STAT 472
Spring 2021
Quiz 4

March 16, 2021

1. Valerie buys a special whole life insurance with the death benefit paid at the end of the year of death. Valerie is age 25. The death benefit is 50,000 for the first 20 years. The death benefit is 200,000 for the second 20 years (ages 45 to 65). The death benefit after age 65 is 100,000.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Calculate the Actuarial Present Value of Valerie's policy.

   Solution:

   $$APV = 50,000A_{25} + 150,000\,{}_{20}E_{25}A_{45} - 100,000\,{}_{40}E_{25}A_{65}$$

   $$= 50,000A_{25} + 150,000\,{}_{20}E_{25}A_{45} - 100,000\,{}_{20}E_{25} \cdot {}_{20}E_{45}A_{65}$$

   $$= (50,000)(0.06147) + (150,000)(0.37373)(0.15161) - (100,000)(0.37373)(0.35994)(0.35477)$$

   $$= 6800.30$$

2. Filza who is (45) buys a 25 year term insurance with a death benefit of 1,000,000 paid at the moment of death.

   You are given that:

   a. Mortality follows the Standard Ultimate Life Table

   b. $i = 0.05$

   c. Deaths are uniformly distributed between integral ages.

   Calculate the Actuarial Present Value of Filza's policy.

   Solution:

$$APV = 1,000,000\bar{A}^{1}_{45:\overline{25|}} = 1,000,000(\bar{A}_{45} - {}_{25}E_{45}\bar{A}_{70})$$

$$1,000,000(i/\delta)(A_{45} - {}_{20}E_{45} \cdot {}_{5}E_{65}A_{70})$$

$$= (1,000,000)(1.0248)[0.15161 - (0.35994)(0.75455)(0.42818)]$$

$$= 36,195.35$$

3. Gigli Life Insurance Company has 900 whole life policies issued to independent lives who are all age 50. The death benefit for each policy is 10,000 paid at the end of the year of death.

   Assuming a normal distribution, calculate the 95% confidence interval for the present value of future losses under these policies.

   Solution:

$$E[Z] = 10,000A_{50} = (10,000)(0.18931) = 1893.10$$
$$Var[Z] = (10,000)^2[{}^2A_{50} - (A_{50})^2] = (10,000)^2[0.05108 - (0.18931)^2] = 1,524,172$$

$$E[Port] = 900(1893.10) = 1,703,790$$
$$Var[Port] = 900(1,524,172)$$

$$C.I. = E[Port] \pm (1.96)\sqrt{Var[Port]}$$

$$= 1,703,790 \pm (1.96)\sqrt{900(1,524,172)} = (1,631,197; 1,776,383)$$

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1. Tara buys a special 40 year term life insurance with the death benefit paid at the end of the year of death. Tara is age 35. The death benefit is 100,000 for the first 20 years. The death benefit is 50,000 for the last 20 years (ages 55 to75). There is no death benefit after age 75.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Calculate the Actuarial Present Value of Tara's policy.

   Solution:

$$APV = 100,000A_{35} - 50,000\,{}_{20}E_{35}A_{55} - 50,000\,{}_{40}E_{35}A_{75}$$

$$= 100,000A_{35} - 50,000\,{}_{20}E_{35}A_{55} - 50,000\,{}_{20}E_{35} \cdot {}_{20}E_{55}A_{75}$$

$$= (100,000)(0.09653) - (50,000)(0.37041)(0.23524) - (50,000)(0.37041)(0.32819)(0.50868)$$

$$= 2204.36$$

2. Amber who is (35) buys a 25 year term insurance with a death benefit of 1,000,000 paid at the moment of death.

   You are given that:

   a. Mortality follows the Standard Ultimate Life Table

   b. $i = 0.05$

   c. Deaths are uniformly distributed between integral ages.

   Calculate the Actuarial Present Value of Amber's policy

   Solution:

   $$APV = 1,000,000\bar{A}^{1}_{35:\overline{25}|} = 1,000,000(\bar{A}_{35} - {}_{25}E_{35}\bar{A}_{60})$$

   $$1,000,000(i/\delta)(A_{35} - {}_{20}E_{35} \cdot {}_{5}E_{55}A_{60})$$

   $$= (1,000,000)(1.0248)[0.09653 - (0.37041)(0.32819)(0.29028)]$$

   $$= 13,657$$

3. Saransh Life Insurance Company has 900 whole life policies issued to independent lives who are all age 60. The death benefit for each policy is 10,000 paid at the end of the year of death.

   Assuming a normal distribution, calculate the amount that Saransh needs to have at time 0 to be 95% confidence that the company will be able to pay the future benefits.

   Solution:

$$E[Z] = 10,000A_{60} = (10,000)(0.29028) = 2902.8$$
$$Var[Z] = (10,000)^2[{}^2A_{60} - (A_{60})^2] = (10,000)^2[0.10834 - (0.29028)^2] = 2,407,752$$

$$E[Port] = 900(2902.8) = 2,612,520$$
$$Var[Port] = 900(2,407,752)$$

$$C.I. = E[Port] + (1.645)\sqrt{Var[Port]}$$

$$= 2,612,520 + 1.645\sqrt{900(2,407,752)} = 2,689,096$$