STAT 472
Spring 2021
Test 3

May 5, 2021

1. Conley Life Insurance Company sells a whole life policy to Andrew who is (60). The policy pays a death benefit of 100,000 at the end of the year of death. The premiums for the policy are paid annually.

   You are given that:

   i. Mortality follows the Standard Ultimate Life Table.

   ii. $i = 0.05$

   iii. Commissions of 40% of premium in year 1 and 8% of premium thereafter.

   iv. Issue expenses of 400 per policy at time 0.

   v. Maintenance expenses of 30 at the beginning of each year including year 1.

   vi. Expense of paying a death claim is 300 and will be incurred at the end of the year of death.

   a. (3 points) Calculate the net premium reserve at the end of the 10^{th} year.

   Solution:

$$P = \frac{100,000A_{60}}{\ddot{a}_{60}} = \frac{(100,000)(0.29028)}{14.9041} = 1947.6520$$

$${}_{10}V^n = 100,000A_{70} - 1947.6520\ddot{a}_{70}$$

$$= (100,000)(0.42818) - (1947.6520)(12.0083) = 19,430.01$$

Or

$${}_{10}V^n = 100,000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right) = (100,000)\left(1 - \frac{12.0083}{14.9041}\right) = 19,429.55$$

b. (5 points) Calculate the gross premium using the equivalence principle.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{60} = 100,000A_{60} + 0.32P + 0.08P\ddot{a}_{60} + 400 + 30\ddot{a}_{60} + 300A_{60}$$

$$P = \frac{100,300A_{60} + 400 + 30\ddot{a}_{60}}{0.92\ddot{a}_{60} - 0.32} = \frac{(100,300)(0.29028) + 400 + (30)(14.9041)}{(0.92)(14.9041) - 0.32} = 2,237.36$$

c. (5 points) Calculate the gross premium reserve at the end of the 10th year.

Solution:

$${}_{10}V^{g} = PVFB + PVFE - PVFP$$

$$= 100,000A_{70} + (0.08)(2237.36)\ddot{a}_{70} + 30\ddot{a}_{70} + 300A_{70} - 2237.36\ddot{a}_{70}$$

$$= (100,300)(0.42818) - [(0.92)(2237.36) - 30](12.0083) = 18,589.16$$

d. (2 points) Calculate the expense reserve at the end of the 10th year.

Solution:

$${}_{10}V^{g} = {}_{10}V^{n} + {}_{10}V^{e}$$

$${}_{10}V^{e} = 18,589.16 - 19430.01 = -840.85$$

e. (5 points) Let L_0^g be the loss at issue random variable for this policy. You can write L_0^g as $Av^{K_{60}+1} + B + Ca_{\overline{K_{60}+1|}}$. Determine $A, B,$ and C .

Solution:

$$L_0^g = 100,300v^{K_{60}+1} + 400 + (0.32)(2237.36) + [30 - (0.92)(2237.36)]a_{\overline{K_{60}+1|}}$$

$$A = 100,300$$
$$B = 1115.96$$
$$C = -2028.37$$

f. (5 points) Calculate the $\sqrt{Var[L_0^g]}$.

Solution:

$$Var[L_0^g] = \left(100,300 + \frac{2028.37}{0.05/1.05}\right)^2 \left({}^2A_{60} - (A_{60})^2\right)$$

$$\sqrt{Var[L_0^g]} = \sqrt{\left(100,300 + \frac{2028.37}{0.05/1.05}\right)^2 \left(0.10834 - (0.29028)^2\right)} = 22,173$$

During the 10th year, actual experience was as follows:

i. Mortality was 90% the Standard Ultimate Life Table.

ii. $i = 0.055$

iii. Commissions 10% of premium.

iv. Maintenance expenses of 42 per policy at the beginning of the year.

v. Expense of paying a death claim was 250.

In calculating profit by source, the company allocates profits first to expenses, then to interest, and finally to mortality.

g. (8 points) Calculate the total profit in the 10th year.

Solution:

$$\text{Profit} = ({}_9V^g + P_9(1-e_9) - X_9^{BOY})(1+i) - (S_{10} + E_{10})q_{x+9} - {}_{10}V^g(1-q_{x+9})$$

$${}_9V^g = 100,000A_{69} + (0.08)(2237.36)\ddot{a}_{69} + 30\ddot{a}_{69} + 300A_{69} - 2237.36\ddot{a}_{69}$$

$$= (100,300)(0.41285) - [(0.92)(2237.36) - 30](12.3302) = 16,398.63$$

$$(16,398.63 + (2237.36)(1-0.1) - 42)(1.055) - (100,000 + 250)(0.9)(0.009294)$$
$$-(18,589.16)(1-(0.9)(0.009294))$$

$$= 108.40$$

h. (8 points) Calculate the profit from interest in the 10th year.

Solution:

$$G^{Expenses} = (16,398.63 + (2237.36)(1-0.1) - 42)(1.05) - (100,000+250)(0.009294) - (18,589.16)(1-(0.009294))$$

$$= -59.35$$

$$G^{Exp\&Int} = (16,398.63 + (2237.36)(1-0.1) - 42)(1.055) - (100,000+250)(0.009294) - (18,589.16)(1-(0.009294))$$

$$= 32.50$$

$$G^{Interest} = G^{Exp\&Int} - G^{Expenses} = 32.50 - (-59.35) = 91.36$$

2. Danielle buys a 21 year term insurance policy with a death benefit of 250,000 paid at the end of the year of death. Danielle is 44 years old. The premiums for this policy are paid annually.

   You are given that:

      i. Mortality follows the Standard Ultimate Life Table.

      ii. $i = 0.05$

   a. (3 Points) Calculate the first year premium under Full Preliminary Term reserves.

      Solution:

      $$P_1^{FPT} = 250,000vq_{44} = (250,000)(1.05)^{-1}(0.00071) = 169.05$$

   b. (5 points) Calculate the premium in year 2 and later under Full Preliminary Term reserves.

      Solution:

      $$P_{x+1}^{FPT} = \frac{(250,000)A^{1}_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}} = \frac{(250,000)(A_{45:\overline{20}|} - {}_{20}E_{45})}{\ddot{a}_{45:\overline{20}|}} = \frac{(250,000)(0.38385 - 0.35994)}{17.9391} = 461.97$$

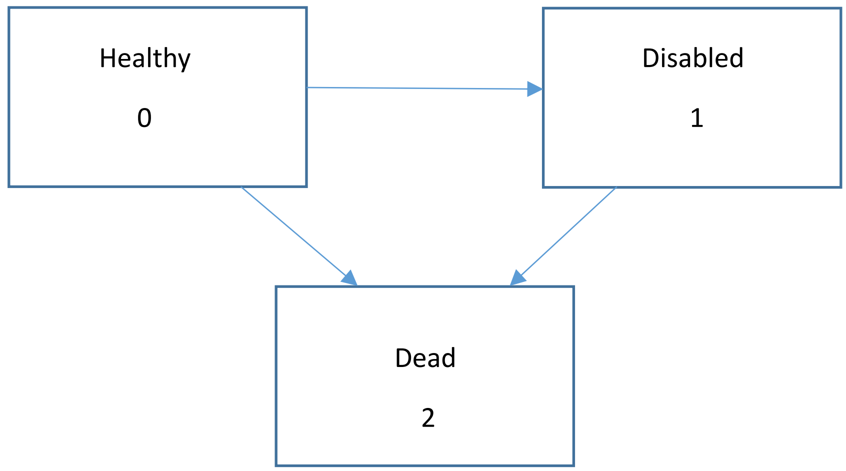
   c. (5 points) Calculate the Full Preliminary Term reserve at the end of the 11th year.

      Solution:

      $${}_{11}V = 250,000A^{1}_{55:\overline{10}|} - 461.97\ddot{a}_{55:\overline{10}|}$$

      $$= (250,000)(0.61813 - 0.59342) - (461.97)(8.0192) = 2472.87$$

3. Tugman Assurance Company uses a multi-state model to price and reserve their permanent disability products. The model is as follows:



You are also given that $\mu_x^{01} = 0.13$ $\mu_x^{02} = 0.04$ $\mu_x^{12} = 0.09$ and $\delta = 0.04$.

Tugman sells a five year term policy that pays 5000 at the moment that a person transitions from healthy to disabled. It also pays a continuous annuity to a person who is disabled at a rate of 12,000 per year.

Premiums are paid continuously.

a. (8 points) Calculate the present value of benefits for the 5000 paid at the moment of transition from Healthy to Disabled.

Solution:

$$5000\int_0^5 e^{-\delta t} \cdot {}_t p_x \cdot \mu_{x+t}^{01} dt = 5000\int_0^5 e^{-\delta t} \cdot e^{(\mu_{x+t}^{01}+\mu_{x+t}^{02})t} \mu_{x+t}^{01} dt$$

$$= 5000\int_0^5 e^{-\delta 0.04t} \cdot e^{(0.13+0.07)t}(0.13)dt = (5000)(0.13)\left(\frac{1-e^{-0.21(5)}}{0.21}\right) = 2012.10$$

b. (8 points) Calculate the present value of the benefits for the annuity paid while disabled.

Solution:

$$12,000\int_0^5 e^{-\delta t} \cdot {}_t p_x^{01} \cdot dt$$

$${}_t p_x^{01} = \int_0^t {}_s p_x^{00} \cdot \mu_{x+s}^{01} \cdot {}_{t-s} p_{x+s}^{11} ds = \int_0^t e^{-0.17s} \cdot (0.13) \cdot e^{-0.09(t-s)} ds$$

$$= 0.13 \cdot e^{-0.09t}\left(\frac{1-e^{-0.08t}}{0.08}\right) = \frac{0.13}{0.08} \cdot \left(e^{-0.09t} - e^{-0.17t}\right)$$

$$(12,000)\left(\frac{0.13}{0.08}\right)\int_0^5 \left(e^{-0.13t} - e^{-0.21t}\right)dt$$

$$= (12,000)\left(\frac{0.13}{0.08}\right)\left[\left(\frac{1-e^{-0.13(5)}}{0.13}\right) - \left(\frac{1-e^{-0.21(5)}}{0.21}\right)\right] = 11,330.21$$

c. (8 points) Calculate the premium paid continuously.

Solution:

$$PVP = PVB$$

$$P\int_0^5 e^{-\delta t} \cdot {}_t p_x^{00} \cdot dt = 2012.10 + 11,330.21$$

$$P\int_0^5 e^{-0.04t} \cdot e^{-0.17t} \cdot dt = P\left(\frac{1-e^{-0.21(5)}}{0.21}\right) = 13,342.31$$

$$P = 4310.18$$

4. Lives insured for nursing home coverage can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is In A Nursing Home
   iii. State 2 is Dead

   State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

   You are given the following matrix of annual transition probabilities.

$$\begin{bmatrix} 0.60 & 0.30 & 0.10 \\ 0.15 & 0.40 & 0.45 \\ 0 & 0 & 1 \end{bmatrix}$$

   An insurance company decides to issue a special two year insurance policy which pays a benefit of 10,000 at the end of the year of death. The policy will also pay 120,000 at the end of each year that the insured is in a nursing home. Finally, the policy will pay 50,000 to everyone who is healthy at the end of 2 years.

   Annual premiums will be paid only by healthy lives during the two years. The premium is determined using the equivalence principle.

   You are also given $v = 0.92$.

   a. (6 points) Calculate the annual premium.

      Solution:

	$Time\ 0$	$Time\ 1$	$Time\ 2$
$State\ 0$	1	0.6	$(0.6)(0.6)+(0.3)(0.15)=0.405$
$State\ 1$	0	0.3	$(0.6)(0.3)+(0.3)(0.4)=0.300$
$State\ 2$	0	0.1	$(0.6)(0.1)+(0.3)(0.45)+0.1=0.295$

$$PVP = PVB$$
$$P(1+v(0.6)) = (10,000)[0.1v+(0.295-0.1)v^2]+(120,000)[0.3v+0.3v^2]+(50,000)[0.405v^2]$$

$$P = \frac{2570.48+63,590.40+17,139.6}{1.552} = 53,672.99$$

b. (5 points) Calculate ${}_1V^{(0)}$.

Solution:

	Time 1	*Time* 2
State 0	1	0.6
State 1	0	0.3
State 2	0	0.1

$${}_1V^{(0)} = PVFB - PVFP =$$

$$(10,000)[0.1v] + (120,000)[0.3v] + (50,000)[0.6v] - 53,672.99 = 7967.01$$

c. (5 points) Calculate ${}_1V^{(1)}$.

Solution:

	Time 1	*Time* 2
State 0	0	0.15
State 1	1	0.4
State 2	0	0.45

$${}_1V^{(1)} = PVFB - PVFP =$$

$$(10,000)[0.45v] + (120,000)[0.4v] + (50,000)[0.15v] - 0 = 55,200$$

5. (6 points) Lives insured for disability income policies can be modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is Disabled
   iii. State 2 is Dead

State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

You are given:

   i. $\mu_x^{01} = 0.10t + 0.02$
   ii. $\mu_x^{02} = 0.12$
   iii. $\mu_x^{12} = 0.2t + 0.02$
   iv. $\mu_x^{10} = 0.15$

An insurance company sells 10,000 policies to independent healthy lives. Using the Euler method with h=1/2, determine how many lives will be healthy at the end of 1 year.

Solution:

We need $10{,}000\,{}_1p_x^{00}$. We know that ${}_0p_x^{00} = 1 \quad {}_0p_x^{01} = 0 \quad {}_0p_x^{02} = 0$

$$ {}_{0.5}p_x^{00} = {}_0p_x^{00} - {}_0p_x^{00}(0.5)(\mu_x^{01} + \mu_x^{02}) + {}_0p_x^{01}(0.5)(\mu_x^{10}) $$
$$ = 1 - (1)(0.5)(0.02 + 0.12) + (0)(0.5)(0.15) = 0.93 $$

$$ {}_{0.5}p_x^{01} = {}_0p_x^{01} - {}_0p_x^{01}(0.5)(\mu_x^{12}) + {}_0p_x^{00}(0.5)(\mu_x^{01}) $$
$$ 0 - (0)(0.5)(0.02) + (1)(0.5)(0.02) = 0.01 $$

$$ {}_{0.5}p_x^{02} = {}_0p_x^{02} + {}_0p_x^{01}(0.5)(\mu_x^{12}) + {}_0p_x^{00}(0.5)(\mu_x^{02}) $$
$$ = 0 + (0)(0.5)(0.02) + 1(0.5)(0.12) = 0.06 $$

$$ {}_1p_x^{00} = {}_{0.5}p_x^{00} - {}_{0.5}p_x^{00}(0.5)(\mu_{x+0.5}^{01} + \mu_{x+0.5}^{02}) + {}_{0.5}p_x^{01}(0.5)(\mu_{x+0.5}^{10}) $$
$$ = 0.93 - (0.93)(0.5)(0.07 + 0.12) + (0.01)(0.5)(0.15) = 0.8424 $$