STAT 475
Quiz 2
Spring 2020
March 26, 2020

1. Michelle, (25), is employed at Goh Company. Goh provides a post-retirement medical benefit to its retirees that covers the first three years following retirement. You are given:

    i. Retirement is assumed to occur at age 65.
    ii. Michelle is assumed to remain employed with Goh until retirement or earlier death.
    iii. Currently, annual health insurance premiums are X at age 60 and increase by 2.000% with each age year.
    iv. Health insurance premium inflation is assumed to be 3.5% per year.
    v. Mortality follows the Standard Ultimate Life Table.
    vi. $i = 0.05$

    The expected present value today of Michelle's benefits under the plan are 6664.99.

    Calculate X.

    Solution:

    First we note that $X = B(60,0)$. Then $PV @ 65 = B(65,40) + v_1 p_{65}^{(\tau)} B(66,41) + v^2{}_2 p_{65}^{(\tau)} B(67,42)$

$$= B(65,40)\left[1 + (1.02)(1.035)(1.05)^{-1}\left(\frac{94,020.3}{94,579.7}\right) + (1.02)^2(1.035)^2(1.05)^{-2}\left(\frac{93,398.10}{94,579.7}\right)\right]$$

$$B(65,40) = B(60+5,40) = B(60,0)(1.02)^5(1.035)^{40}$$

$$PV @ 25 = (1.05)^{-40}{}_{40}p_{25}^{(\tau)}[PV @ 65]$$

$$6664.99 = (1.05)^{-40}\left(\frac{94,579.7}{99,871.1}\right)B(60,0)(1.02)^5(1.035)^{40}\left[\begin{array}{l} 1 + (1.02)(1.035)(1.05)^{-1}\left(\frac{94,020.3}{94,579.7}\right) \\ \quad + (1.02)^2(1.035)^2(1.05)^{-2}\left(\frac{93,398.10}{94,579.7}\right) \end{array}\right]$$

$$B(60,0) = \frac{6664.99}{1.762766749} = 3780.98$$

2. For a four year term insurance product, you are given the following:

Time t	$\Pr_t$	${}_t p_x^{(\tau)}$	Annual Premium
0	-120	1	200
1	$\Pr_1$	0.9	200
2	+70	0.8	200
3	+80	0.7	200
4	+60	0.6	

The internal rate of return on this product is 22.5%.

Calculate the Profit Margin using a discount rate of 10%.

Solution:

Time t	$\Pr_t$	${}_t p_x^{(\tau)}$	π_t
0	-120	1	-120)(1)=-120
1	$\Pr_1$	0.9	($\Pr_1$)(1)= $\Pr_1$
2	+70	0.8	(70)(0.9) = 63
3	+80	0.7	(80)(0.8) = 64
4	+60	0.6	(60)(0.7) = 42

$$NPV\ @\ 22.5\% = 0$$

$$0 = 120 + \Pr_1(1.225)^{-1} + 63(1.225)^{-2} + (64)(1.225)^{-3} + (42)(1.225)^{-4} \text{ ==> } \Pr_1 = 30.08$$

$$NPV\ @10\% = 120 + \Pr_1(1.10)^{-1} + 63(1.10)^{-2} + (64)(1.10)^{-3} + (42)(1.10)^{-4} = 36.18$$

$$PVP\ @10\% = 200 + (200)(0.9)(1.10)^{-1} + (200)(0.8)(1.10)^{-2} + (200)(0.7)(1.10)^{-3} = 601.05$$

$$PM = \frac{36.18}{601.05} = 0.0602$$