STAT 475
Test 3
Spring 2018

April 19, 2018

1. Madison who is (60) buys a fully discrete participating whole life policy with a death benefit of 100,000 and annual premiums of 4500.

   The reserves for this policy are net premium reserves based on the Illustrative Life Table and an annual interest rate of 6%. Cash values are 85% of the reserve.

   Madison uses the dividends to purchase compound reversionary bonuses. At the end of the 9^{th} year, the previous dividends have been used to purchase 20,000 of paid up death benefit.

   The following is the actual experience during the 10^{th} policy year:

   i. Mortality was 90% of the Illustrative Life Table

   ii. The actual interest rate earned was 6.8%.

   iii. Lapses were 5%.

   iv. Expenses were 100 per policy at the start of the policy year plus 8% of the gross premium.

   v. A dividend equal to 80% of the profit in year 10 will be paid to the policyholders who were inforce at the start of the year.

   a. (10 points) The reserve at the end of the ninth year is 30,700 to the nearest 25. This includes the reserve for the death benefit related to the past reversionary bonuses. Calculate it to the nearest 1.

   Solution:

$${}_9V = 100,000\left(1 - \frac{\ddot{a}_{69}}{\ddot{a}_{60}}\right) + 20,000A_{69}$$

$$= (100,000)\left(1 - \frac{8.8387}{11.1454}\right) + (20,000)(0.49970)$$

$$= 30,690.43$$

b. (10 points) Calculate the reserve at the end of the 10th year.

Solution:

$$ {}_{10}V = 100,000\left(1-\frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right)+20,000A_{70} $$

$$ =(100,000)\left(1-\frac{8.5693}{11.1454}\right)+(20,000)(0.51495) $$

$$ =33,412.57 $$

c. (16 points) The profit before the dividend at the end of the 10th year is 1550 to the nearest 25. Calculate it to the nearest 1.

Solution:

$$ \mathrm{Pr}_{10} = ({}_9V + P_{10} - E_{10})(1+i_{10}) - Sq_{x+9} - {}_{10}CV(1-q_{x+9})(w_{10}) - {}_{10}V(1-q_{x+9})(1-w_{10}) $$

$$ \begin{aligned} &=[30,690.43+4500(0.92)-100](1.068)-(100,000+20,000)(0.9)(0.03037) \\ &\qquad -(0.85)(33,412.57)[1-(0.9)(0.03037)](0.05)-(33,412.57)([1-(0.9)(0.03037)](1-0.05) \end{aligned} $$

$$ =1556.58 $$

d. (2 points) Calculate the dividend at the end of the 10th year.

Solution:

$$Div_{10} = (\Pr_{10})(0.80) = (1556.58)(0.8) = 1245.26$$

e. (6 points) Calculate the additional death benefit that will be purchased with this dividend.

Solution:

$$\text{Additonal Death Benefit} = \frac{Div_{10}}{A_{60+10}} = \frac{1245.26}{0.51495} = 2418.22$$

2. Lisa purchases a Type B Universal Life policy. Lisa is (50). The Additional Death Benefit for Lisa's policy is 250,000. The cost of insurance for Lisa's policy is 100% of the mortality rates in the Illustrated Life Table. Additionally, you are given that:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Discount Rate for COI	Annual Credited Interest Rate
1	5000	25%	60	4%	6.0%
2	5000	5%	60	4%	5.8%

a. (12 points) Determine the Account Value at the end of the second year.

Solution:

$$AV_0 = 0$$

$$AV_1 = (AV_0 + P - E - COI)(1 + i^c)$$

$$= [0 + (5000)(1 - 0.25) - 60 - (250,000)(1.04)^{-1}(0.00592)](1.06) = 2402.94$$

$$AV_2 = [2402.94 + (5000)(1 - 0.05) - 60 - (250,000)(1.04)^{-1}(0.00642)](1.058)$$

$$= 5871.55$$

If Lisa pays 5000 at the beginning each year for 20 years and the annual credited rate is 4% every year, then at the end of 20 years, Lisa has an account value of 40,000.

b. (12 points) If Lisa wanted to have 100,000 at the end of 20 years, she should have paid P at the beginning of each year for 20 years. Determine P.

Solution:

$$(0.95)(5000\ddot{s}_{\overline{20|}}) - 0.2(5000)(1+i)^{20}$$
$$- AccumulatedOtherExpenses - AccumulatedCOI = 40,000$$

$$\text{Let } P = 5000 + X$$

$$(0.95)(5000 + X)(\ddot{s}_{\overline{20|}}) - 0.2(5000 + X)(1+i)^{20}$$
$$- AccumulatedOtherExpenses - AccumulatedCOI = 100,000$$

$$(0.95)X(\ddot{s}_{\overline{20|}}) + (0.95)(5000)(\ddot{s}_{\overline{20|}}) - 0.2(X)(1+i)^{20} - (0.2)(5000)(1+i)^{20}$$
$$- AccumulatedOtherExpenses - AccumulatedCOI = 100,000$$

$$(0.95)X(\ddot{s}_{\overline{20|}}) - 0.2(X)(1+i)^{20} + 40,000 = 100,000$$

$$X = \frac{60,000}{(0.95)(\ddot{s}_{\overline{20|}}) - 0.2(1+i)^{20}} = 2070.21$$

$$P = 5000 + 2070.21 = 7070.21$$

3. (14 points) For a fully continuous whole life of 500,000 on (20), you are given:

   a. The gross premium reserve at $t = 10$ is 20,000.
   b. The gross premium is paid at a rate of 2300 per year.
   c. The force of interest is 5% .
   d. $\mu_{20+t} = 0.001 + 0.0006t$
   e. The following expenses payable continuously:
      i. 60% of premium in the first year and 4% of premium in years 2 and later;
      ii. 140 per policy in the first year and 40 per policy in years 2 and later; and
      iii. 500 payable at the moment of death.

   Estimate the gross premium reserve at $t = 10.5$ using Euler's method with $h = 0.5$.

   Solution:

$$_{10.5}V = {}_{10}V + 0.5\left[{}_{10}V \cdot \delta + P - E - (S + E) \cdot \mu_{10}\right] =$$

$$20,000 + 0.5\left[(20,000)(0.05) + (2300)(0.96) - 40 - (500,000 + 500 - 20,000)(0.001 + 0.0006(10))\right]$$

$$= 19,902.25$$

4. Dylan who is (45) purchases a fully discrete whole life policy with a death benefit of 10,000. You are given that mortality follows the Illustrative Life Table and that interest is equal to 6%.

Let Z be the present value random variable for Dylan's insurance.

a. (6 points) Calculate the $Var(Z)$.

Solution:

$$Var(Z) = (10,000)^2\left[{}^2A_{45} - (A_{45})^2\right]$$

$$= (10,000)^2\left[0.06802 - (0.20120)^2\right] = 2,753,856$$

Dylan's policy has a gross premium of 300. The expenses for this policy are:

i. 20% of premium in the first year and 5% thereafter

ii. 1000 per policy in the first year and 50 per policy thereafter

iii. 500 per policy payable at the end of the year of death

Let L_0^g be the loss at issue random variable based on the gross premium for Dylan's insurance.

b. (12 points) Calculate the $Var(L_0^g)$.

Solution:

$$L_0^g = 10{,}500v^{K_x+1} + 950 + (0.15)(300) + (50)\ddot{a}_{\overline{K_x+1|}} - (0.95)(300)\ddot{a}_{\overline{K_x+1|}}$$

$$= 10{,}500v^{K_x+1} + 995 - (235)\left(\frac{1-v^{K_x+1}}{d}\right)$$

$$= \left[10{,}500 + \frac{235}{d}\right]v^{K_x+1} + 995 - \left(\frac{235}{d}\right)$$

$$Var\left(L_0^g\right) = Var\left(\left[10{,}500 + \frac{235}{d}\right]v^{K_x+1} + 995 - \left(\frac{235}{d}\right)\right)$$

$$= \left[10{,}500 + \frac{235}{d}\right]^2 Var\left(v^{K_x+1}\right) = \left[10{,}500 + \frac{235}{d}\right]\left({}^2A_{45} - (A_{45})^2\right)$$

$$\left[10{,}500 + \frac{235}{0.06/1.06}\right]\left[0.06802 - (0.20120)^2\right] = 5{,}911{,}739$$