STAT 475
Test 1
Spring 2020

March 10, 2020

1. (10 points) Tague (65) and Jake (55) are business partners and buy a survivor life insurance policy with a death benefit of 500,000 paid at the end of the year of the second death. The policy has an annual premium paid until the first death.

   You are given that mortality follows the Standard Ultimate Life Table and interest is at 5%.

   Calculate the annual net premium for this policy.

   Solution:

   $$PVP = PVB$$

   $$P\ddot{a}_{55:65} = 500,000A_{\overline{55:65}}$$

   $$P(12.8328) = (500,000)(A_{55} + A_{65} - A_{55:65}) = (500,000)(0.23524 + 0.35477 - 0.38891)$$

   $$P = \frac{(500,000)(0.23524 + 0.35477 - 0.38891)}{12.8328} = 7835.39$$

2. (10 points) Jordyn (55) and her spouse (60) buy a survivor annuity. The annuity will pay 50,000 at the beginning of each year where both lives are alive. It will pay 40,000 at the beginning of each year if only Jordyn is alive. It will pay 20,000 at the beginning of each year if only Jordyn's spouse is alive.

   You are given that mortality follows the Standard Ultimate Life Table and interest is at 5%.

   You are also given that $\ddot{a}_{\overline{55:60}} = 17.2140$.

   Calculate the actuarial present value of this annuity.

   Solution:

   $$APV = (40,000)\ddot{a}_{55} + (20,000)\ddot{a}_{60} + (50,000 - 40,000 - 20,000)\ddot{a}_{55:60}$$

   $$= (40,000)(16.0599) + (20,000)(14.9041) - (10,000)(\ddot{a}_{55} + \ddot{a}_{60} - \ddot{a}_{\overline{55:60}})$$

   $$= (40,000)(16.0599) + (20,000)(14.9041) - (10,000)(16.0599 + 14.9041 - 17.2140)$$

   $$= 802,978$$

3. (10 points) Zach (48) and Ernest (60) are good friends and often go on vacation together. As such, Zach and Ernest are subject to common shock deaths. Zach and Ernest purchase a 10 year **pure** endowment policy that will pay 100,000 at the end of 10 years if they are both alive or 60,000 at the end of 10 years if only one is alive. If both are dead, no payment will be made.

You are given that ${}_{10}p_{48} = 0.97 \text{ and } {}_{10}p_{60} = 0.90$. This mortality does not include the common shock deaths. The force of mortality from common shock is $\mu_{x+t}^{CommonShock} = 0.012$.

You are also given that interest is at 5%.

Calculate the actuarial present value of this pure endowment.

Solution:

There are several ways to do this. Here is one way.

$$APV = 100,000v^{10}{}_{10}p_{\overline{48:60}}^{CS} + 60,000v^{10}({}_{10}p_{48:60}^{CS} - {}_{10}p_{\overline{48:60}}^{CS})$$

$$= (100,000)(1.05)^{-10}(0.97)(0.90)(e^{-10(0.012)})$$
$$+ (60,000)(1.05)^{-10}\left[\left(0.97e^{-10(0.012)} + 0.9e^{-10(0.012)} - (0.97)(0.90)(e^{-10(0.012)})\right) - (0.97)(0.90)(e^{-10(0.012)})\right]$$

$$= 47,534.17 + 4051.02 = 51,858.19$$

4. Molly is a participant in a defined benefit plan at Galle Actuarial Consultants. You are given:

   - Molly was born on March 10, 1980.
   - Molly was hired by Galle Actuarial Consultants on March 10, 2005 with an annual salary of 60,000.
   - Molly has received a 2% salary increase on each March 10 from 2006 through 2020.
   - Molly's retirement benefit is 1.5% of the three year final average salary for each year of service.

   In completing your calculations, you should use the following assumptions:

   - Molly's salary will increase by 2% on each future March 10.
   - If Molly remains employed by Galle, she will retire at exact age 65.
   - There is a 4% probability that Molly will leave employment (for any reason) with Galle each year.
   - The only benefit provided by the defined benefit plan is a retirement benefit.
   - The interest rate prior to retirement will be 5%.
   - The retirement benefit will be paid annually with the first payment at age 65. The benefit will be a single life annuity based on Molly's life. Assume that mortality after retirement follows the Standard Ultimate Life Table with interest at 5%.

   a. (8 points) Molly's projected retirement benefit is 76,000 to the nearest 1000. Calculate her projected retirement benefit to the nearest 1.

      Solution:

$$\text{Projected Benefit} = (0.015)(40)(60,000)\left(\frac{1.02^{37} + 1.02^{38} + 1.02^{39}}{3}\right) = 76,412.74$$

b. (4 points) Calculate Molly's replacement ratio.

Solution:

$$R = \frac{Benefit}{FinalSalary} = \frac{76,412.74}{(60,000)(1.02)^{39}} = 0.58831$$

c. (8 points) Using the Projected Unit Cost Method, calculate the Accrued Actuarial Liability as of March 10, 2020.

Solution:

$$AccruedBenefit = (0.015)(15)(60,000)\left(\frac{1.02^{37} + 1.02^{38} + 1.02^{39}}{3}\right) = 28,654.78$$

$$AAL_{2020} = (AccBen)(\ddot{a}_{65})({}_{25}p_{40}^{(\tau)})v^{25} = (28,654.78)(13.5498)(0.96)^{25}(1.05)^{-25} = 41,321.71$$

5. Amirul, who is now 70 years old, is entering a Continuing Care Retirement Community (CCRC) under a Full Lifecare (Type A) contract. He will move into an Independent Living Unit (ILU). Amirul pays a one-time fee of *F* immediately on entry and a level monthly fee of *M* at the start of each month that he is in the CCRC, including the first.

You are given:

(i) The entry fee, *F*, is equal to 25% of the expected present value of all future costs.

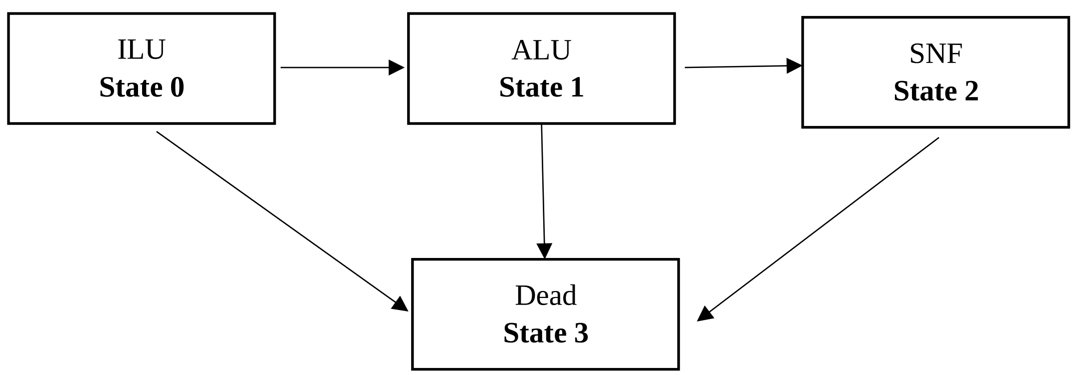
(ii) The CCRC operates three types of accommodation; they are listed here, with the monthly costs incurred at the beginning of the month by the CCRC for each resident in each category:

Independent Living Unit (ILU):	6,000
Assisted Living Unit (ALU):	10,000
Specialized Nursing Facility (SNF):	15,000

(iii) $i = 0.05$

(iv) The monthly fee M is determined so that the expected present value of the monthly costs is equal to the expected present value of M plus the expected present value of F.

(v) The CCRC uses the following multiple state model to determine the fee structure.



(vi) The following actuarial functions have been evaluated for the model at $i = 0.05$

x	$\ddot{a}_x^{(12)00}$	$\ddot{a}_x^{(12)01}$	$\ddot{a}_x^{(12)02}$	$\ddot{a}_x^{(12)11}$	$\ddot{a}_x^{(12)12}$	$\ddot{a}_x^{(12)22}$	$A_x^{(12)03}$
70	9.5210	1.7037	0.4942	10.1754	0.9960	9.1961	0.4294
75	7.4080	2.1230	0.5122	8.2234	1.2121	7.4238	0.5231

a. (8 Points) You are given that *F* is 245,000 to the nearest 1000. Calculate *F* to the nearest 1.

Solution:

$$F = (0.25)(PVB) = (0.25)(12)(6000\ddot{a}_{70}^{(12)00} + 10,000\ddot{a}_{70}^{(12)01} + 15,000\ddot{a}_{70}^{(12)02})$$

$$= (0.25)(12)[(6000)(9.5210) + (10,000)(1.7037) + (15,000)(0.4942)] = 244,728$$

b. (8 points) Calculate M .

Solution:

$$F + 12M(\ddot{a}_{70}^{(12)00} + \ddot{a}_{70}^{(12)01} + \ddot{a}_{70}^{(12)02}) = PVB = (12)(6000\ddot{a}_{70}^{(12)00} + 10,000\ddot{a}_{70}^{(12)01} + 15,000\ddot{a}_{70}^{(12)02})$$

$$(12M)[9.5210 + 1.7037 + 0.4942]$$
$$= (12)[(6000)(9.5210) + (10,000)(1.7037) + (15,000)(0.4942)] - 244,728$$

$$M = 5220.80$$

c. (8 points) Calculate ${}_5V^{(1)}$, the reserve five years after entry, assuming Amirul is in state 1.

Solution:

$${}_5V^{(1)} = PVFB - PVFP = (12)[10,000\ddot{a}_{75}^{(12)11} + 15,000\ddot{a}_{75}^{(12)12}] - 12(M)[\ddot{a}_{75}^{(12)11} + \ddot{a}_{75}^{(12)12}]$$

$$= (12)[10,000(8.2234) + 15,000(1.2121)] - (12)(5220.80)[8.2234 + 1.2121] = 613,855.70$$

6. Purdue Life Insurance Company (PLIC) wants to develop a multiple decrement model to use in pricing a special life annuity. The multiple decrement model will have two decrements – (d) death and (r) remarriage.

   However, PLIC does not have sufficient data to develop a multiple decrement table. Therefore, PLIC will use an independent mortality table and an independent remarriage table to develop a multiple decrement table.

   The Chief Actuary, Alisa, derives the following independent mortality and remarriage tables:

x	$q_x'^{(d)}$		x	$q_x'^{(r)}$
90	0.20		90	0.40
91	0.30		91	0.20
92	0.50		92	0.10

   a. (8 points) Alisa asks David to derive the multiple decrement rates for age 90. David assumes that decrements are uniformly distributed in each independent single decrement table. Complete the following table with double decrement rates accurate to 3 decimal places.

x	$q_x^{(d)}$	$q_x^{(r)}$
90		

   Solution:

$$q_{90}^{(d)} = q_{90}'^{(d)}[1-0.5q_{90}'^{(r)}] = (0.2)[1-0.5(0.4)] = 0.16$$

$$q_{90}^{(r)} = q_{90}'^{(r)}[1-0.5q_{90}'^{(d)}] = (0.4)[1-0.5(0.2)] = 0.36$$

b. (8 points) Alisa asks Brett to derive the multiple decrement rates for age 91. Brett assumes that decrements are uniformly distributed within independent single decrement table for mortality. He further assumes that all marriages occur at age 91.75. Complete the following table with double decrement rates accurate to 3 decimal places.

x	$q_x^{(d)}$	$q_x^{(r)}$
91		

Solution:

Let $l_{91} = 1000$ ==> $l_{92} = (1000)(p_{91}^{\prime(d)})(p_{91}^{\prime(r)}) = (1000)[(1-0.3)(1-0.2)] = 560$

If there are no remarriages, then there are 300 deaths in the year <== $(1000)(0.3) = 300$

Since there are no remarriages during the first 0.75 of the year (all remarriages occur at time 0.75), then there are 300(0.75) deaths during the first three quarters of the year since deaths are uniformily distributed $= 225$.

That leaves $1000 - 225 = 775$ as exposed to remarriage. That means that the number that remarry $= (775)(0.2) = 155$.

Therefore, we have 440 total decrements = 1000 - 560 = 440. Of these, 155 are remarriages and 225 are deaths in the first three quarters of the year. The rest (440 - 155 - 225 = 60) are the deaths in the last quarter of the year. Therefore:

$$q_{91}^{(d)} = \frac{225+60}{1000} = 0.285$$

$$q_{91}^{(r)} = \frac{155}{1000} = 0.155$$

7. (10 points) For a fully continuous whole life of 500,000 on (60), you are given:

   a. The gross premium reserve at $t = 20$ is 150,000.
   b. The gross premium reserve is estimated to be 154,462.20 at $t = 20.5$ using Euler's method with $h = 0.5$.
   c. The gross premium is paid at a rate of P per year during the 21st year.
   d. The force of interest is 5% .
   e. $\mu_{79.5} = 0.055$ $\quad\mu_{80} = 0.056$ $\quad\mu_{80.5} = 0.057$
   f. The following expenses payable continuously:
      i. 60% of premium in the first year and 10% of premium in years 2 and later;
      ii. 500 per policy in the first year and 30 per policy in years 2 and later; and
      iii. 1000 payable at the moment of death.

Calculate P .

Solution:

$${}_{20.5}V = {}_{20}V + (0.5)\left[\delta \cdot {}_{20}V + P - 0.1P - 30 - \mu_{80}\{500,000 - 1000 - {}_{20}V\}\right]$$

$$154,462.2 = 150,000 + (0.5)\left[(0.05)(150,000) + 0.9P - 30 - (0.056)\{500,000 - 1000 - 150,000\}\right]$$

$$P = 23,456$$