STAT 472
Fall 2021
Quiz 3

September 21, 2021

1. You are given:

   a. X is the probability that (90) dies between ages 92.2 and 93.6 which is ${}_{2.2|1.4}q_{90}$ given a constant force of mortality.

   b. Y is the probability that (90) dies between ages 92.2 and 93.6 given uniform distribution of deaths between integral ages.

   c. Mortality follows the Standard Ultimate Life Table.

   Calculate $(100,000)(X-Y)$.

   Solution:

$${}_{2.2|1.4}q_{90} = \frac{l_{92.2} - l_{93.6}}{l_{90}}$$

$$l_{90} = 41{,}841.1$$

$$\text{Under CFM } l_{92.2} = \left(l_{92}\right)^{1-0.2}\left(l_{93}\right)^{0.2} = (33{,}379.9)^{0.8}(29{,}183.8)^{0.2} = 32{,}494.989$$

$$\text{Under CFM } l_{93.6} = \left(l_{93}\right)^{1-0.6}\left(l_{94}\right)^{0.6} = (29{,}183.8)^{0.4}(25{,}094.3)^{0.6} = 26{,}656.418$$

$$\text{Under CFM } {}_{2.2|1.4}q_{90} = \frac{32{,}494.989 - 26{,}656.418}{41{,}841.1} = 0.139541529 = X$$

$$\text{Under UDD } l_{92.2} = \left(l_{92}\right)(1-0.2) + \left(l_{93}\right)(0.2) = (33{,}379.9)(0.8) + (29{,}183.8)(0.2) = 32{,}540.68$$

$$\text{Under CFM } l_{93.6} = \left(l_{93}\right)(1-0.6) + \left(l_{94}\right)(0.6) = (29{,}183.8)(0.4) + (25{,}094.3)(0.6) = 26{,}730.10$$

$$\text{Under CFM } {}_{2.2|1.4}q_{90} = \frac{32{,}540.68 - 26{,}730.10}{41{,}841.1} = 0.138872544 = Y$$

$$(100{,}000)(X-Y) = (100{,}000)(0.139541529 - 0.138872544) = 66.89$$

2. You are given the following two-year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
80	0.03	0.11	0.12	82
81	0.09	0.13	0.15	83
82	0.11	0.18	0.22	84
83	0.13	0.20	0.25	85
84	0.14	0.21	0.29	86

You are given that $e_{[82]+2} = 3$.

Calculate $e_{[82]}$.

Solution:

$$e_{[82]+1} = p_{[82]+1}(1+e_{[82]+2}) = (1-0.18)(1+3) = 3.28$$

$$e_{[82]} = p_{[82]}(1+e_{[82]+1}) = (1-0.11)(1+3.28) = 3.81$$