STAT 472
Spring 2021
Test 1
March 4, 2021

1. For a new iPhone, the distribution function of its future lifetime is $F_0(t) = 0.01t^2$ for $0 \le t \le 10$
.

   a. Calculate the complete future lifetime of a new iPhone.

   Solution:

$$E[T_0] = \int_0^{10} {}_t p_0 \cdot dt = \int_0^{10} \left(1 - 0.01t^2\right) \cdot dt = \left[t - \frac{0.01t^3}{3} \right]_0^{10} = 10 - \frac{0.01(10)^3}{3} = 6.66667$$

   b. Trevor buys a used iPhone that is two years old. What is the probability that Trevor's iPhone will still be alive (functioning) at the end of three years when it is five years old.

   Solution:

$${}_3 p_2 = \frac{S_0(5)}{S_0(2)} = \frac{1 - F_0(5)}{1 - F_0(2)} = \frac{1 - (0.01)(5^2)}{1 - (0.01)(2^2)} = \frac{0.75}{0.96} = 0.78125$$

2. You are given:

   i. $A_{70} = 0.6435$

   ii. $A_{71} = 0.7000$

   iii. $v = 0.90$

   a. Calculate q_{70} .

   Solution:

   $$A_{70} = vq_{70} + vp_{70}A_{71}$$

   $$0.6435 = (0.9)(q_{70}) + (0.9)(1 - q_{70})(0.700)$$

   $$0.6435 = 0.9q_{70} + 0.63 - 0.63q_{70} \text{ ==> } q_{70} = \frac{0.6435 - 0.63}{0.9 - 0.63} = 0.05$$

   b. Calculate ${}_{0.5}q_{70.2}$ assuming that there is a constant force of mortality between ages 70 and 71.

   Solution:

   $${}_{0.5}q_{70.2} = \frac{l_{70.2} - l_{70.7}}{l_{70.2}}$$

   $$l_{70} = 1000; l_{71} = (1000)(1 - 0.05) = 950$$

   $${}_{0.5}q_{70.2} = \frac{l_{70.2} - l_{70.7}}{l_{70.2}} = \frac{(1000)^{1-0.2}(950)^{0.2} - (1000)^{1-0.7}(950)^{0.7}}{(1000)^{1-0.2}(950)^{0.2}} = 0.02532$$

3. You are given:

   a. $\mu_{x+0.4} = 0.13$

   b. $q_{x+1} = q_x + 0.02$

   c. Deaths are uniformly distributed between integral ages.

   Calculate ${}_{0.2|0.5}q_{x+0.7}$.

   Solution:

$$\text{Under UDD} ==> \mu_{x+s} = \frac{q_x}{1-s\cdot q_x} ==> 0.13 = \frac{q_x}{1-(0.4)q_x}$$

$$==> 0.13 - 0.052q_x = q_x ==> q_x = \frac{0.13}{1.052} = 0.123574$$

$$q_{x+1} = q_x + 0.02 = 0.143574$$

$$l_x = 1000; l_{x+1} = (1000)(1-0.123574) = 876.426; l_{x+2} = (876.426)(1-0.143574) = 750.594$$

$${}_{0.2|0.5}q_{x+0.7} = \frac{l_{x+0.9} - l_{x+1.4}}{l_{x+0.7}}$$

$$= \frac{(1000)(1-0.9) + (876.426)(0.9) - \left[(876.426)(1-0.4) + (750.594)(0.4)\right]}{(1000)(1-0.7) + (876.426)(0.7)} = 0.06863$$

4. You are given the following one year select and ultimate mortality table:

$[x]$	$q_{[x]}$	q_{x+1}	$x+1$
80	0.05	0.10	81
81	0.07	0.12	82
82	0.10	0.15	83
83	0.13	0.19	84

Calculate $e_{[81]:\overline{3}|}$.

Solution:

$$e_{[81]:\overline{3}|} = {}_1p_{[81]} + {}_2p_{[81]} + {}_3p_{[81]}$$

$$= (1-0.07) + (1-1.07)(1-0.12) + (1-1.07)(1-0.12)(1-0.15)$$

$$= 2.44404$$

5. The actuaries at Purdue have set short term improvement factors for population mortality based on the experience in 2019 and 2020, and long term factors based on projected values in 2030 and 2031. Actuaries calculate the appropriate improvement factors for intermediate years using a cubic spline which takes the form of $C_a(x,t) = at^3 + bt^2 + ct + d$.

   You are given the following information:

   (i) There are no cohort effects.

   (ii) $\varphi(35, 2019) = 0.027 \quad \varphi(35, 2020) = 0.025$

   (iii) $\varphi(35, 2030) = 0.010 \quad \varphi(35, 2031) = 0.010$

   (iv) $a = 0.00001$

   (v) $b = -0.00005$

   a. Determine the parameters c and d to be used to calculate the improvement factors.

      Solution:

$$C_a(x,0) = \varphi(35,2020) \text{==>} a(0)^3 + b(0)^2 + c(0) + d = 0.025 \text{==>} d = 0.025$$

$$C_a'(x,0) = \varphi(35,2020) - \varphi(35,2019)$$

$$3at^2 + 2bt + c = 0.025 - 0.027 \text{==>} 3a(0)^2 + 2b(0) + c = -0.002 \text{==>} c = -0.002$$

Continued on the next page.

b. Calculate the improvement factor applying to a life age 35 in 2024 accurate to five decimal places.

Solution:

$$\varphi(35, 2024) = C_a(35, 4) = (0.00001)(4^3) - 0.00005(4^2) - 0.002(4) + 0.025 = 0.01684$$

c. It is important to reflect mortality improvement in actuarial models to protect from longevity risk. Define longevity risk and give two examples.

Solution:

Longevity risk is the risk of living longer than expected.

One example is for pension plans. The pension benefits are paid for as long as a person lives. If people live longer than expected, then the pension plan may have to pay out more money than anticipated.

Another example would be if a person is retired and they live longer than expected, they could run out of money to live on.

There are many other examples that could have been used.

6. Arthur is (80) and buys a three year endowment insurance with a death benefit of 10,000 paid at the end of the year of death.

   a. Write the random variable for Arthur's endowment insurance.

   Solution:

$$Z = \begin{cases} 10,000v^{K_{80}+1} & \text{for } K_x \le 2 \\ 10,000v^3 & \text{for } K_x \ge 3 \end{cases}$$

You are given that:

   i. $q_{80} = 0.08$

   ii. $q_{81} = 0.10$

   iii. $v = 0.93$

   b. Calculate the Actuarial Present Value of Arthur's endowment insurance.

   Solution:

$$l_{80} = 1000; l_{81} = (1000)(1-0.08) = 920; l_{82} = (920)(1-0.1) = 828$$

$$A_{80:\overline{3|}} = \frac{(1000-920)(0.93)+(920-828)(0.93)^2+828(0.93)^3}{1000} = 0.81997840$$

Note that for the 828 people alive at the beginning of the third year, it does not matter if they live or die because they will get paid at the end of the third year.

$$Answer = (10,000)(0.81997840) = 8199.78$$

7. Filza is 95 and buys a whole life policy with a death benefit of 100,000 paid at the end of the year of death.

   You are given the following mortality table and $i = 0.08$.

x	q_x
95	0.2
96	0.5
97	0.8
98	1.0

   Let Z be the present value random variable of Filza's whole life insurance.

   Calculate the $\sqrt{Var[Z]}$.

   Solution:

$$l_{95} = 100; l_{96} = (100)(1-0.2) = 80; l_{97} = (80)(1-0.5) = 40; l_{98} = (40)(1-0.8) = 8; l_{99} = 0$$

$$A_{95} = \frac{20(1.08)^{-1} + 40(1.08)^{-2} + 32(1.08)^{-3} + 8(1.08)^{-4}}{100} = 0.840949$$

$$^{2}A_{95} = \frac{20(1.08)^{-2} + 40(1.08)^{-4} + 32(1.08)^{-6} + 8(1.08)^{-8}}{100} = 0.710355$$

$$Var[Z] = (100,000)^2(0.710355 - 0.840949^2) = 31,597,793.99$$

$$\sqrt{Var[Z]} = 5621$$

8. Dylan is (35) and buys a 10 year term policy with a death benefit of 500,000 paid at the end of the year of death.

   You are given that mortality follows the illustrative life table with interest at 5%.

   a. Calculate the Expected Present Value of this term policy.

      Solution:

      $$500,000A^{1}_{35:\overline{10}|} = 500,000(A_{35:\overline{10}|} - {}_{10}E_{35}) = (500,000)(0.61464 - 0.061069) = 1975$$

   b. Explain why a 10 year term insurance has an actuarial present value that is less than a 20 year term insurance.

      Solution:

      A ten year term provides a death benefit if a person dies in the first 10 years while a 20 year term provides a death benefit if the person dies in the first 20 years. Therefore, the 20 year term provides the benefits of the 10 year term plus additional benefits if you die during the last ten years. Therefore, the APV of the 20 year term must be greater.

9. Jiaying is (60) and buys a pure endowment of 25,000 to be paid at the end of 10 years if she is alive at that time.

   You are given that $\mu_{60+t} = 0.01t$. You are also given that $\delta = 0.06$.

   Calculate the Actuarial Present Value of Jiaying's pure endowment.

   Solution:

$$APV = 25{,}000\,{}_{10}E_{60} = (25{,}000)v^{10}\,{}_{10}p_{60}$$

$$v^{10} = e^{-0.06(10)}$$

$${}_{10}p_{60} = e^{-\int_0^{10} \mu_{60+t}dt} = e^{-\int_0^{10} 0.01t \cdot dt} = e^{-\left[0.005t^2\right]_0^{10}} = e^{-0.5}$$

$$APV = (25{,}000)e^{-0.6}e^{-0.5} = 8321.78$$