STAT 472
Test 2
Spring 2021

April 1, 2021

1. (10 points) Andrew is (45) and purchases a whole life policy with a death benefit of 200,000 paid at the moment of death. Premiums are paid annually during Andrew's lifetime on the policy.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. Deaths are uniformly distributed between integral ages.

   iii. $i = 0.05$

   Calculate the premium for Andrew's policy.

   Solution:

   $$PVP = PVB ==> P\ddot{a}_{45} = 200,000\bar{A}_{45} = (200,000)(i/\delta)A_{45}$$

   $$==> P(17.8162) = 200,000(1.02480)(0.15161)$$

   $$P = 1744.14$$

2. (10 points) Trevor is (60) and is receiving a life annuity with payments at the beginning of each year. The payments are not level and are as follows:

    i. Payments of 10,000 for the first 10 payments;

    ii. Payments of 25,000 for the next 10 payments; and

    iii. Payments of 5000 for any payments after the first 20.

    You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

    Calculate the Actuarial Present Value of this annuity.

    Solution:

$$APV = 10,000\ddot{a}_{60} + 15,000 \cdot {}_{10}E_{60} \cdot \ddot{a}_{70} - 20,000 \cdot {}_{20}E_{60} \cdot \ddot{a}_{80}$$

$$= (10,000)(14.9041) + (15,000)(0.57864)(12.0083) - (20,000)(0.29508)(8.5484)$$

$$= 202,819.00$$

3. (10 points) Valerie is 70 and purchases a whole life policy with a death benefit of 1000 paid at the end of the year of death.

   You are given:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. $i = 0.07$

   Calculate the $\Pr(Z < 500)$

   Solution:

$$Z = 1000v^{K_{70}+1}$$

$$1000v^{K_{70}+1} = 500 ==> v^{K_{70}+1} = 0.5$$

$$K_{70} + 1 = \frac{\ln(0.5)}{\ln[(1.07)^{-1}]} = 10.24$$

$$K_{70} = 9.24 ==> \text{Round Up} ==> 10$$

$$\Pr[Z < 500] = {}_{10}p_{70} = \frac{l_{80}}{l_{70}} = \frac{75,657.2}{91,082.4} = 0.830646$$

4. (10 points) A 3 year temporary life annuity due to (x) makes payment a payment of 300 at the beginning of year one, 400 at the beginning of year 2, and 200 at the beginning of year 3.

   Let Y be the present value for this annuity.

   You are given:

   a. $q_x = 0.2$

   b. $q_{x+1} = 0.3$

   c. $q_{x+2} = 0.4$

   d. $v = 0.92$

   Calculate the $Var(Y)$.

   Solution:

Case	**Present Value**	**Probability**
Die Year 1	300	0.2
Die Year 2	300 + 400v = 668	(0.8)(0.3) = 0.24
Live 2 Years	300 + 400v +200v^2 = 837.28	(0.8)(0.7) = 0.56

$$E[Y] = 300(0.2) + 668(0.24) + 837.28(0.56) = 689.1968$$

$$E[Y^2] = (300)^2(0.2) + (668)^2(0.24) + (837.28)^2(0.56) = 517,674.9271$$

$$Var[Y] = 517,674.9271 - (689.1968)^2 = 42,682.70$$

5. Diego is (65) and is the recipient of an annuity that pays 5000 at the beginning of each year for the rest of his life.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Let Y be the present value random variable for Diego's annuity.

   a. (1 point) Calculate the $E[Y]$.

      Solution:

      $$E[Y] = 5000\ddot{a}_{65} = (5000)(13.5498) = 67,749$$

   b. (7 points) Calculate the $Var[Y]$.

      Solution:

      $$Var[Y] = (5000)^2 \left[\frac{{}^2A_{65} - (A_{65})^2}{d^2} \right] = (5000)^2 \left[\frac{0.15420 - (0.35477)^2}{(0.05/1.05)^2} \right] = 312,429,174$$

   Lin Life Insurance Company has sold 625 annuities identical to Diego's annuity to 625 independent lives.

   c. (7 points) Calculate the 90% confidence interval for the present value of this annuity portfolio using the normal distribution.

      Solution:

      $$E[Port] = 625(67,749) = 42,343,125$$

      $$\sqrt{Var[Port]} = \sqrt{(625)(312,429,174)} = 441,891.65$$

      $$CI = 42,343,125 \pm (1.645)(441,891.65)$$

      $$(41,616,213 \ ; \ 43,070,037)$$

6. (10 points) Sam who is (35) buys a 15 year temporary life annuity that pays 2000 at the beginning of each a year that she is alive. The payments stop at the end of 15 years even if Sam is still alive.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Calculate the actuarial present value of Sam's annuity.

   Solution:

$$APV = 2000\ddot{a}_{35:\overline{15|}} = (2000)(\ddot{a}_{35} - {}_{15}E_{35} \cdot \ddot{a}_{50})$$

$$= (2000)\left[18.9728 - (1.05)^{-15}\left(\frac{98,576.4}{99,556.7}\right)(17.0245)\right] = 21,728.72$$

7. (10 points) Dylan who is (68) receives an annuity as his pension benefit. The annuity is a 10 year certain and life annuity which pays 50,000 at the beginning of each year. The first 10 payments are guaranteed to be made even if Dylan dies prior to the payment. After 10 payments, payments are only made if Dylan is alive.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Calculate the Actuarial Present Value of Dylan's pension benefit.

   Solution:

$$APV = 50{,}000\ddot{a}_{\overline{68:\overline{10|}}} = (50{,}000)(\ddot{a}_{\overline{10|}} + {}_{10}E_{68} \cdot \ddot{a}_{78})$$

$$= (50{,}000)\left(\frac{1-(1.05)^{-10}}{0.5/1.05} + (0.52981)(9.2598)\right) = 650{,}687.82$$

8. Jackson is (50) and has 1,000,000 that he wants to invest in an annuity that makes quarterly payments of X at the beginning quarter for the rest of his life.

Jackson approaches Gigli Life Insurance Company. Gigli calculates the amount of the benefit using the two term Woolhouse formula with mortality equal to the Standard Ultimate Life Table and interest equal to 5%.

a. (6 points) Calculate X from Gigli.

Solution:

$$1,000,000 = 4X\ddot{a}_{50}^{(4)} = 4X\left(\ddot{a}_{50} - 3/8\right) = 4X\left(17.0245 - 3/8\right)$$

$$X = 15,015.47$$

Jackson also approaches Balson Life Insurance Company. Balson calculates the amount of the benefit assuming uniform distribution of deaths between integral ages with mortality equal to the Standard Ultimate Life Table and interest equal to 5%.

b. (6 points) Calculate X from Balson.

Solution:

$$1,000,000 = 4X\ddot{a}_{50}^{(4)} = 4X\left(\alpha(4)\ddot{a}_{50} - \beta(4)\right) = 4X\left((1.00019)(17.0245) - 0.38272\right)$$

$$X = 15,019.51$$

c. (3 points) In real life, life insurance companies use the 2 term Woolhouse formula. List three reasons that Woolhouse is used.

Solution:

Valid Reasons would include:

- Simplicity of Calculation
- Produces a smaller payment that 3 term woolhouse or UDD
- Mortality does not actually follow UDD
- Woolhouse tends to provide a more accurate estimate than UDD

9. (10 points) You are given:

a. $A_{70} = 0.400$

b. $i = 0.06$

c. $q_{70} = 0.02$ and $q_{71} = 0.025$

Calculate $\ddot{a}_{71}$.

Solution:

$$\ddot{a}_{71} = \frac{1 - A_{71}}{d} = \frac{1 - 0.412244898}{0.06/1.06} = 10.3837$$

$$A_{70} = vq_{70} + v(1 - q_{70})A_{71}$$

$$0.400 = (1.06)^{-1}(0.02) + (1.06)^{-1}(1 - 0.02)A_{71}$$

$$A_{71} = \frac{0.400 - (1.06)^{-1}(0.02)}{(1.06)^{-1}(1 - 0.02)} = 0.412244898$$