Chapter 8

1. You are given a multiple decrement model with decrements of (1) death by natural causes and (2) death by accidental causes.

You are also given:

- \[ \mu_s^{(1)} = 0.031 \]
- \[ \mu_s^{(2)} = 0.015 \]
- \[ \delta = 0.05 \]

a. Calculate the annual net benefit premium rate paid continuously for a whole life policy issued to (x) that pays 100,000 at the moment of death when death is from an accident and pays 70,000 at the moment of death when death is from natural causes.

b. A 10 year term insurance policy issued to (x) pays 50,000 at the moment of death for any death plus an additional 40,000 at the moment of death if the death is from accidental causes. The annual net benefit premium is paid continuously. Calculate the annual net benefit premium rate.

c. A fully discrete whole policy issued to (x) pays 100,000 upon a death from natural causes. It also pays 300,000 upon death from accidental causes. The net benefit premium is paid annually for 20 years during the lifetime of the insured. Calculate the annual net benefit premium.
2. Mayfawny purchases a whole life insurance policy.

There are three ways that Mayfawny’s policy can terminate:
   a. Death (1)
   b. Diagnosis of a critical illness (2); and
   c. Lapse (3).

The policy pays a death benefit of 10,000 at the moment of death. The policy will also pay a critical illness benefit of 20,000 if Mayfawny is diagnosed with a critical illness. Only one benefit will be paid.

There is no benefit paid upon lapse.

You are also given:
   i. \( \mu_x^{(1)} = 0.01 \)
   ii. \( \mu_x^{(2)} = 0.015 \)
   iii. \( \mu_x^{(3)} = 0.06 \)
   iv. \( \delta = 0.035 \)

Mayfawny pays a net premium continuously for her lifetime as long as the policy is in force. The net premium is determined using the equivalence principle.

Calculate the net premium paid by Mayfawny.

3. Jeff is receiving a salary paid continuously for as long as he is in employed at Purdue. Jeff can leave employment through death (1), retirement (2), or disability (3). Once Jeff leaves employment, the salary stops.

You are given:
   i. The salary pays at an annual rate of 70,000 per year.
   ii. \( \delta = 0.05 \)
   iii. Jeff is currently age 59.
   iv. Jeff will retire at age 65 if he is still teaching. He will not retire prior to age 65.
   v. \( \mu_t^{(1)} = 0.01 + 0.001t \)
   vi. \( \mu_t^{(3)} = 0.03 - 0.001t \)

Calculate the present value of Jeff’s future earnings while employed at Purdue.
4. You are given the following table where decrement (1) is death, decrement (2) is lapse, and decrement (3) is diagnosis of critical illness:

<table>
<thead>
<tr>
<th>x</th>
<th>$d_x^{(1)}$</th>
<th>$d_x^{(2)}$</th>
<th>$d_x^{(3)}$</th>
<th>$p_x^{(1)}$</th>
<th>$p_x^{(2)}$</th>
<th>$l_x^{(1)}$</th>
<th>$d_x^{(1)}$</th>
<th>$d_x^{(2)}$</th>
<th>$d_x^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.02</td>
<td>0.15</td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.03</td>
<td>0.06</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.04</td>
<td>0.04</td>
<td>0.020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>0.05</td>
<td>0.03</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>0.06</td>
<td>0.02</td>
<td>0.030</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Complete the table using a radix of 10,000.
b. Calculate:
   i. $3 \frac{q_{55}}{p_{55}}$
   ii. $2 \frac{q_{56}}{q_{56}}$
   iii. $\frac{1}{2} \frac{q_{56}}{q_{56}}$
   iv. The probability that a person age 55 will decrement from death or critical illness before age 60.
c. Assuming uniform distribution of each decrement between integer ages, calculate:
   i. $0.25 q_{55}^{(2)}$
   ii. $0.5 p_{56}^{(2)}$
   iii. $0.5 p_{56.8}^{(3)}$
   iv. $0.5 q_{56.8}^{(1)}$
d. Assuming a constant force of decrement for each decrement between integer ages, calculate:
   i. $0.25 q_{55}^{(2)}$
   ii. $0.5 p_{56}^{(2)}$
   iii. $0.5 p_{56.8}^{(3)}$
   iv. $0.5 q_{56.8}^{(1)}$
5. A fully discrete 3 year term pays a benefit of 1000 upon any death. It pays an additional 1000 (for a total of 2000) upon death from accident. You are given:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x^{(1)}$</th>
<th>$q_x^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.030</td>
<td>0.010</td>
</tr>
<tr>
<td>21</td>
<td>0.025</td>
<td>0.020</td>
</tr>
<tr>
<td>22</td>
<td>0.020</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Decrement (1) is death from accidental causes while decrement (2) is death from non-accidental causes.

The annual effective interest rate is 10%.

a. Calculate the level annual net premium for this insurance.

b. Calculate the net premium reserve at the end of year 0, 1, 2, and 3.

6. You are given:
   a. $q_x^{(1)} = 0.200$
   b. $q_x^{(2)} = 0.080$
   c. $q_x^{(3)} = 0.125$

Assuming that each decrement is uniformly distributed over each year of age in the associated single decrement table, calculate $q_x^{(1)}$.

7. You are given:
   a. $q_x^{(1)} = 0.200$
   b. $q_x^{(2)} = 0.080$
   c. $q_x^{(3)} = 0.125$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate $q_x^{(1)}$.

8. You are given the following for a double decrement table:
   a. $q_x^{(1)} = 0.200$
   b. $q_x^{(2)} = 0.080$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate $0.4q_x^{(1)} + 0.4$. 
9. You are given:
   a. \( q_x^{r(1)} = 0.200 \)
   b. \( q_x^{r(2)} = 0.080 \)

Decrement (1) is uniformly distributed over the year. Decrement (2) occurs at time 0.6.

Calculate \( q_x^{r(1)} \) and \( q_x^{r(2)} \).

10. For a double decrement table with \( l_{40}^r = 2000 \):

<table>
<thead>
<tr>
<th>x</th>
<th>( q_x^{(1)} )</th>
<th>( q_x^{(2)} )</th>
<th>( q_x^{r(1)} )</th>
<th>( q_x^{r(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.24</td>
<td>0.10</td>
<td>0.25</td>
<td>y</td>
</tr>
<tr>
<td>41</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
<td>2y</td>
</tr>
</tbody>
</table>

Calculate \( l_{42}^r \).

11. You are given the following excerpt from a double decrement table:

<table>
<thead>
<tr>
<th>x</th>
<th>( l_x^{(r)} )</th>
<th>( q_x^{(1)} )</th>
<th>( q_x^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>---</td>
<td>0.025</td>
<td>0.030</td>
</tr>
<tr>
<td>54</td>
<td>5000</td>
<td>--</td>
<td>0.040</td>
</tr>
<tr>
<td>55</td>
<td>4625</td>
<td>0.055</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Calculate \( \Delta q_{53}^{(1)} \).
12. For iPhones, the phone may cease service for mechanical failure or for other reasons (lost, stolen, dropped in a pitcher of beer, etc). You are given the following double decrement table:

<table>
<thead>
<tr>
<th>Year of Service</th>
<th>For an iPhone at the beginning of the year of service, probability of</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mechanical Failure</td>
<td>Failure for Other Reasons</td>
<td>Survival through the year of service</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.30</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>0.40</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
</tr>
</tbody>
</table>

You are also given:

c. The number of iPhones that terminate for other reasons in year 3 is 40% of the number of iPhones that survive to the end of year 2.

d. The number of iPhones that terminate for other reasons in year 2 is 80% of the number of iPhones that survive to the end of year 2.

Calculate the probability that an iPhone will cease to function due to mechanical failure during the three year period following its entry into service.

13. Your actuarial student has constructed a multiple decrement table using independent mortality and lapse tables. The multiple decrement table values, where decrement $d$ is death and decrement $w$ is lapse, are as follows:

<table>
<thead>
<tr>
<th>$l_{60}^{(3)}$</th>
<th>$d_{60}^{(d)}$</th>
<th>$d_{60}^{(w)}$</th>
<th>$l_{61}^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>950,000</td>
<td>2,580</td>
<td>94,742</td>
<td>852,678</td>
</tr>
</tbody>
</table>

You discover that an incorrect value of $q_{60}^{(w)}$ was taken from the independent lapse table. The correct value is 0.05.

Decretments are uniformly distributed over each year of age in the multiple decrement table.

You correct the multiple decrement table, keeping $l_{60}^{(3)} = 950,000$.

Calculate the correct values of $d_{60}^{(w)}$. 
14. A person age 60 is subject to three decrements. You are given:

i. The following excerpt from a triple decrement table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q_x^{(1)} )</th>
<th>( q_x^{(2)} )</th>
<th>( q_x^{(3)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.05</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>61</td>
<td>0.00</td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>

ii. Decrement 1 occurs exactly one quarter of the way through the year.

iii. \( q_x^{(2)} = t q_x^{(2)} \) for integer \( x \) and \( 0 \leq x \leq 1 \).

iv. \( q_x^{(3)} = \begin{cases} 0, & \text{for integer } x \text{ and } 0 \leq t \leq 0.5 \\ 2(t-0.5)q_x^{(3)}, & \text{for integer } x \text{ and } 0.5 \leq t \leq 1 \end{cases} \)

a. Calculate \( \Phi^{(r)}_{60} \).

**Solution:**

\[
\Phi^{(r)}_{60} = \Phi^{(r)}_{60} \cdot \Phi^{(r)}_{61} = (1-0.05-0.10-0.08)(1-0.00-0.14-0.12) = 0.5698
\]
b. Calculate $0.8 P_{60}^{(r)}$.

**Solution:**

Let $l_{60}^{(r)} = 1000$

\[
d_{60}^{(1)} = l_{60}^{(r)} \cdot q_{60}^{(1)} = (1000)(0.05) = 50. \text{ These all occur at time 0.25.}
\]

\[
d_{60}^{(2)} = l_{60}^{(r)} \cdot q_{60}^{(2)} = (1000)(0.10) = 100. \text{ These decrements are uniformly distributed throughout the year.}
\]

\[
d_{60}^{(3)} = l_{60}^{(r)} \cdot q_{60}^{(3)} = (1000)(0.08) = 80. \text{ These decrements begin at time 0.5 and are uniformly distributed throughout the second half of the year.}
\]

\[
0.8 P_{60}^{(r)} = \frac{l_{60}^{(r)}}{l_{60}^{(r)}}
\]

\[
1000 - 50 \text{ [decrement 1 all occurred at time 0.25]}
- (100)(0.8) \text{ [decrement 2 is uniformly distributed and 0.8 of the year has passed]}
- (80)(0.6) \text{ [decrement 3 is uniformly distributed during the second half of the year and 0.6 of the second half of the year has passed]}
= \frac{1000}{1000} = 0.822
\]
Answers

1. 
   a. 3670  
   b. 2900  
   c. 8279.41

2. 400

3. 324,529.14

4. 
   a. 

<table>
<thead>
<tr>
<th>x</th>
<th>$q_s^{(1)}$</th>
<th>$q_s^{(2)}$</th>
<th>$q_s^{(3)}$</th>
<th>$p_s^{(r)}$</th>
<th>$l_s^{(r)}$</th>
<th>$d_s^{(1)}$</th>
<th>$d_s^{(2)}$</th>
<th>$d_s^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.02</td>
<td>0.15</td>
<td>0.010</td>
<td>0.820</td>
<td>10,000</td>
<td>200</td>
<td>1500</td>
<td>100</td>
</tr>
<tr>
<td>56</td>
<td>0.03</td>
<td>0.06</td>
<td>0.015</td>
<td>0.895</td>
<td>8,200</td>
<td>246</td>
<td>492</td>
<td>123</td>
</tr>
<tr>
<td>57</td>
<td>0.04</td>
<td>0.04</td>
<td>0.020</td>
<td>0.900</td>
<td>7,339</td>
<td>293.56</td>
<td>293.56</td>
<td>146.78</td>
</tr>
<tr>
<td>58</td>
<td>0.05</td>
<td>0.03</td>
<td>0.025</td>
<td>0.895</td>
<td>6,605.1</td>
<td>330.255</td>
<td>198.153</td>
<td>165.1275</td>
</tr>
<tr>
<td>59</td>
<td>0.06</td>
<td>0.02</td>
<td>0.030</td>
<td>0.890</td>
<td>5,911.56845</td>
<td>354.69387</td>
<td>118.23129</td>
<td>177.346935</td>
</tr>
</tbody>
</table>

   b. 
      i. 0.66051
      ii. 0.0958
      iii. 0.026978
      iv. 0.21368

   c. 
      i. 0.0375
      ii. 0.9475
      iii. 0.94776
      iv. 0.01173

   d. 
      i. 0.04034
      ii. 0.94604
      iii. 0.94763
      iv. 0.01139

5. 
   a. 63.64
   b. The reserve is zero at the end of each year.

6. 0.180167

7. 0.180520

8. 0.085951
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.0704</td>
</tr>
<tr>
<td>10</td>
<td>802.56</td>
</tr>
<tr>
<td>11</td>
<td>0.058075</td>
</tr>
<tr>
<td>12</td>
<td>0.35</td>
</tr>
<tr>
<td>13</td>
<td>47,433</td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a. 0.5698</td>
</tr>
<tr>
<td></td>
<td>b. 0.8220</td>
</tr>
</tbody>
</table>