement factors for population mortality based on the experience in 2016 and 2017, and long term factors based on projected values in 2027 and 2028. Actuaries calculate the appropriate improvement factors for intermediate years using a cubic spline.

You are given the following information:

(i) There are no cohort effects.

(ii) $\varphi(35,2016)=0.035 \quad \varphi(35,2017)=0.037$

(iii) $\varphi(35,2027)=0.015 \quad \varphi(35,2028)=0.015$

Assuming that the cubic spline takes the form of $C_a(x,t)=at^3+bt^2+ct+d$, determine the parameters to be used to calculate the improvement factors.

Solution:

Set 2017 to be $t=0$. The spline function is $C_a(x,t)=at^3+bt^2+ct+d$ and the first derivative is $C_a'(x,t)=3at^2+2bt+c$. Then

$$C_a(35,0)=d=0.037 \quad C_a'(35,0)=(0.037-0.035)=0.002=c$$
$$C_a(35,10)=1000a+100b+10c+d=0.015 \Rightarrow -0.042=1000a+100b$$
$$C_a'(35,10)=300a+20b+c=0 \Rightarrow -0.002=300a+20b$$
$$\Rightarrow a=0.000064 \quad b=-0.00106$$

$$a=0.000064 \quad b=-0.00106 \quad c=0.002 \quad d=0.037$$

Calculate the improvement factor applying to a life age 35 in 2022.

Solution

$$C_a(35,t)=0.000064t^3-0.00106t^2+0.002t+0.037$$

$$C_a(35,5)=0.000064(5)^3-0.00106(5)^2+0.002(5)+0.037=0.0285$$

4. The actuaries at Purdue have set short term improvement factors for population mortality based on the experience in 2019 and 2020, and long term factors based on projected values in 2030 and 2031. Actuaries calculate the appropriate improvement factors for intermediate years using a cubic spline which takes the form of $C_c(x+t,t) = at^3 + bt^2 + ct + d$.

You are given the following information:

(iv) There are no age based effects.

(v) $\varphi(34, 2019) = 0.027 \quad \varphi(35, 2020) = 0.025$

(vi) $\varphi(45, 2030) = 0.010 \quad \varphi(46, 2031) = 0.010$

a. Determine the parameters a, b, c and d to be used to calculate the improvement factors.

Solution:

Set 2020 to be $t = 0$. The spline function is $C_c(x+t,t) = at^3 + bt^2 + ct + d$ and the first derivative is $C_a'(x,t) = 3at^2 + 2bt + c$. Then

$$C_c(35,0) = d = 0.025 \quad C_a'(35,0) = (0.025 - 0.027) = -0.002 = c$$
$$C_a(45,10) = 1000a + 100b + 10c + d = 0.010 \Rightarrow 0.005 = 1000a + 100b$$
$$C_a'(45,10) = 300a + 20b + c = 0 \Rightarrow 0.002 = 300a + 20b$$
$$\Rightarrow a = 0.00001 \quad b = -0.00005$$

$$a = 0.00001 \quad b = -0.00005 \quad c = -0.002 \quad d = 0.025$$

b. Calculate the improvement factor applying to a life age 39 in 2024 accurate to five decimal places.

Solution:

$$C_c(35+t,t) = 0.00001t^3 - 0.00005t^2 - 0.002t + 0.025$$

$$C_a(39,4) = 0.00001(4)^3 - 0.00005(4)^2 - 0.002(4) + 0.025 = 0.01684$$