1. (10 points) For a special whole life insurance policy on (30), you are given:

    i.   Death benefits are payable at the moment of death.
    ii.   The amount of benefit is the following:
            a. 50,000 if death occurs within the first 10 years
            b. 25,000 if death occurs within the following 10 years
            c. 75,000 if death occurs after the first 20 years
    iii.   Mortality follows the Standard Ultimate Life Table
    iv.   Deaths are uniformly distributed between integral ages.
    v. $i = 0.05$

    Calculate the Actuarial Present Value of this policy.

    Solution:

    $$APV = 50,000\bar{A}_{30} - 25,000\,_{10}E_{30}\bar{A}_{40} + 50,000\,_{20}E_{30}\bar{A}_{50}$$

    $$= 50,000\left(\frac{i}{\delta}\right)A_{30} - 25,000\,_{10}E_{30}\left(\frac{i}{\delta}\right)A_{40} + 50,000\,_{20}E_{30}\left(\frac{i}{\delta}\right)A_{50}$$

    $$= (50,000)(1.02480)(0.07698) - (25,000)(0.61152)(1.02480)(0.12106)$$
    $$+ (50,000)(0.37254)(1.02480)(0.18931)$$

    $$= 5661.52$$

2. (10 points) Avena Life Insurance Company has 900 whole life policies that are sold to independent lives each age 70. The death benefit of each policy is 10,000 with the death benefit paid at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the amount of funds that Avena must have on hand at time 0 to be 95% confidence he will be able to pay future death benefits assuming a normal distribution.

Solution:

$$E[OnePolicy] = 10,000A_{70} = (10,000)(0.42818) = 4281.80$$

$$Var[OnePolicy] = (10,000)^2 \left[{}^2A_{70} - (A_{70})^2 \right] = (10,000)^2 \left[0.21467 - (0.42818)^2 \right]$$

$$= 3,133,188.76$$

$$E[Portfolio] = (900)(4281.80) = 3,853,620$$

$$Var[Portfolio] = (900)(3,133,188.76)$$

$$\text{Amount Needed} = 3,853,620 + (1.645)\sqrt{(900)(3,133,188.76)} = 3,940,974$$