STAT 472
Quiz 5
Spring 2021

April 15, 2021

1. The Pierce Life Insurance Company sells a 20 year term policy with a death benefit of 80,000 to (63). The death benefit is payable at the end of the year of death. The premiums for the policy are paid annually for 10 years.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table

   ii. $i = 0.05$

   iii. The expenses for the policy:

      1. 30% of premium the first year and 5% of premium thereafter. This expense stops when premiums stop.

      2. 100 per policy in the first year and 30 per policy in years 2 and later.

   a. (6 points) Calculate the gross premium for this policy if the gross premium is determined using the equivalence principle.

   Solutions:

   $$PVP = PVB + PVE$$

   $$P\ddot{a}_{63:\overline{10}|} = 80,000A^{1}_{63:\overline{20}|} + 0.25P + 0.05P\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|}$$

   $$P = \frac{80,000A^{1}_{63:\overline{20}|} + 70 + 30\ddot{a}_{63:\overline{20}|}}{0.95\ddot{a}_{63:\overline{10}|} - 0.25}$$

   $$= \frac{80,000(0.42298 - 0.26674) + 70 + 30(12.1174)}{0.95(7.8960) - 0.25} = 1783.53$$

Pierce decides to charge a premium of 1800. Let L_0^g be the loss at issue random variable based on the gross premium for this policy.

b. (6 points) Calculate the $E(L_0^g)$.

Solutions:

$$E(L_0^g) = PVB + PVE - PVP$$

$$= 80,000A^{1}_{63:\overline{20}|} + 0.25(1800) + 0.05(1800)\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|} - (1800)\ddot{a}_{63:\overline{10}|}$$

$$80,000(0.42298 - 0.26674) + 0.25(1800)$$
$$+ 0.05(1800)(7.8960) + 70 + 30(12.1174) - (1800)(7.8960)$$

$$= -119.44$$

c. (4 points) Explain why the expected value is negative.

Solution:

The premium has increased and is greater than the premium under the equivalence principle where the expected value is equal to zero. Therefore, we expect a gain which is a negative loss.

d. (4 points) If Pierce wanted an expected profit of 500 on the policy, determine the gross premium that should be charged. Note that a profit of 500 is equivalent to $E(L_0^g) = -500$.

Solutions:

$$E(L_0^g) = PVB + PVE - PVP$$

$$-500 = 80,000A^{1}_{63:\overline{20}|} + 0.25(P) + 0.05(P)\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|} - (P)\ddot{a}_{63:\overline{10}|}$$

$$-500 = 80,000(0.42298 - 0.26674) + 0.25(P)$$
$$+ 0.05(P)(7.8960) + 70 + 30(12.1174) - (P)(7.8960)$$

$$P = \frac{500 + 80,000(0.42298 - 0.26674) + 70 + 30(12.1174)}{0.95(7.8960) - 0.25} = 1852.48$$