## STAT 475 <br> TEST 1

Spring 2018
February 15, 2018

1. Denis and Lucy are partners in H\&R Consulting. Denis is (45) and Lucy is (35). They have purchased an insurance policy from Clark Life Insurance Company. The policy pays the following benefits:
i. A death benefit of 500,000 paid at the end of the year of the second death; and
ii. A reversionary annuity which pays 100,000 at the beginning of each year that Denis is alive but Lucy is dead.

Premiums for this policy are paid annually as long as at least one of the partners is alive.
You are given:
i. Mortality follows the Illustrative Life Table.
ii. Denis and Lucy are assumed to be independent lives.
iii. $\quad i=0.06$.
a. (6 points) The present value of benefits is 114,000 to the nearest 1000 . Calculate it to the nearest 1.
b. (6 points) Calculate the annual premium for the policy using the equivalence principle.

Clark Life Insurance Company offers a rider on the policy. The rider will pay the premium on the life insurance policy if only one of the partners is alive. This is equivalent to a survivor annuity that pays zero when both Denis and Lucy are alive but pays the premium amount when only Lucy or only Denis is alive.
c. (6 points) Calculate the Actuarial Present Value of the benefit under this rider.

Premiums for this rider are paid annually, but only while both partners are alive.
d. (6 points) Calculate the annual premium for this rider.

Denis and Lucy decide to purchase the rider from Clark Life Insurance Company.
e. ( 6 points) Calculate the probability that the premium at the beginning of the $20^{\text {th }}$ year will be paid by the rider.
2. Bryan is a participant in a defined benefit plan at Dai Actuarial Consultants. The retirement benefit is $2 \%$ of the three year final average salary for each year of service.

You are given:
i. Bryan was born on February $15,1980$.
ii. Bryan was hired by Dai Actuarial Consultants on February 15, 2008.
iii. Bryan has earned or will earned the following total salary during each timeframe listed in the table:

| Date | Total Salary Earned |
| :---: | :---: |
| February 15, 2008 to February 15, 2015 | 210,000 |
| February 15, 2015 to February 15, 2016 | 40,000 |
| February 15, 2016 to February 15, 2017 | 44,000 |
| February 15, 2017 to February 15,2018 | 50,000 |
| February 15, 2018 to February 15, 2019 | 60,000 |

In completing your calculations, you should use the following assumptions:
i. Bryan's salary will increase by $3 \%$ on each future February 15 beginning with February 15, 2019.
ii. If Bryan remains employed by Dai, he will retire at exact age 65 .
iii. There is a $5 \%$ probability that Bryan will leave employment (for any reason) with Dai in each of the next ten years measured from February 15, 2018. After that, the probability that Bryan will leave employment is $2 \%$ each year until he is age 65.
iv. The only benefit provided by the defined benefit plan is a retirement benefit.
$v$. The interest rate prior to retirement is $4.5 \%$.
vi. The retirement benefit will be paid annually with the first payment at age 65 . The benefit will be a single life annuity based on Bryan's life.
vii. $\quad \ddot{a}_{65}=10$
a. (6 points) Calculate Bryan's projected annual retirement benefit at age 65 using a Projected Unit Cost method.
b. (6 points) Calculate Bryan's projected replacement ratio.

Dai hires Raza Retirement Consulting to complete a valuation on this plan as of February 15, 2018 using the Projected Unit Cost Method.
c. (6 points) Calculate the Accrued Benefit for Bryan using the Projected Unit Cost method.
d. (6 points) Calculate the Accrued Actuarial Liability for Bryan as of February 15, 2018 using the Projected Unit Cost method.
e. (6 points) Calculate the Normal Contribution on February 15, 2018 for the year ended February 15, 2019 using the Projected Unit Cost Method.
3. Jain Life Insurance Company sells a two year term life insurance policy to (80) that pays 100,000 at the end of the year of death if death is from natural causes and pays 200,000 at the end of the year of death if death is from accidental causes.

Sanchita prices this policy using a multiple decrement table where decrement (1) is death from natural causes, decrement (2) is death from accidental causes, and decrement (3) is lapse.

If the policy lapses, no benefits are paid.
Premiums for this policy are paid annually as long as the policy is in force.
You are given:
i. The following multiple decrement table:

| x | $q_{x}^{(1)}$ | $q_{x}^{(2)}$ | $q_{x}^{(3)}$ |
| :---: | :---: | :---: | :---: |
| 80 | 0.08 | 0.02 | 0.20 |
| 81 | 0.12 | 0.04 | 0.00 |

ii. $\quad v=0.92$
a. (6 points) The Actuarial Present Value of Benefits is 22,900 to the nearest 100. Calculate it to the nearest 1.
b. ( 6 points) Calculate the annual net benefit premium for this policy using the equivalence principle.
c. (18 points) Let $L_{0}$ be the present value loss at issue random variable for this policy. In order to calculate the $\operatorname{Var}\left[L_{0}\right]$, you must use first principles. There are six possible values of the random variable. Complete the table below by describing each case, the value of the random variable in that case, and probability that the case will happen. You do NOT need to calculate the variance. Show your work.

| Case | Description of Case |  | Value of $L_{0}$ |
| :---: | :--- | :--- | :--- |
| 1 |  |  | Probability |
|  |  |  |  |
| 2 |  |  |  |
| 5 |  |  |  |
|  |  |  |  |
|  |  |  |  |


|  |  |  |  |
| :--- | :--- | :--- | :--- |
| 6 |  |  |  |
|  |  |  |  |

4. (10 points) You are given the following independent single decrement rates: $q_{50}^{\prime(1)}=0.10$ and $q_{50}^{\prime(2)}=0.20$

Decrement (1) occurs uniformly throughout the year. Decrement (2) occurs three quarters of the way throughout the year.

Complete the following multiple decrement table. Be sure to show your work.

| Age | $l_{x}^{(\tau)}$ | $d_{x}^{(1)}$ | $d_{x}^{(2)}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 50 | 10,000 |  |  |
|  |  |  |  |

