1. Jack is an employee of the H3 (Hsu, He, and Heckler) Consulting firm. Jack began working at exact age 25 at H3 and today is his 40th birthday.

   H3 has a defined benefit plan which pays a pension benefit of 2% of final 3 year average salary for each year of service. All salary increase occur on birthdates. Jack salary just increased to 200,000 on his 40th birthday. It is assumed that salary increases will occur on Jack's birthday and that each increase will be 3%. Further, it is assumed that Jack will retire at age 60.

   a. Jack's projected annual pension benefit at age 60 is 238,400 to the nearest 100. Calculate it to the nearest 1.

      Solution:

      $$(0.02)(35)\frac{200,000\left[(1.03)^{17}+(1.03)^{18}+(1.03)^{19}\right]}{3}=238,410$$

   Jack's pension benefit will be paid as an annual life annuity due. Assume that mortality after retirement follows the Standard Ultimate Life Table after retirement. Further the interest rate for calculations after retirement is 5%.

   b. Calculate the amount that H3 would need to have at age 60 to fund Jack's benefit. This is the present value of future benefits at age 60.

      Solution:

      $$APV=(238,410)\ddot{a}_{60}=(238,410)(14.9041)=3,553,287$$

Prior to retirement, decrements from H3 are assumed to follow the Standard Service Table with interest at 8%. Under the Standard Service Table, all lives that reach exact age 60 are assumed to retire at exact age 60. There are no benefits other than pension benefits.

c. Using the Projected Unit Cost Method, Jack's accrued benefit is 102,200 to the nearest 100. Calculate Jack's accrued benefit to the nearest 1.

Solution:

$$AccBen = (0.02)(15)\frac{200,000\left[(1.03)^{17}+(1.03)^{18}+(1.03)^{19}\right]}{3} = 102,176$$

d. Using the Projected Unit Cost Method, calculate Jack's Actuarial Accrued Liability on his 40th birthday.

Solution:

$$AAL = (AccBen)(\ddot{a}_{60})({}_{20}p_{40}^{(\tau)})(v^{20})$$

$$= (102,176)(14.9041)\left(\frac{93,085.4}{169,205.8}\right)(1.08)^{-20} = 179,740$$

e. Using the Projected Unit Cost Method, calculate Jack's normal cost for the period between is 40th and 41st birthday.

Solution:

Under PUC:

$$Normal\ \mathrm{Cos}\,t = \frac{AAL}{Yrs\ Of\ \ Service} = \frac{179,740}{15} = 11,983$$

Or

$$(AccBen)_{16} = (\alpha)(FAS^{Projected})(SrvYrs) = (0.02)\frac{200,000\left[(1.03)^{17}+(1.03)^{18}+(1.03)^{19}\right]}{3}(16) = 108,988$$

$${}_{16}V = (108,988)(14.9041)\left(\frac{93,085.4}{160,707.9}\right)(1.08)^{-19} = 218,009$$

$$179,740 + C_{15} = 0 + (218,009)\left(\frac{160,707.9}{169,205.8}\right)(1.08)^{-1} ==> C_{15} = 11,983$$

H3 also provides a survivor whole life insurance policy to Jack upon his retirement at age 60. The policy provides a death benefit of 500,000 at the end of the year of death of the survivor of Jack or his spouse. Premiums are paid annually by H3 while both Jack and his spouse are alive. Assume that Jack's spouse is 10 years younger than Jack. Remember that mortality after Jack's retirement follows the Standard Ultimate Life Table and that $i = 5\%$.

f. Calculate the net annual premium for this policy.

Solution:

$$P = \frac{500,000(A_{60} + A_{50} - A_{50:60})}{\ddot{a}_{50:60}} = \frac{500,000(0.29028 + 0.18931 - 0.32048)}{14.2699} = 5575.02$$

At age 60 when Jack retires, he will also receive a survivor life annuity due with annual payments as an additional payment over and above the retirement benefit. The annuity will pay 50,000 at the beginning of each year while Jack and his spouse are alive. It will pay 37,500 while only Jack is alive. It will pay 25,000 while only his spouse is alive.

g. Calculate the actuarial present value of this annuity.

Solution:

$$APV = 37,500\ddot{a}_{60} + 25,000\ddot{a}_{50} + (50,000 - 37,500 - 25,000)\ddot{a}_{50:60}$$

$$= (37,500)(14.9041) + (25,000)(17.0245) - (12,500)(14.2699) = 806,142.50$$

H3 also provides Retiree Healthcare to Jack upon his retirement at age 60. You are given:

i. The annual benefit premium for retiree healthcare cover for a life age 60 today is $B(60,0)=5000$.

ii. $c=1.03,\ j=6\%,\ i=5\%$

iii. As stated above, retirements follow the Standard Service Table except all lives at exact age 60 will retire at that point.

iv. $\ddot{a}_{60|i^*}=35$

h. Calculate i^*.

Solution:

$$i^* = \frac{1+i}{(c)(1+j)} - 1 = \frac{1.05}{(1.03)(1.06)} - 1 = -0.0382854$$

i. Calculate the AVTHB for Jack at age 40.

Solution:

$$AVTHB = B(60,20)v^{20}\,{}_{20}p_{40}^{(\tau)}(\ddot{a}_{60|i^*})$$

$$= (5000)(1.06)^{20}(1.05)^{-20}\left(\frac{93,085.4}{169,205.8}\right)(35) = 116,368.56$$

Let N be the random variable that represents the number of lives age 40 out of 10,000 lives that will receive the Retiree Healthcare benefit.

j. Calculate the $Var[N]$

Solution:

$$Var[N] = npq = (1000)\left(\frac{93,085.4}{169,205.8}\right)\left(1 - \frac{93,085.4}{169,205.8}\right) = 2474.87$$

2. The Tan Life Insurance Company sells a two year term policy to x which pays a death benefit of 10,000 at the end of the year of death and has annual premiums. You are given the following additional information:

    i. $i = 0.08$

    ii. $q_x = 0.10$ and $q_{x+1} = 0.20$

    iii. Reserves are net premium reserves.

    iv. ${}_1V = 505.05$

    v. The gross premium for the policy is 1800.

    vi. Pre-Issue expenses are 15% of premium and 100 per policy.

    vii. Actual maintenance expenses for the policy are 5% of the premium and 50 per policy at the beginning of each year.

    viii. Withdrawals at the end of the first year are 25%. There are no withdrawals at the end of the second year.

    ix. The cash value at the end of the first year is 400.

    x. $\Pr_0 = -370$ and $\Pr_2 = 338.25$

a. The value of $\Pr_1$ to the nearest 10 is 360. Calculate $\Pr_1$ to the nearest 0.01.

Solution:

$$\Pr_1 = ({}_0V + P - Exp)(1+i) - q_x(DB) - CV(1-q_x)(w_x) - {}_1V(1-q_x)(1-w_x)$$

$$= [0 + 1800 - (0.05)(1800) - 50](1.08) - (10,000)(0.1)$$
$$- (400)(1-0.1)(0.25) - (505.05)(1-0.1)(1-0.25)$$

$$= 361.89$$

b. Calculate the Profit Margin for this policy using an annual effective interest rate of 12%.

Solution:

$$PM = \frac{\pi_0 + \pi_1 v + \pi_2 v^2}{P(1 + v \cdot {}_1p_x^{(\tau)})} = \frac{\Pr_0 + \Pr_1 v + \Pr_2 \cdot {}_1p_x^{(\tau)} \cdot v^2}{P(1 + v \cdot {}_1p_x^{(\tau)})}$$

$$= \frac{-370 + 361.89(1.12)^{-1} + 338.75(0.9)(0.75)(1.12)^{-2}}{1800[1 + (1.12)^{-1}(0.9)(0.75)]} = 0.0468$$