STAT 472
Spring 2021
Quiz 2

February 4, 2021

1. You are given:

   a. $e_{40:\overline{10}|} = 9.2$

   b. $e_{50:\overline{10}|} = 8.6$

   c. $e_{60} = 24$

   d. ${}_{10}p_{40} = 0.96$

   e. ${}_{10}p_{50} = 0.92$

   Calculate e_{40} .

   Solution:

   $$e_{50} = e_{50:\overline{10}|} + {}_{10}p_{50} \cdot e_{60} = 8.6 + (0.92)(24) = 30.68$$

   $$e_{40} = e_{40:\overline{10}|} + {}_{10}p_{50} \cdot e_{50} = 9.2 + (0.96)(30.68) = \boxed{38.6528}$$

2. You are given that ${}_t p_{90} = 1 - 0.04t^2$ for $0 \le t \le 5$.

   Calculate μ_{91}.

   Solution:

$$\mu_{90+t} = -\frac{\frac{d}{dt}\,{}_t p_{90}}{{}_t p_{90}} = \frac{0.08t}{1-0.04t^2}$$

$$\mu_{91} = \frac{0.08(1)}{1-0.04(1)^2} = \boxed{0.0833333}$$

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1. You are given that $\mu_{80+t} = 0.04t$.

   Calculate ${}_{10}p_{80}$.

   Solution:

$$ {}_{10}p_{80} = e^{-\int_0^{10} \mu_{80+t} \cdot dt} = e^{-\int_0^{10} 0.04t \cdot dt} = e^{-\left[0.02t^2\right]_0^{10}} = e^{-2} = \boxed{0.13534} $$

2. You are given that ${}_t p_{90} = 1 - 0.04t^2$ for $0 \le t \le 5$.

Calculate the $Var[T_{90}] - Var[K_{90}]$.

Solution:

$$Var[T_{90}] = E[T_{90}{}^2] - \left(E[T_{90}]\right)^2 = 12.5 - (3.33333)^2 = 1.38889$$

$$E[T_{90}] = \int_0^5 {}_t p_{90} \cdot dt = \int_0^5 (1 - 0.04t^2) \cdot dt = \left[t - \frac{0.04t^3}{3} \right]_0^5 = \left[5 - \frac{0.04(5)^3}{3} \right] = 3.33333$$

$$E[T_{90}{}^2] = 2\int_0^5 t \cdot {}_t p_{90} \cdot dt = 2\int_0^5 (t)(1 - 0.04t^2) \cdot dt = 2\left[\frac{t^2}{2} - \frac{0.04t^4}{4} \right]_0^5 = 2\left[\frac{(5)^2}{2} - \frac{0.04(5)^4}{4} \right] = 12.5$$

$$Var[K_{90}] = E[K_{90}{}^2] - \left(E[K_{90}]\right)^2 = 9.2 - (2.8)^2 = 1.36$$

$$E[K_{90}] = \sum_{k=1}^5 {}_t p_{90} = \sum_{k=1}^5 (1 - 0.04t^2) =$$

$$(1 - 0.04(1)^2) + (1 - 0.04(2)^2) + (1 - 0.04(3)^2) + (1 - 0.04(4)^2) + (1 - 0.04(5)^2) = 2.8$$

$$E[K_{90}{}^2] = 2\sum_{k=1}^5 t \cdot {}_t p_{90} - \sum_{k=1}^5 {}_t p_{90} = \left[2\sum_{k=1}^5 t(1 - 0.04t^2) \right] - 2.8$$

$$= (1)(1 - 0.04(1)^2) + (2)(1 - 0.04(2)^2) + (3)(1 - 0.04(3)^2) + (4)(1 - 0.04(4)^2) + (5)(1 - 0.04(5)^2) - 2.8$$

$$= 9.2$$

$$Var[T_{90}] - Var[K_{90}] = 1.38889 - 1.36 = \boxed{0.02889}$$