STAT 472
Test 2
Fall 2021
November 4, 2021

1. (8 points) Carolyn who is (40) buys a 20 year term insurance with non-level death benefits. The death benefit paid at the end of the year of death is:

   i. 250,000 for death in the first 10 years; and

   ii. 150,000 for death in the second 10 years.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. $i = 0.05$

   Calculate the Actuarial Present Value of Benefits for this policy.

   Solution:

   $$APV = 250,000A_{40} - 100,000\,{}_{10}E_{40} \cdot A_{50} - 150,000\,{}_{20}E_{40} \cdot A_{60}$$

   $$(250,000)(0.12106) - (100,000)(0.60920)(0.18931) - (150,000)(0.36663)(0.29028)$$

   $$= 2768.43$$

2. A 30 year endowment insurance is sold to Niken who is (50). The death benefit of 40,000 is paid at the moment of death. Monthly premiums are paid for 5 years.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. $i = 0.05$

   iii. Deaths are uniformly distributed between integral ages.

   a. (5 Points) The Actuarial Present Value of Benefits is 10,550 to the nearest 10. Calculate the Actuarial Present Value to the nearest 1.

   Solution:

$$APV = 40,000\bar{A}_{50:\overline{30}|} = 40,000[\bar{A}_{50} - {}_{30}E_{50} \cdot \bar{A}_{80} + {}_{30}E_{50}]$$

$$\boxed{{}_{30}E_{50} = {}_{20}E_{50} \cdot {}_{10}E_{70} = (0.34824)(0.50994) = 0.1775815}$$

$$(40,000)\{(1.0248)[0.18931 - (0.1775815)(0.59293)] + 0.1775815\} = 10,547.27$$

   b. (5 points) Calculate the monthly net premiums paid for 5 years using the Equivalence Principle.

   Solution:

$$PVP = PVB$$

$$12P\ddot{a}^{(12)}_{50:\overline{5}|} = 10,547.27$$

$$P = \frac{10,547.27}{12[\{\alpha(12)\ddot{a}_{50} - \beta(12)\} - {}_{5}E_{50}\{\alpha(12)\ddot{a}_{55} - \beta(12)\}}$$

$$= \frac{10,547.27}{12[\{(1.00020)(17.0245) - 0.46651\} - (0.77772)\{(1.00020)(16.0599) - 0.46651\}}$$

$$= 198.33$$

3. (10 points) You are given:

   i. $q_{55} = 0.02$

   ii. $A_{55:\overline{3}|} = 0.8434228$

   iii. $i = 0.06$

Calculate q_{56} .

Solution:

$$A_{55:\overline{3}|} = vq_{55} + v^2 p_{55} q_{56} + v^3 p_{55} p_{56}$$

$$0.8434228 = (1.06)^{-1}(0.02) + (1.06)^{-2}(1-0.02)q_{56} + (1.06)^{-3}(1-0.02)(1-q_{56})$$

$$0.8434228 - (1.06)^{-1}(0.02) = (1.06)^{-2}(0.98)q_{56} + (1.06)^{-3}(0.98)(1-q_{56})$$

$$0.8434228 - (1.06)^{-1}(0.02) - (1.06)^{-3}(0.98) = (1.06)^{-2}(0.98)q_{56} - (1.06)^{-3}(0.98)q_{56}$$

$$q_{56} = \frac{0.8434228 - (1.06)^{-1}(0.02) - (1.06)^{-3}(0.98)}{(1.06)^{-2}(0.98) - (1.06)^{-3}(0.98)} = 0.035$$

4. (10 points) A whole life annuity due with annual payments of 1000 is sold to Kelly who is 50.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.06$

Let Y be the present value random variable for this annuity.

Calculate the $\Pr(Y \geq 15,000)$.

Solution:

$$Y = 1000\ddot{a}_{\overline{K_{50}+1|}} = 1000\left[\frac{1-v^{K_{50}+1}}{0.06/1.06}\right]$$

$$\Pr(Y \geq 15,000) ==> Set\ Y = 15,000 ==> 1000\left[\frac{1-v^{K_{50}+1}}{0.06/1.06}\right] = 15,000$$

$$\frac{1-v^{K_{50}+1}}{0.06/1.06} = 15 ==> v^{K_{50}+1} = 1-(15)(0.06/1.06) ==> K_{50}+1 = \frac{\ln\left[1-(15)(0.06/1.06)\right]}{\ln\left[1.06^{-1}\right]} = 32.45$$

$$K_{50} = 31.45 ==> \text{Round Up to 32}$$

$$\Pr(Y \geq 15,000) = {}_{32}p_{50} = \frac{l_{82}}{l_{50}} = \frac{70,507.2}{98,576.4} = 0.71525$$

We use p because the value of an annuity increases the longer that the annuitant is alive.

5. Jeff is retiring at age 66. He has 2,000,000 to invest in an annuity and he wants to consider his options.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. $i = 0.05$

   iii. Use the two factor Woolhouse formula to calculate monthly annuities.

   If Jeff elects to take an annual whole life annuity due, his payments will be P_{Annual} assuming the Actuarial Present Value of the payments is 2,000,000.

   a. (2 points) The payment P_{Annual} is 150,900 to the nearest 100. Determine P_{Annual} to the nearest 0.01.

      Solution:

$$P_{Annual} = \frac{2,000,000}{\ddot{a}_{66}} = \frac{2,000,000}{13.2557} = 150,878.49$$

   b. (5 points) Calculate the $\sqrt{Var[Y]}$ where Y is the present value random variable for this annuity.

      Solution:

$$\sqrt{Var[Y]} = \sqrt{(150,878.49)^2 \left(\frac{{}^2A_{66} - (A_{66})^2}{d^2} \right)}$$

$$= (150,878.49) \frac{\sqrt{0.16507 - (0.36878)^2}}{0.05/1.05} = 540,230$$

If Jeff elects to take a monthly whole life annuity due, his monthly payments will be $P_{Monthly}$ assuming the Actuarial Present Value of the payments is 2,000,000.

c. (4 points) Determine $P_{Monthly}$.

Solution:

$$P_{Monthly} = \frac{2,000,000}{12\ddot{a}_{66}^{(12)}} = \frac{2,000,000}{12(\ddot{a}_{66} - 11/24)} = \frac{2,000,000}{12(13.2557 - 11/24)} = 13,023.51$$

If Jeff elects to take an annual whole life and certain annuity due with 10 payments guaranteed, the annual payments will be $P_{L\&C}$ assuming the Actuarial Present Value of the payments is 2,000,000.

d. (5 points) Determine $P_{L\&C}$.

Solution:

$$P_{L\&C} = \frac{2,000,000}{\ddot{a}_{\overline{\overline{66:\overline{10}|}}}} = \frac{2,000,000}{\ddot{a}_{\overline{10}|} + {}_{10}E_{66} \cdot \ddot{a}_{76}}$$

$$= \frac{2,000,000}{\frac{1-(1.05)^{-10}}{0.05/1.05} + (0.54609)(9.9674)} = 147,591.46$$

e. (3 points) Please list one advantage and one disadvantage of selecting this option.

Solution:

Advantage – If Jeff dies in the next ten years, his beneficiaries will still receive payments for 10 years.

Disadvantage, the payment is smaller than the payment for a life only annuity.

If Jeff elects to take a deferred whole life annuity due with annual payments starting at age 80, his annual payments will be $P_{Deferred}$ assuming the Actuarial Present Value of the payments is 2,000,000.

f. (4 points) Determine $P_{Deferred}$.

Solution:

$$P_{Deferred} = \frac{2,000,000}{{}_{14}E_{66} \cdot \ddot{a}_{80}} = \frac{2,000,000}{v^{14} \cdot {}_{14}p_{66} \cdot \ddot{a}_{80}} = \frac{2,000,000}{(1.05)^{-14}\left(\dfrac{75,657.2}{94,020.3}\right)(8.5484)} = 575,660.88$$

g. (3 points) Please list two advantages and one disadvantage of selecting this option.

Solution:

Advantage – The annual payments on the deferred annuity are much larger than on the other options.

Advantage – The annuity mitigates longevity risk as if Jeff lives a long time he will receive large payments which will prevent him from running out of money due to a long life.

Disadvantage – If Jeff dies in the next 14 years, he and his beneficiaries will receive nothing.

6. A whole life insurance policy is issued to (75). The death benefit is 20,000 paid at the end of the year of death. Annual premiums are payable for life. All premiums are determined using the Equivalence Principle.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.05$

iii. L_0^n is the loss at issue random variable based on the net premiums.

iv. L_0^g is the loss at issue random variable based on the gross premiums.

a. (2 points) The annual net benefit premium is 990 to the nearest 10. Calculate the annual net premium for this policy to the nearest 0.01.

Solution:

$$PVP = PVB \Longrightarrow P\ddot{a}_{75} = 20,000A_{75}$$

$$P = \frac{(20,000)(0.50868)}{10.3178} = 986.02$$

b. (5 points) Calculate $\sqrt{Var[L_0^n]}$

Solution:

$$Var[L_0^n] = \left[S + \frac{P}{d}\right]^2 \left({}^2A_{75} - (A_{75})^2\right) = \left[20,000 + \frac{986.02}{0.05/1.05}\right]^2 \left(0.29079 - (0.50868)^2\right)$$

$$\sqrt{Var[L_0^n]} = 7285.73$$

You are given the expenses for this policy are :

i. 60% of premium in the first year and 8% of premium thereafter.

ii. 500 per policy issue expense in the first year only.

iii. Maintenance expense of 50 every year including the first year.

c. (6 points) Calculate the annual gross premium for this policy.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{75} = 20,000A_{75} + 0.52P + 0.08P\ddot{a}_{75} + 500 + 50\ddot{a}_{75}$$

$$P\ddot{a}_{75} = \frac{20,000A_{75} + 500 + 50\ddot{a}_{75}}{0.92\ddot{a}_{75} - 0.52}$$

$$P = \frac{(20,000)(0.50865) + 500 + 50(10.3175)}{(0.92)(10.3175) - 0.52} = 1247.10$$

d. (3 points) Without doing any calculations, explain why the $Var[L_0^g]$ is greater than $Var[L_0^n]$.

Solution:

The additional variance is due to expenses varying by duration. Expenses in and of itself does not add to the variance as no additional variance would result if expenses were level in every year. Similarly, use of gross premium does not increase the variance unless the expenses vary by year.

7. (10 points) The Carsen Life Insurance Company sells a 2 year life annuity **immediate** to Sally who is (x). The annuity payment at the end of the first year is 10,000. The annuity payment at the end of the second year is 8000.

Sally pays a net single premium of 14,151.96 calculated using the equivalence principle.

Let L_0^n be the loss at issue random variable.

You are given that:

i. $v = 0.9$

ii. $q_{x+t} = 0.05 + 0.04t \qquad \text{for } t = 0, 1, 2, 3, 4, 5$

Calculate $Var[L_0^n]$.

Solutions:

Case	L_0^n	Probability
Die year 1	0 – 14,151.96 = -14,151.96	0.05
Die year 2	10,000v – 14,151.96 = -5,151.96	(0.95)(0.09) = 0.0855
Live 2 years	10,000v + 8000v^2 – 14,151.96 = 1328.04	(0.95)(0.91) = 0..8645

$$E[L_0^n] = 0$$

$$E[(L_0^n)^2] = (-14{,}151.96)^2(0.05) + (-5{,}151.96)^2(0.0855) + (1328.04)^2(0.8645) = 13{,}808{,}009$$

$$Var[L_0^n] = E[(L_0^n)^2] - \left(E[L_0^n]\right)^2 = 13{,}808{,}009$$

8. (10 points) You are given:

i. $\ddot{a}_{40} = 14$

ii. $\ddot{a}_{50} = 13$

iii. ${}_{1}E_{40} = 0.935$

iv. ${}_{10}E_{40} = 0.5$

v. $i = 0.06$

Calculate $1000A^{1}_{41:\overline{9|}}$.

Solution:

$$A^{1}_{41:\overline{9|}} = A_{41} - {}_{9}E_{41} \cdot A_{50}$$

$$A_{50} = 1 - d\ddot{a}_{50} = 1 - (0.06/1.06)(13) = 0.261450943$$

$$\ddot{a}_{40} = 1 + vp_{40}\ddot{a}_{41} ==> 14 = 1 + (0.935)\ddot{a}_{41} ==> \ddot{a}_{41} = \frac{14-1}{0.935} = 13.90374332$$

$$A_{41} = 1 - d\ddot{a}_{41} = 1 - (0.06/1.06)(13.90374332) = 0.212995661$$

$${}_{10}E_{40} = {}_{1}E_{40} \cdot {}_{9}E_{41} ==> {}_{9}E_{41} = \frac{{}_{10}E_{40}}{{}_{1}E_{40}} = \frac{0.5}{0.935}$$

$$1000A^{1}_{41:\overline{9|}} = 1000(A_{41} - {}_{9}E_{41} \cdot A_{50}) = 1000\left[0.212995661 - \left(\frac{0.5}{0.935}\right)0.261450943\right] = 71.74$$