# STAT 475
# Quiz 1
# Spring 2019

**January 29, 2019**

Elsie, who is now 70 years old, is entering a Continuing Care Retirement Community (CCRC) under a Full Lifecare (Type A) contract. She will move into an Independent Living Unit (ILU). Elsie pays a one-time fee of $F$ immediately on entry and a level monthly fee of $M$ at the start of each month that she is in the CCRC, including the first.

You are given:

(i) The entry fee, $F$, is equal to 1/3 of the expected present value of all future costs.

(ii) The CCRC operates three types of accommodation; they are listed here, with the monthly costs incurred at the beginning of the month by the CCRC for each resident in each category:

Independent Living Unit (ILU): 4,000

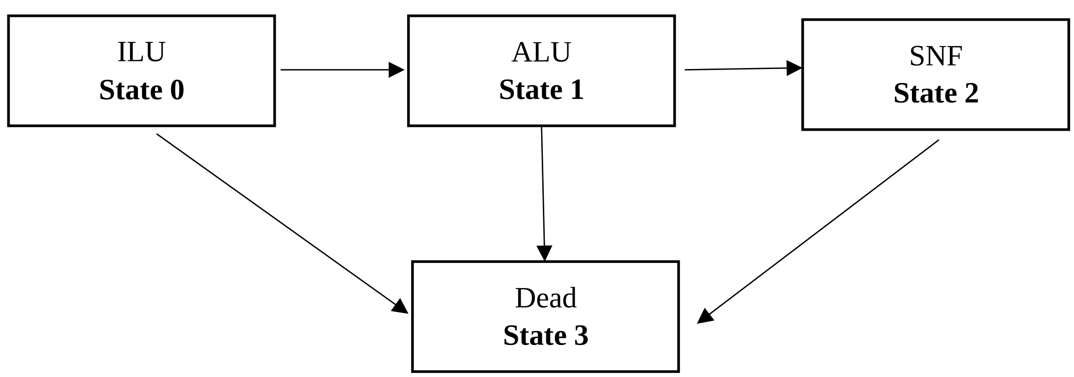
Assisted Living Unit (ALU): 8,000

Specialized Nursing Facility (SNF): 13,000

(iii) $i = 0.05$

(iv) The monthly fee $M$ is determined so that the expected present value of the monthly costs is equal to the expected present value of $M$ plus the expected present value of $F$.

(v) The CCRC uses the following multiple state model to determine the fee structure.




(vi) The following actuarial functions have been evaluated for the model at $i = 0.05$

| $x$ | $\ddot{a}_x^{(12)00}$ | $\ddot{a}_x^{(12)01}$ | $\ddot{a}_x^{(12)02}$ | $\ddot{a}_x^{(12)11}$ | $\ddot{a}_x^{(12)12}$ | $\ddot{a}_x^{(12)22}$ | $A_x^{(12)03}$ |
|---|---|---|---|---|---|---|---|
| 70 | 9.5210 | 1.7037 | 0.4942 | 10.1754 | 0.9960 | 9.1961 | 0.4294 |
| 75 | 7.4080 | 2.1230 | 0.5122 | 8.2234 | 1.2121 | 7.4238 | 0.5231 |

*Please note that this quiz is worth 20 points. You can earn 25 points total with a perfect score.*

(i) *(4 points)* You are given that $F$ is 233,000 to the nearest 1000. Calculate $F$ to the nearest 1.

**Solution:**

$$EPV \text{ of Costs} = (12)(4000)\ddot{a}_{70}^{(12)00} + (12)(8000)\ddot{a}_{70}^{(12)01} + (12)(13,000)\ddot{a}_{70}^{(12)02}$$

$$= (12)(4000)(9.5210) + (12)(8000)(1.7037) + (12)(13,000)(0.4942)$$

$$= 697,658.4$$

$$F = \frac{1}{3}(EPV) = \frac{1}{3}(697,658.4) = 232,553$$

(ii) *(5 points)* Calculate $M$.

**Solution:**

$$F + EPV \text{ of M} = EPV \text{ of Costs} \implies EPV \text{ of M} = \frac{2}{3}\left(EPV \text{ of Costs}\right)$$

$$(12)(M)(\ddot{a}_{70}^{(12)00} + \ddot{a}_{70}^{(12)01} + \ddot{a}_{70}^{(12)02}) = \frac{2}{3}\left(697,658.4\right)$$

$$M = \frac{\frac{2}{3}\left(697,658.4\right)}{(12)(9.5210 + 1.7037 + 0.4942)} = 3307.38$$

(iii) *(5 points)* Calculate ${}_5V^{(0)}$, the reserve five years after entry, assuming Elsie is in state 0.

**Solution:**

${}_5V^{(0)} = EPV$ of Future Costs $-$ EPV of Future Payment

$$= (12)(4000)\ddot{a}_{75}^{(12)00} + (12)(8000)\ddot{a}_{75}^{(12)01} + (12)(13,000)\ddot{a}_{75}^{(12)02} - (12)(3307.38)(\ddot{a}_{75}^{(12)00} + \ddot{a}_{75}^{(12)01} + \ddot{a}_{75}^{(12)02})$$

$$= (12)(4000)(7.4080) + (12)(8000)(2.1230) + (12)(13,000)(0.5122) - (12)(3307.38)(7.4080 + 2.1230 + 0.5122)$$

$$= 240,696$$

(iv) *(5 points)* Calculate ${}_5V^{(1)}$, the reserve five years after entry, assuming Elsie is in state 1.

**Solution:**

${}_5V^{(1)} = EPV$ of Future Costs $-$ EPV of Future Payment

$$= (12)(8000)\ddot{a}_{75}^{(12)11} + (12)(13,000)\ddot{a}_{75}^{(12)12} - (12)(3307.38)(\ddot{a}_{75}^{(12)11} + \ddot{a}_{75}^{(12)12})$$

$$= (12)(8000)(8.2234) + (12)(13,000)(1.2121) - (12)(3307.38)(8.2234 + 1.2121)$$

$$= 604,053$$

(v) *(6 points)* You are given that

$$ {}_{\frac{1}{12}}p_{70}^{00} = 0.94937 \qquad {}_{\frac{1}{12}}p_{70}^{01} = 0.00906 \qquad {}_{\frac{1}{12}}p_{70}^{02} = 0.00003 $$

$$ {}_{1/12}V^{(1)} = 1,510,500 \qquad {}_{1/12}V^{(2)} = 715,600 $$

Calculate ${}_{\frac{1}{12}}V^{(0)}$ .

**Solution:**

We must use the recursive formula. Since we are doing the recursion at time 0 we must include both F and M as both are paid at time 0. Additionally, we must recognize that ${}_0V^{(0)} = 0$ since at time 0, the EPV of future costs is equal to the EPV of future payments of F + M.

$$ \left({}_0V^{(0)} + F + M - 4000\right)(1.05)^{1/12} = {}_{1/12}\,p_{70}^{00} \cdot {}_{1/12}V^{(0)} + {}_{1/12}\,p_{70}^{01} \cdot {}_{1/12}V^{(1)} + {}_{1/12}\,p_{70}^{02} \cdot {}_{1/12}V^{(2)} $$

$$ (0 + 232,553 + 3307.38 - 4000)(1.05)^{1/12} $$
$$ = (0.94937)\,{}_{1/12}V^{(0)} + (0.00906)(1,510,500) + (0.00003)(715,600) $$

$$ {}_{1/12}V^{(0)} = \frac{(0 + 232,553 + 3307.38 - 4000)(1.05)^{1/12} - (0.00906)(1,510,500) + (0.00003)(715,600)}{0.94937} $$

$$ = 230,783 $$