Long-Term Actuarial Mathematics

Solutions to Sample Multiple Choice Questions

November 4, 2019

Versions:

July 2, 2018	Original Set of Questions Published.
July 24, 2018	Correction to question 6.25.
August 10, 2018	Correction to question S4.1, S4.3, S4.4, and S4.5.
September 17, 2018	Added 72 questions from the 2016 and 2017 multiple choice MLC exams. Corrected solutions to questions 6.15 and 9.7.
October 1, 2018	Corrected solution to question 7.27.
October 13, 2018	Corrected rendering of certain symbols that appeared incorrectly in October 1 version. Correction to questions 4.21, 6.4, 6.7, 6.27, 6.32, 7.7, 7.29, 7.30, 10.18, S2.1, and S4.1.
January 1, 2019	Corrected solutions to questions 3.10, 8.24, and 10.15.
February 6, 2019	Questions 4.5, 5.3, 5.5, and 5.8 were previously misclassified so they were renumbered to move them to the correct chapter.
March 6, 2019	Clarified solutions for questions S4.3, S4.4, and S4.5.
July 31, 2019	Questions 6.6 and 6.17 were previously misclassified so they were renumbered to move them to the correct chapter. Also, correct minor typos in solution to 8.8.
November 4, 2019	Correction to the solution for question 2.1.

Question 1.1
Answer C

Answer C is false. If the purchaser of a single premium immediate annuity has higher mortality than expected, this reduces the number of payments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, single premium life annuities do not need to be underwritten.

The other items are true.

A➔ Life insurance is typically underwritten to prevent adverse selection as higher mortality than expected will result in the Actuarial Present Value of the benefits being higher than expected.

B➔ In some cases such as direct marketed products for low face amounts, there may be very limited underwriting. The actuary would assume that mortality will be higher than normal, but the expenses related to selling the business will be low and partially offset the extra mortality.

D➔ If the insured's occupation or hobby is hazardous, then the insured life may be rated.

E➔ If the purchaser of the pure endowment has higher mortality than expected, this reduces the number of endowments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, pure endowments do not need to be underwritten.

Question 1.2
Answer E

Insurers have an increased interest in combining savings and insurance products so Item E is false.

The other items are all true.

Question # 2.1
Answer: B

Since $S_0(t) = 1 - F_0(t) = \left(1 - \frac{t}{\omega}\right)^{\frac{1}{4}}$, we have $\ln[S_0(t)] = \frac{1}{4}\ln\left[\frac{\omega - t}{\omega}\right]$.

Then $\mu_t = -\frac{d}{dt}\log S_0(t) = \frac{1}{4}\ \frac{1}{\omega - t}$, and $\mu_{65} = \frac{1}{180} = \frac{1}{4}\ \frac{1}{\omega - 65} \Rightarrow \omega = 110.$

$$e_{106} = \sum_{t=1}^{3} {}_t p_{106}, \text{ since } {}_4 p_{106} = 0$$

$${}_t p_{106} = \frac{S_0(106+t)}{S_0(106)} = \frac{\left(1 - \frac{106+t}{110}\right)^{1/4}}{\left(1 - \frac{106}{110}\right)^{1/4}} = \left(\frac{4-t}{4}\right)^{1/4}$$

$$e_{106} = \sum_{i=1}^{i=4} {}_t p_{106} = \frac{1}{4^{0.25}}\left(1^{0.25} + 2^{0.25} + 3^{0.25}\right) = 2.4786$$

Question # 2.2
Answer: D

This is a mixed distribution for the population, since the vaccine will apply to all once available.

Available? (A)	$\Pr(A)$	${}_2p \mid A$	S = # of survivors: $E(S \mid A)$	$Var(S \mid A)$	$E(S^2 \mid A)$
Yes	0.2	0.9702	97,020	2,891	9,412,883,291
No	0.8	0.9604	96,040	3,803	9,223,685,403
			$E(S)$		$E(S^2)$
			96,236		9,261,524,981
			$Var(S)$	157,285	
			$SD(S)$	397	

As an example, the formulas for the "No" row are

Pr(No) = 1 – 0.2 = 0.8

${}_2p$ given No = (0.98 during year 1)(0.98 during year 2) = 0.9604.

$E(S \mid \text{No}), Var(S \mid \text{No})$ and $E(S^2 \mid \text{No})$ are just binomial, $n = 100,000$; p(success) = 0.9604

$E(S)$, $E(S^2)$ are weighted averages,
$Var(S) = E(S^2) - E(S)^2$

Or, by the conditional variance formula:

$$
\begin{aligned}
Var(S) &= Var[E(S \mid A)] + E[Var(S \mid A)] \\
&= 0.2(0.8)(97,020 - 96,040)^2 + 0.2(2,891) + 0.8(3,803) \\
&= 153,664 + 3,621 = 157,285
\end{aligned}
$$

$StdDev(S) = 397$

Question # 2.3
Answer: A

$$f_x\left(t\right) = -\frac{d}{dt}S_x\left(t\right) = -\frac{d}{dt}\left(e^{-\frac{B}{\ln c}\left(c^x\right)\left(c^t-1\right)}\right)$$

$$= -e^{-\frac{B}{\ln c}\left(c^x\right)\left(c^t-1\right)}\cdot\left(-\frac{B}{\ln c}\cdot c^x\right)\cdot c^t\cdot\ln c$$

$$= e^{-\frac{B}{\ln c}\left(c^x\right)\left(c^t-1\right)}\cdot Bc^{x+t}$$

$$= 0.00027\times 1.1^{x+t}\cdot e^{-\frac{0.00027}{\ln(1.1)}\left(1.1^x\right)\left(1.1^t-1\right)}$$

$$f_{50}(10) = 0.00027\times 1.1^{50+10}\cdot e^{-\frac{0.00027}{\ln(1.1)}\left(1.1^{50}\right)\left(1.1^{10}-1\right)} = 0.04839$$

Alternative Solution:

$$f_x(t) = {}_tp_x\cdot\mu_{x+t}$$

Then we can use the formulas given for Makeham with $A = 0,\ B = 0.00027$ and $c = 1.1$

$$f_x(t) = \left(e^{-\frac{0.00027}{\ln(1.1)}\left(1.1^{50}\right)\left(1.1^{10}-1\right)}\right)\left(0.00027\times 1.1^{50+10}\right) = 0.04839$$

Question # 2.4
Answer: E

$$\mathring{e}_{75:\overline{10}|} = \int_{t=0}^{t=10} {}_t p_{75} dt \quad \text{where} \quad {}_t p_x = \frac{{}_{t+x} p_0}{{}_x p_0} = \frac{1-\dfrac{(t+x)^2}{10000}}{1-\dfrac{x^2}{10000}} = \frac{10000-(t+x)^2}{10000-x^2} \quad \text{for } 0 < t < 100-x$$

$$= \int_0^{10} \frac{10000-75^2-150t-t^2}{10000-75^2} dt$$

$$= \frac{1}{4375} \cdot \left[4375t - 75t^2 - \frac{t^3}{3} \right]_{t=0}^{t=10} = 8.21$$

Question # 2.5
Answer: B

$$\begin{aligned} e_{40} &= e_{40:\overline{20}|} + {}_{20} p_{40} \cdot e_{60} \\ &= 18 + (1-0.2)(25) \\ &= 38 \end{aligned}$$

$$e_{40} = e_{40:\overline{1}|} + p_{40} \cdot e_{41}$$

$$==> e_{41} = \frac{e_{40} - e_{40:\overline{1}|}}{p_{40}} = \frac{e_{40} - p_{40}}{p_{40}} = \frac{38-0.997}{0.997} = 37.11434$$

Question # 2.6
Answer: C

$$\mu_x = -\frac{d}{d_x} lnS_0(x) = -\frac{1}{3}\frac{d}{d_x} ln\left(1-\frac{x}{60}\right)$$

$$= \frac{1}{180}\left(1-\frac{x}{60}\right)^{-1} = \frac{1}{3(60-x)}$$

Therefore, $1000\mu_{35} = (1000)\dfrac{1}{3(25)} = \dfrac{1000}{75} = 13.3.$

Question # 2.7
Answer: B

$$ {}_{20}q_{30} = \frac{S_0(30) - S_0(50)}{S_0(30)} = \frac{\left(1 - \frac{30}{250}\right) - \left(1 - \left[\frac{50}{100}\right]^2\right)}{1 - \frac{30}{250}} = \frac{\frac{220}{250} - \frac{3}{4}}{\frac{220}{250}} $$

$$ = \frac{440 - 375}{440} = \frac{65}{440} = \frac{13}{88} = 0.1477 $$

Question # 2.8
Answer: C

The 20-year female survival probability = $e^{-20\mu}$
The 20-year male survival probability = $e^{-30\mu}$

We want 1-year female survival = $e^{-\mu}$

Suppose that there were *M* males and 3*M* females initially. After 20 years, there are expected to be $Me^{-30\mu}$ and $3Me^{-20\mu}$ survivors, respectively. At that time we have:

$$ \frac{3Me^{-20\mu}}{Me^{-30\mu}} = \frac{85}{15} \Rightarrow e^{10\mu} = \frac{85}{45} = \frac{17}{9} \Rightarrow e^{-\mu} = \left(\frac{9}{17}\right)^{1/10} = 0.938 $$

Question # 3.1
Answer: B

Under constant force over each year of age, $l_{x+k} = (l_x)^{1-k}(l_{x+1})^k$ for x an integer and $0 \le k \le 1$.

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}}$$

$$l_{[60]+0.75} = (80,000)^{0.25}(79,000)^{0.75} = 79,249$$
$$l_{[60]+2.75} = (77,000)^{0.25}(74,000)^{0.75} = 74,739$$
$$l_{[60]+5.75} = (67,000)^{0.25}(65,000)^{0.75} = 65,494$$

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}} = \frac{74,739 - 65,494}{79,249} = 0.11679$$

$$1000\,{}_{2|3}q_{[60]+0.75} = 116.8$$

Question # 3.2
Answer: D

$$l_{65+1} = 1000 - 40 = 960$$
$$l_{66+1} = 955 - 45 = 910$$

$$\mathring{e}_{[65]} = \int_0^1 {}_tp_{[65]}dt + p_{[65]}\int_0^1 {}_tp_{66}\,dt + p_{[65]}p_{66}\,\mathring{e}_{67}$$
$$15.0 = \left[1 - \left(\frac{1}{2}\right)\left(\frac{40}{1000}\right)\right] + \frac{960}{1000}\left[1 - \left(\frac{1}{2}\right)\left(\frac{50}{960}\right)\right] + \left(\frac{960}{1000}\right)\left(\frac{910}{960}\right)\mathring{e}_{67}$$
$$\mathring{e}_{67} = \frac{15(1000) - (980 + 935)}{910} = 14.37912$$
$$\mathring{e}_{[66]} = \int_0^1 {}_tp_{[66]}dt + p_{[66]}\,\mathring{e}_{67} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)\mathring{e}_{67}$$
$$\mathring{e}_{[66]} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)(14.37912) = 14.678$$

Note that because deaths are uniformly distributed over each year of age, $\int_0^1 {}_tp_x dt = 1 - 0.5q_x$.

Question # 3.3
Answer: E

$$ {}_{2.2}q_{[51]+0.5} = \frac{l_{[51]+0.5} - l_{53.7}}{l_{[51]+0.5}} $$

$$ l_{[51]+0.5} = 0.5l_{[51]} + 0.5l_{[51]+1} = 0.5(97,000) + 0.5(93,000) = 95,000 $$

$$ l_{53.7} = 0.3l_{53} + 0.7l_{54} = 0.3(89,000) + 0.7(83,000) = 84,800 $$

$$ {}_{2.2}q_{[51]+0.5} = \frac{95,000 - 84,800}{95,000} = 0.1074 $$

$$ 10,000\,{}_{2.2}q_{[51]+0.5} = 1,074 $$

Question # 3.4
Answer: B

Let *S* denote the number of survivors.

This is a binomial random variable with $n = 4000$ and success probability $\dfrac{21,178.3}{99,871.1} = 0.21206$

$$ E(S) = 4,000(0.21206) = 848.24 $$

The variance is $Var(S) = (0.21206)(1-0.21206)(4,000) = 668.36$

$$ StdDev(S) = \sqrt{668.36} = 25.853 $$

The 90% percentile of the standard normal is 1.282

Let *S** denote the normal distribution with mean 848.24 and standard deviation 25.853. Since *S* is discrete and integer-valued, for any integer *s*,

$$ \begin{aligned} \Pr(S \geq s) &= \Pr(S > s - 0.5) \approx \Pr(S^* > s - 0.5) \\ &= \Pr\left(\frac{S^* - 848.24}{25.853} > \frac{s - 0.5 - 848.24}{25.853}\right) \\ &= \Pr\left(Z > \frac{s - 0.5 - 848.24}{25.853}\right) \end{aligned} $$

For this probability to be at least 90%, we must have $\dfrac{s - 0.5 - 848.24}{25.853} < -1.282$

$$ \Rightarrow s < 815.6 $$

So $s = 815$ is the largest integer that works.

Question # 3.5
Answer: E

Using UDD

$$l_{63.4} = (0.6)66,666 + (0.4)(55,555) = 62,221.6$$
$$l_{65.9} = (0.1)(44,444) + (0.9)(33,333) = 34,444.1$$
$$_{3.4|2.5}q_{60} = \frac{l_{63.4} - l_{65.9}}{l_{60}} = \frac{62,221.6 - 34,444.1}{99,999} = 0.277778 \qquad \text{(a)}$$

Using constant force

$$l_{63.4} = l_{63}\left(\frac{l_{64}}{l_{63}}\right)^{0.4} = l_{63}^{0.6}\, l_{64}^{0.4}$$
$$= \left(66,666^{0.6}\right)\left(55,555^{0.4}\right)$$
$$= 61,977.2$$
$$l_{65.9} = l_{65}^{0.1}\, l_{66}^{0.9} = \left(44,444^{0.1}\right)\left(33,333^{0.9}\right)$$
$$= 34,305.9$$

$$_{3.4|2.5}q_{60} = \frac{61,977.2 - 34,305.9}{99.999}$$
$$= 0.276716 \qquad \text{(b)}$$

$$100,000(a-b) = 100,000(0.277778 - 0.276716) = 106$$

Question # 3.6
Answer: D

$$e_{[61]} = e_{[61]:\overline{3}|} + {}_3p_{[61]}(e_{64})$$

$$p_{[61]} = 0.90,$$
$${}_2p_{[61]} = 0.9(0.88) = 0.792,$$
$${}_3p_{[61]} = 0.792(0.86) = 0.68112$$
$$e_{[61]:\overline{3}|} = \sum_{k=1}^{3} {}_kp_{[61]} = 0.9 + 0.792 + 0.68112 = 2.37312$$

$$e_{[61]} = 2.37312 + 0.68112e_{64} = 2.37312 + 0.68112(5.10) = 5.847$$

Question # 3.7
Answer: B

$$
\begin{aligned}
{}_{2.5}q_{[50]+0.4} &= 1 - {}_{2.5}p_{[50]+0.4} = 1 - {}_{2.9}p_{[50]} / \left(p_{[50]}\right)^{0.4} \\
&= 1 - \left\{ p_{[50]}\ p_{[50]+1} \left(p_{52}\right)^{0.9} \right\} / \left(1 - q_{[50]}\right)^{0.4} \\
&= 1 - \left\{ \left(1 - q_{[50]}\right)\left(1 - q_{[50]+1}\right)\left(1 - q_{52}\right)^{0.9} \right\} / \left(1 - q_{[50]}\right)^{0.4} \\
&= 1 - \left\{ (1 - 0.0050)(1 - 0.0063)(1 - 0.0080)^{0.9} \right\} / (1 - 0.0050)^{0.4} \\
&= 0.01642
\end{aligned}
$$

$$1000\ {}_{2.5}q_{[50]+0.4} = 16.42$$

Question # 3.8
Answer: B

$$E(N) = 1000\left({}_{40}p_{35} + {}_{40}p_{45}\right) = 1000\left(\frac{85,203.5}{99,556.7} + \frac{61,184.9}{99,033.9}\right) = 1473.65$$

$$Var(N) = 1000\ {}_{40}p_{35}\left(1 - {}_{40}p_{35}\right) + 1000\ {}_{40}p_{45}\left(1 - {}_{40}p_{45}\right) = 359.50$$

$$\text{Since } 1473.65 + 1.645\sqrt{359.50} = 1504.84$$

$$N = 1505$$

Question # 3.9
Answer: E

From the SULT, we have:

$$ {}_{25}p_{20} = \frac{\ell_{45}}{\ell_{20}} = \frac{99,033.9}{100,000.0} = 0.99034 $$
$$ {}_{25}p_{45} = \frac{\ell_{70}}{\ell_{45}} = \frac{91,082.4}{99,033.9} = 0.91971 $$

The expected number of survivors from the sons is 1980.68 with variance 19.133.

The expected number of survivors from fathers is 1839.42 with variance 147.687.

The total expected number of survivors is therefore 3820.10.

The standard deviation of the total expected number of survivors is therefore
$\sqrt{19.133+147.687} = \sqrt{166.82} = 12.916$

The 99th percentile equals $3820.10+(2.326)(12.916)=3850$

Question # 3.10
Answer: C

The number of left-handed members at the end of each year *k* is:

$$ L_0 = 75 \ \text{ and } \ L_1 = (75)(0.75) $$
$$ \text{Thereafter, } L_k = L_{k-1}\times 0.75 + 35\times 0.75 = 75\times 0.75^k + 35\times\left(0.75+0.75^2+\ldots 0.75^{k-1}\right) $$

Similarly, the number of right-handed members after each year *k* is:

$$ R_0 = 25 \ \text{ and } \ R_1 = (25)(0.5) $$
$$ \text{Thereafter, } R_k = R_{k-1}\times 0.50 + 15\times 0.50 = 25\times 0.50^k + 15\times\left(0.50+0.50^2+\ldots 0.50^{k-1}\right) $$

At the end of year 5, the number of left-handed members is expected to be 89.5752, and the number of right-handed members is expected to be 14.8435.

The proportion of left-handed members at the end of year 5 is therefore

$$ \frac{89.5752}{89.5752+14.8438} = 0.8578 $$

Question # 3.11
Answer: B

$${}_{2.5}q_{50} = {}_{2}q_{50} + {}_{2}p_{50}\ {}_{0.5}q_{52} \ = 0.02 + (0.98)\left(\frac{0.5}{2}\right)(0.04) \ = 0.0298$$

Question # 3.12
Answer: C

$${}_{3.5}p_{[61]} - {}_{3.5}p_{[60]+1} = {}_{0.5}p_{64}\left({}_{3}p_{[61]} - {}_{3}p_{[60]+1}\right)$$

$$= \left(\frac{\ell_{65}}{\ell_{64}}\right)^{0.5}\left(\frac{\ell_{64}}{\ell_{[61]}} - \frac{\ell_{64}}{\ell_{[60]+1}}\right)$$

$$= \left(\frac{4016}{5737}\right)^{0.5}\left(\frac{5737}{8654} - \frac{5737}{9600}\right)$$

$$= 0.05466$$

Question # 3.13
Answer: B

$$\overset{0}{e}_{[58]+2} = e_{[58]+2} + 0.5$$

$$e_{[58]+2} = p_{[58]+2}(1 + e_{61}) = p_{[58]+2}\left[1 + \frac{e_{60}}{p_{60}} - 1\right]$$

$$= \frac{\ell_{61}}{\ell_{[58]+2}} \times \frac{e_{60}}{p_{60}} = \frac{2210}{3548} \times \frac{1}{(2210/3904)} = \frac{3904}{3549} = 1.100338$$

$$\overset{0}{e}_{[58]+2} = 1.100338 + 0.5 = 1.6$$

Question # 4.1
Answer: A

$$E[Z] = 2 \cdot A_{40} - {}_{20}E_{40}\, A_{60} = (2)(0.36987) - (0.51276)(0.62567) = 0.41892$$

$$E[Z^2] = 0.24954 \quad \text{which is given in the problem.}$$

$$Var(Z) = E[Z^2] - (E[Z])^2 = 0.24954 - 0.41892^2 = 0.07405$$

$$SD(Z) = \sqrt{0.07405} = 0.27212$$

An alternative way to obtain the mean is $E[Z] = 2A^{\ 1}_{40:\overline{20|}} + {}_{20|}A_{40}$. Had the problem asked for the evaluation of the second moment, a formula is

$$E[Z^2] = (2^2)\left({}^2A^{\ 1}_{40:\overline{20|}}\right) + (v^2)^{20}\left({}_{20}p_{40}\right)\left({}^2A_{60}\right)$$

Question # 4.2
Answer: D

<table>
<tr><td>Half-year</td><td>PV of Benefit</td><td rowspan="5">$PV > 277{,}000$ if and only if (x) dies in the 2nd or 3rd half years.</td></tr>
<tr><td>1</td><td>$300{,}000v^{0.5} = (300{,}000)(1.09)^{-1} = 275{,}229$</td></tr>
<tr><td>2</td><td>$330{,}000v^{1} = (330{,}000)(1.09)^{-2} = 277{,}754$</td></tr>
<tr><td>3</td><td>$360{,}000v^{1.5} = (360{,}000)(1.09)^{-3} = 277{,}986$</td></tr>
<tr><td>4</td><td>$390{,}000v^{2} = (390{,}000)(1.09)^{-4} = 276{,}286$</td></tr>
</table>

Under CF assumption, ${}_{0.5}p_x = {}_{0.5}p_{x+0.5} = (0.84)^{0.5} = 0.9165$ and ${}_{0.5}p_{x+1} = {}_{0.5}p_{x+1.5} = (0.77)^{0.5} = 0.8775$ Then the probability of dying in the 2nd or 3rd half-years is

$$\left({}_{0.5}p_x\right)\left(1 - {}_{0.5}p_{x+0.5}\right) + \left(p_x\right)\left(1 - {}_{0.5}p_{x+1}\right) = (0.9165)(0.0835) + (0.84)(0.1225) = 0.1794$$

Question # 4.3
Answer: D

$$A_{60:\overline{3}|} = q_{60}v + (1-q_{60})q_{60+1}v^2 + (1-q_{60})(1-q_{60+1})v^3 = 0.86545$$

$$q_{60+1} = \frac{A_{60:\overline{3}|} - q_{60}v - (1-q_{60})v^3}{(1-q_{60})v^2 - (1-q_{60})v^3} = \frac{0.86545 - \dfrac{0.01}{1.05} - \dfrac{0.99}{1.05^3}}{\dfrac{0.99}{1.05^2} - \dfrac{0.99}{1.05^3}} = 0.017 \text{ when } v = 1/1.05.$$

The primes indicate calculations at 4.5% interest.

$$\begin{aligned} A'_{60:\overline{3}|} &= q_{60}v' + (1-q_{60})q_{60+1}v'^2 + (1-q_{60})(1-q_{60+1})v'^3 \\ &= \frac{0.01}{1.045} + \frac{0.99(0.017)}{1.045^2} + \frac{0.99(0.983)}{1.045^3} \\ &= 0.87777 \end{aligned}$$

Question # 4.4
Answer: A

$$\begin{aligned} Var(Z) &= E(Z^2) - E(Z)^2 \\ E(Z) &= E\left[(1+0.2T)(1+0.2T)^{-2}\right] = E\left[(1+0.2T)^{-1}\right] \\ &= \int_0^{40} \frac{1}{(1+0.2t)} f_T(t)dt = \frac{1}{40}\int_0^{40} \frac{1}{1+0.2t}dt \\ &= \frac{1}{40}\ \frac{1}{0.2}\ \ln(1+0.2t)\Big|_0^{40} = \frac{1}{8}\ln(9) = 0.27465 \\ E(Z^2) &= E\{(1+0.2T)^2[(1+0.2T)^{-2}]^2\} = E[(1+0.2T)^{-2}] \\ &= \int_0^{40} \frac{1}{(1+0.2t)^2}\ f_T(t) = \frac{1}{40}\ \frac{1}{0.2}\left[\frac{-1}{(1+0.2t)}\right]_0^{40} \\ &= \frac{1}{8}\left(1-\frac{1}{9}\right) = \frac{1}{9} = 0.11111 \\ Var(Z) &= 0.11111 - (0.27465)^2 = 0.03568 \end{aligned}$$

Question # 4.5

Question 4.5 was misclassified and therefore was moved to Question 8.26.

Question # 4.6
Answer: B

Time	Age	q_x^{SULT}	Improvement factor	q_x
0	70	0.010413	100.00%	0.010413
1	71	0.011670	95.00%	0.011087
2	72	0.013081	90.25%	0.011806

$v = 1/1.05 = 0.952381$

$$
\begin{aligned}
EPV &= 1{,}000[0.010413v + 0.989587(0.011087)v^2 + 0.989587(0.988913)(0.011806)v^3] \\
&= 29.85
\end{aligned}
$$

Question # 4.7
Answer: C

We need to determine ${}_{3|2.5}q_{90}$.

$$
{}_{3|2.5}q_{90} = \frac{l_{90+3} - l_{90+3+2.5}}{l_{90}} = \frac{l_{93} - l_{95.5}}{l_{90}} = \frac{l_{93} - (l_{95} - 0.5d_{95})}{l_{90}} = \frac{825 - [600 - 0.5(240)]}{1{,}000} = 0.3450
$$

where $l_{90} = 1{,}000$, $l_{93} = 825$, $l_{97} = \frac{d_{97}}{q_{97}} = \frac{72}{1} = 72$, $l_{96} = \frac{l_{97}}{p_{96}} = \frac{72}{0.2} = 360$, $l_{95} = \frac{l_{96}}{p_{95}} = \frac{360}{1-0.4} = 600$, and $d_{95} = l_{95} - l_{96} = 600 - 360 = 240$.

Question # 4.8
Answer: C

Let A_{51}^{SULT} designate A_{51} using the Standard Ultimate Life Table at 5%.

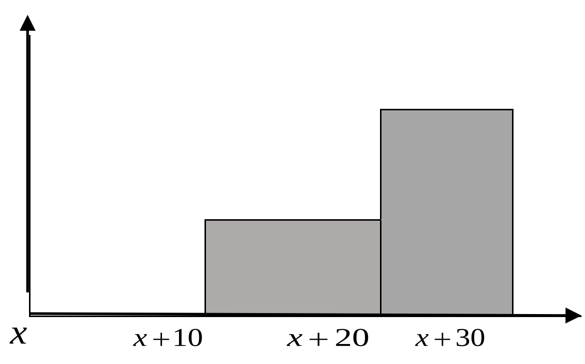
$$\begin{aligned}\text{APV (insurance)} &= 1000\left(\frac{1}{1.04}\right)\left(q_{50} + p_{50}\, A_{51}^{SULT}\right)\\ &= 1000\left(\frac{1}{1.04}\right)\left[0.001209 + (1-0.001209)(0.19780)\right]\\ &= 191.12\end{aligned}$$

Question # 4.9
Answer: D

$$\begin{aligned}A_{35} &= A^{1}_{35:\overline{15}|} + A_{35:\overline{15}|}^{\;\;\;\;1}\, A_{50}\\ 0.32 &= 0.25 + 0.14\, A_{50}\\ A_{50} &= \frac{0.07}{0.14} = 0.50\end{aligned}$$

Question # 4.10
Answer: D

Drawing the benefit payment pattern:



$$E[Z] = {}_{10}E_x \bullet \bar{A}_{x+10} + {}_{20}E_x \bullet \bar{A}_{x+20} - 2\; {}_{30}E_x \bullet \bar{A}_{x+30}$$

Question # 4.11
Answer: A

$$Var\left(Z_2\right)=(1000)^2\left[{}^2A_{x:\overline{n|}}-\left(A_{x:\overline{n|}}\right)^2\right]=15,000$$

$$=(1000)^2\left({}^2A^{1}_{x:\overline{n|}}+{}^2A_{x:\overset{\;\;1}{\overline{n|}}}\right)-(1000)^2\left[A^{1}_{x:\overline{n|}}+A_{x\overset{\;\;1}{\overline{n|}}}\right]^2$$

$$=(1000)^2\;{}^2A^{1}_{x:\overline{n|}}+(1000)^2\;{}^2A_{x:\overset{\;\;1}{\overline{n|}}}-(1000)^2\left(A^{1}_{x:\overline{n|}}\right)^2-(1000)^2\left(A_{x\overset{\;\;1}{\overline{n|}}}\right)^2$$
$$-2(1000)^2\left(A^{1}_{x:\overline{n|}}\right)\left(A_{x\overset{\;\;1}{\overline{n|}}}\right)$$

$$=(1000)^2\left[{}^2A^{1}_{x:\overline{n|}}-\left(A^{1}_{x:\overline{n|}}\right)^2\right]+\left(1000^2\;{}^2A_{x:\overset{\;\;1}{\overline{n|}}}\right)-\left(1000A_{x:\overset{\;\;1}{\overline{n|}}}\right)^2$$
$$-\left(1000A^{1}_{x:\overline{n|}}\right)^2-(2)\left(1000A^{1}_{x:\overline{n|}}\right)\left(1000A_{x\overset{\;\;1}{\overline{n|}}}\right)$$

$$=V\left(Z_1\right)+(1000)\left(1000\;{}^2A_{x:\overset{\;\;1}{\overline{n|}}}\right)-\left(1000A_{x\overset{\;\;1}{\overline{n|}}}\right)^2-\left(1000A^{1}_{x:\overline{n|}}\right)^2$$
$$-(2)\left(1000A^{1}_{x:\overline{n|}}\right)\left(1000A_{x\overset{\;\;1}{\overline{n|}}}\right)$$

$$15,000=Var\left(Z_1\right)+(1000)(136)-(209)^2-2(528)(209)$$

Therefore, $Var\left(Z_1\right)=15,000-136,000+43,681+220,704=143,385.$

Question # 4.12
Answer: C

$Z_3=2Z_1+Z_2$ so that $\mathrm{Var}\left(Z_3\right)=4\mathrm{Var}\left(Z_1\right)+\mathrm{Var}\left(Z_2\right)+4Cov\left(Z_1,Z_2\right)$

where $Cov\left(Z_1,Z_2\right)=\underbrace{E\left[Z_1\,Z_2\right]}_{=0}-E\left[Z_1\right]E\left[Z_2\right]=-(1.65)(10.75)$

$$\begin{aligned}\mathrm{Var}\left(Z_3\right)&=4(46.75)+50.78-4(1.65)(10.75)\\&=166.83\end{aligned}$$

Question # 4.13
Answer: C

$$ {}_{2|2}A_{65} = \underbrace{v^3}_{\text{payment year 3}} \underbrace{{}_2 p_{[65]}}_{\text{Lives 2 years}} \times \underbrace{q_{[65]+2}}_{\text{Die year 3}} $$

$$ + \underbrace{v^4}_{\text{payment year 4}} \underbrace{{}_3 p_{[65]}}_{\text{Lives 3 years}} \times \underbrace{q_{65+3}}_{\text{Die year 4}} $$

$$ = \left(\frac{1}{1.04}\right)^3 (0.92)(0.9)(0.12) $$

$$ + \left(\frac{1}{1.04}\right)^4 (0.92)(0.9)(0.88)(0.14) $$

$$ = 0.088 + 0.087 = 0.176 $$

The actuarial present value of this insurance is therefore $2000 \times 0.176 = 352$.

Question # 4.14
Answer: E

Out of 400 lives initially, we expect $400\,{}_{25}p_{60} = 400\frac{l_{85}}{l_{60}} = 400\left(\frac{61{,}184.9}{96{,}634.1}\right) = 253.26 \text{ survivors}$

The standard deviation of the number of survivors is $\sqrt{400\,{}_{25}p_{60}\left(1-{}_{25}p_{60}\right)} = 9.639$

To ensure 86% funding, using the normal distribution table, we plan for
$253.26 + 1.08(9.639) = 263.67$

The initial fund must therefore be $F = (264)(5000)\left(\frac{1}{1.05}\right)^{25} = 389{,}800.$

Question # 4.15
Answer: E

$$E[Z] = \int_0^\infty b_t \cdot v^t \cdot {}_t p_x \cdot \mu_{x+t} \; dt = \int_0^\infty e^{0.02t} \cdot e^{-0.06t} \cdot e^{-0.04t} \cdot 0.04 \; dt$$

$$= 0.04\int_0^\infty e^{-0.08t} dt = \frac{0.04}{0.08} = \frac{1}{2}$$

$$E\left[Z^2\right] = \int_0^\infty \left(b_t \cdot v^t\right)^2 \cdot {}_t p_x \cdot \mu_{x+t} \; dt = \int_0^\infty \left(e^{0.04t}\right)\left(e^{-0.12t}\right)\left(0.04e^{-0.04}\right) dt = \frac{0.04}{0.12} = \frac{1}{3}$$

$$Var[Z] = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12} = 0.0833$$

Question # 4.16
Answer: D

$$A^{\,1}_{[50]:\overline{3}|} = vq_{[50]} + v^2 p_{[50]} q_{[50]+1} + v^3 p_{[50]} p_{[50]+1} q_{52}$$

where: $v = \dfrac{1}{1.04}$

$$q_{[50]} = 0.7(0.045) = 0.0315$$
$$p_{[50]} = 1 - q_{[50]} = 0.9685$$
$$q_{[50]+1} = 0.8(0.050) = 0.040$$
$$p_{[50]+1} = 1 - q_{[50]+1} = 0.960$$
$$q_{52} = 0.055$$

So: $A^{\,1}_{[50]:\overline{3}|} = 0.1116$

Question # 4.17
Answer: A

The median of K_{48} is the integer m for which

$$P(K_{48} < m) \le 0.5 \text{ and } P(K_{48} > m) \le 0.5.$$

This is equivalent to finding m for which

$$\frac{l_{48+m}}{l_{48}} \ge 0.5 \text{ and } \frac{l_{48+m+1}}{l_{48}} \le 0.5.$$

Based on the SULT and $l_{48}(0.5) = (98,783.9)(0.5) = 49,391.95$, we have $m = 40$ since

$$l_{88} \ge 49,391.95 \text{ and } l_{89} \le 49,391.95.$$

So: $APV = 5000A_{48} + 5000\,{}_{40}E_{48}A_{88} = 5000A_{48} + 5000 \cdot {}_{20}E_{48} \cdot {}_{20}E_{68} \cdot A_{88}$
$= 5000\left(0.17330\right) + 5000\left(0.35370\right)(0.20343)\left(0.72349\right) = 1126.79$

Question # 4.18
Answer: A

The present value random variable $\mathbf{PV} = 1{,}000{,}000e^{-0.05T}$, $2 \le T \le 10$ is a decreasing function of T so that its 90th percentile is

$1{,}000{,}000e^{-0.05p}$ where p is the solution to $\int_2^p 0.4t^{-2}dt = 0.10$.

$$\int_2^p 0.4t^{-2}dt = -0.4\left(\frac{t^{-1}}{-1}\right)\Bigg|_2^p = 0.4\left(\frac{1}{2} - \frac{1}{p}\right) = 0.10$$

$$p = 4$$

$$1{,}000{,}000e^{-0.05\times 4} = 81{,}873.08$$

Question # 4.19
Answer: B

$$q_{80}^{Ming} = 0.8q_{80}^{SULT} = 0.0261264 \Rightarrow p_{80}^{Ming} = 0.9738736$$

$$A_{80}^{Ming} = vq_{80}^{Ming} + vp_{80}^{Ming} A_{81}^{SULT}$$

$$= (1.05)^{-1}(0.0261264) + (1.05)^{-1}(0.9738736)(0.60984) = 0.59051$$

$$100,000A_{80}^{Ming} = 59,051$$

Question # 4.20
Answer: B

$$Var(Z) = 0.10E[Z] \Rightarrow v^{50}\, {}_{25}p_x(1 - {}_{25}p_x) = 0.10 \cdot v^{25}\, {}_{25}p_x$$

$$\Rightarrow \frac{(1-0.57)}{(1+i)^{50}} = 0.10 \times \frac{1}{(1+i)^{25}}$$

$$\Rightarrow (1+i)^{25} = \frac{0.43}{0.10} = 4.3 \Rightarrow i = 0.06$$

Question # 4.21
Answer: C

The earlier the death (before year 30), the larger the loss. Since we are looking for the 95th percentile of the present value of benefits random variable, we must find the time at which 5% of the insureds have died. The present value of the death benefit for that insured is what is being asked for.

$$l_{45} = 99,033.9 \Rightarrow 0.95 l_{45} = 94,082.2$$
$$l_{65} = 94,579.7$$
$$l_{66} = 94,020.3$$

So, the time is between ages 65 and 66, i.e. time 20 and time 21.

$$l_{65} - l_{66} = 94,579.7 - 94,020.3 = 559.4$$
$$l_{65+t} - l_{66} = 94,579.7 - 94,082.2 = 497.5$$
$$497.5 / 559.4 = 0.8893$$

The time just before the last 5% of deaths is expected to occur is: 20 + 0.8893 = 20.8893

The present value of death benefits at this time is:

$$100,000 e^{-20.8893(0.05)} = 35,188$$

Question # 4.22
Answer: C

$$\left(\bar{I}\bar{a}\right)_{40:\overline{t}|} = \int_0^t s\, {}_s p_{40} v^s ds \Rightarrow \frac{d\left(\bar{I}\bar{a}\right)_{40:\overline{t}|}}{dt} = t\, {}_t p_{40} v^t$$

At $t = 10.5$,

$$10.5\, {}_{10.5}E_{40} = 10.5\, {}_{10}p_{40}\, {}_{0.5}p_{50} v^{10.5}$$
$$= 10.5\, {}_{10}E_{40}\, {}_{0.5}p_{50} v^{0.5}$$
$$= 10.5 \times 0.60920 \times (1 - 0.5 \times 0.001209)(0.975900073)$$
$$= 6.239$$

Question # 5.1
Answer: A

$$E(Y) = \bar{a}_{\overline{10}|} + e^{-\delta(10)} e^{-\mu(10)} \bar{a}_{x+10}$$

$$= \frac{\left(1 - e^{-0.6}\right)}{0.06} + e^{-0.7} \frac{1}{0.07}$$

$$= 14.6139$$

$$Y > E(Y) \Rightarrow \left(\frac{1 - e^{-0.06T}}{0.06} \right) > 14.6139$$

$$\Rightarrow T > 34.90$$

$$\Pr[Y > E(Y)] = \Pr(T > 34.90) = e^{-34.90(0.01)} = 0.705$$

Question # 5.2
Answer: B

$$A_{x:\overline{n}|}^{\ \ \ 1} = {}_nE_x$$

$$A_x = A^1_{x:\overline{n}|} + {}_nE_x A_{x+n}$$

$$0.3 = A^1_{x:\overline{n}|} + (0.35)(0.4) \Rightarrow A^1_{x:\overline{n}|} = 0.16$$

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + {}_nE_x = 0.16 + 0.35 = 0.51$$

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{1 - 0.51}{(0.05/1.05)} = 10.29$$

$$a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x = 10.29 - 0.65 = 9.64$$

Question # 5.3

Question 5.3 was misclassified and therefore was moved to Question 9.14.

Question # 5.4
Answer: A

$\overset{\circ}{e}_{40} = \dfrac{1}{\mu} = 50$ So receive *K* for 50 years guaranteed and for life thereafter.

$$10,000 = K\left[\bar{a}_{\overline{50|}} + {}_{50|}\bar{a}_{40}\right]$$

$$\bar{a}_{\overline{50|}} = \int_0^{50} e^{-\delta t} = \frac{1-e^{-50\delta}}{\delta} = \frac{1-e^{-50(0.01)}}{0.01} = 39.35$$

$${}_{50|}\bar{a}_{40} = {}_{50}E_{40}\bar{a}_{40+50} = e^{-(\delta+\mu)50}\frac{1}{\mu+\delta} = e^{-1.5}\frac{1}{0.03} = 7.44$$

$$K = \frac{10,000}{39.35+7.44} = 213.7$$

Question # 5.5

Question 5.5 was misclassified and therefore was moved to Question 6.51.

Question # 5.6
Answer: D

Let Y_i be the present value random variable of the payment to life *i*.

$$E[Y_i] = \ddot{a}_x = \frac{1-A_x}{d} = 11.55 \qquad Var[Y_i] = \frac{{}^2A_x - \left(A_x\right)^2}{d^2} = \frac{0.22-0.45^2}{\left(0.05/1.05\right)^2} = 7.7175$$

Then $Y = \sum_{i=1}^{100} Y_i$ is the present value of the aggregate payments.

$E[Y] = 100E[Y_i] = 1155$ and $Var[Y] = 100Var[Y_i] = 771.75$

$$\Pr[Y \le F] = \Pr\left[Z \le \frac{F-1155}{\sqrt{771.75}}\right] = 0.95 \Rightarrow \frac{F-1155}{\sqrt{771.75}} = 1.645$$

$$\Rightarrow F = 1155 + 1.645\sqrt{771.75} = 1200.699$$

Question # 5.7

Answer: C

$$\ddot{a}^{(2)}_{35:\overline{30}|} \approx \ddot{a}_{35:\overline{30}|} - \frac{(m-1)}{2m}\left(1 - v^{30}\, {}_{30}p_{35}\right)$$

$$\ddot{a}_{35:\overline{30}|} = \frac{1 - A_{35:\overline{30}|}}{d} = \frac{1 - A^{1}_{35:\overline{30}|} - {}_{30}E_{35}}{d}$$

$$= \frac{1 - \left(A_{35} - {}_{30}E_{35} \bullet A_{65}\right) - {}_{30}E_{35}}{d}$$

Since ${}_{30}E_{35} = v^{30}\, {}_{30}p_{35} = 0.2722$, then

$$\ddot{a}_{35:\overline{30}|} = \frac{1 - \left(A_{35} - v^{30}\, {}_{30}p_{35} \bullet A_{65}\right) - v^{30}\, {}_{30}p_{35}}{d}$$

$$= \frac{1 - (0.188 - (0.2722)(0.498)) - 0.2722}{(0.04/1.04)}$$

$$= 17.5592$$

$$\ddot{a}^{(2)}_{35:\overline{30}|} \approx 17.5592 - \frac{1}{4}(1 - 0.2722) = 17.38$$

$$1000\,\ddot{a}^{(2)}_{35:\overline{30}|} \approx 1000 \times 17.38 = 17{,}380$$

Question # 5.8

Question 5.8 was misclassified and therefore was moved to Question 9.15.

Question # 5.9
Answer: C

$$\ddot{a}_{[x]:\overline{n}|} = 1 + vp_{[x]}\ddot{a}_{x+1:\overline{n-1}|} = 1 + (1+k)\left(vp_x \ddot{a}_{x+1:\overline{n-1}|}\right) = 1 + (1+k)\left(\ddot{a}_{x:\overline{n}|} - 1\right)$$

Therefore, we have

$$k = \frac{\ddot{a}_{[x]:\overline{n}|} - 1}{\ddot{a}_{x:\overline{n}|} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015$$

Question # 5.10
Answer: C

The expected present value is:

$$\ddot{a}_{\overline{5}|} + {}_5E_{55}\ddot{a}_{60} = 4.54595 + 0.77382 \times 14.9041 = 16.07904$$

The probability that the sum of the undiscounted payments will exceed the expected present value is the probability that at least 17 payments will be made. This will occur if (55) survives to age 71. The probability is therefore:

$${}_{16}p_{55} = \frac{\ell_{71}}{\ell_{55}} = \frac{90{,}134.0}{97{,}846.2} = 0.92118$$

Question # 5.11
Answer: A

$$\ddot{a}_{45}^{S} = 1 + vp_{45}^{S}\ddot{a}_{46}^{SULT}$$

$$p_{45}^{S} = e^{-\int_0^1 \mu_{45+t}^{S}dt} = e^{-\int_0^1 (\mu_{45+t}^{SULT}+0.05)dt} = e^{-\int_0^1 (\mu_{45+t}^{SULT})dt}\,e^{-\int_0^1 (0.05)dt} = p_{45}^{SULT}\cdot e^{-0.05} = \left(\frac{98{,}957.6}{99{,}033.9}\right)e^{-0.05} = 0.9504966$$

$$\ddot{a}_{45}^{S} = 1 + vp_{45}^{S}\ddot{a}_{46}^{SULT} = 1 + (1.05)^{-1}(0.9504966)(17.6706) = 17.00$$

$$100\ddot{a}_{45}^{S} = 1700$$

Question # 6.1
Answer: D

The equation of value is given by

Actuarial Present Value of Premiums = Actuarial Present Value of Death Benefits.

The death benefit in the first year is $1000+P$. The death benefit in the second year is $1000+2P$.

The formula is $P\ddot{a}_{80:2} = 1000A^{\,1}_{80:\overline{2}|} + P(IA)^{\,1}_{80:\overline{2}|}$.

Solving for P we obtain $P = \dfrac{1000A^{\,1}_{80:\overline{2}|}}{\ddot{a}_{80:\overline{2}|} - (IA)^{\,1}_{80:\overline{2}|}}$.

$$\ddot{a}_{80:\overline{2}|} = 1 + p_{80}v = 1 + \frac{0.967342}{1.03} = 1.93917$$

$$1000A^{\,1}_{80:\overline{2}|} = 1000\left(vq_{80} + v^2 p_{80}q_{81}\right) = 1000\left(\frac{0.032658}{1.03} + \frac{(0.967342)(0.036607)}{1.03^2}\right) = 65.08552$$

$$(IA)^{\,1}_{80:\overline{2}|} = vq_{80} + 2v^2 p_{80}q_{81} = \frac{0.032658}{1.03} + (2)\frac{(0.967342)(0.036607)}{1.03^2} = 0.09846$$

$$P = \frac{65.08552}{1.93917 - 0.09846} = 35.36 \rightarrow D$$

Question # 6.2
Answer: E

$$G\ddot{a}_{x:\overline{10}|} = 100,000A^{1}_{x:\overline{10}|} + G(IA)^{1}_{x:\overline{10}|} + 0.45G + 0.05G\ddot{a}_{x:\overline{10}|} + 200\ddot{a}_{x:\overline{10}|}$$

$$G = \frac{(100,000)(0.17094) + 200(6.8865)}{(1-0.05)(6.8865) - 0.96728 - 0.45} = 3604.23$$

Question # 6.3
Answer: C

Let *C* be the annual contribution, then $C = \dfrac{{}_{20}E_{45}\,\ddot{a}_{65}}{\ddot{a}_{45:\overline{20}|}}$

Let K_{65} be the curtate future lifetime of (65). The required probability is

$$\Pr\left(\frac{C\ddot{a}_{45:\overline{20}|}}{{}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\frac{{}_{20}E_{45}\,\ddot{a}_{65}}{\ddot{a}_{45:\overline{20}|}}\frac{\ddot{a}_{45:\overline{20}|}}{{}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\ddot{a}_{65} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(13.5498 > \ddot{a}_{\overline{K_{65}+1}|}\right)$$

Thus, since $\ddot{a}_{\overline{21}|} = 13.4622$ and $\ddot{a}_{\overline{22}|} = 13.8212$ we have

$$\Pr\left(\ddot{a}_{\overline{K_{65}+1}|} < 13.5498\right) = \Pr\left(K_{65}+1 \le 21\right) = 1 - {}_{21}p_{65} = 1 - \frac{l_{86}}{l_{65}} = 1 - \frac{57,656.7}{94,579.7} = 0.390$$

Question # 6.4
Answer: E

Let X_i be the present value of a life annuity of 1/12 per month on life i for $i = 1, 2, ..., 200$.

Let $S = \sum_{i=1}^{200} X_i$ be the present value of all the annuity payments.

$$E[X_i] = \ddot{a}_{62}^{(12)} = \frac{1 - A_{62}^{(12)}}{d^{(12)}} = \frac{1 - 0.4075}{0.05813} = 10.19267$$

$$Var\left(X_i\right) = \frac{{}^2A_{62}^{(12)} - \left(A_{62}^{(12)}\right)^2}{\left(d^{(12)}\right)^2} = \frac{0.2105 - (0.4075)^2}{(0.05813)^2} = 13.15255$$

$$E[S] = (200)(180)(10.19267) = 366{,}936.12$$
$$Var(S) = (200)(180)^2(13.15255) = 85{,}228{,}524$$

With the normal approximation, for $\Pr(S \le M) = 0.90$
$$M = E[S] + 1.282\sqrt{Var(S)} = 366{,}936.12 + 1.282\sqrt{85{,}228{,}524} = 378{,}771.45$$

So $\pi = \dfrac{378{,}771.45}{200} = 1893.86$

Question # 6.5
Answer: D

Let *k* be the policy year, so that the mortality rate during that year is q_{30+k-1}. The objective is to determine the smallest value of *k* such that

$$v^{k-1}\left({}_{k-1}p_{30}\right)(1000P_{30}) < v^k\left({}_{k-1}p_{30}\right)q_{30+k-1}(1000)$$
$$P_{30} < vq_{30+k-1}$$
$$\frac{0.07698}{19.3834} < \frac{q_{29+k}}{1.05}$$
$$q_{29+k} > 0.00417$$
$$29 + k > 61 \Rightarrow k > 32$$

Therefore, the smallest value that meets the condition is 33.

Question # 6.6

Question 6.6 was misclassified and therefore was moved to Question 8.27.

Question # 6.7
Answer: C

There are four ways to approach this problem. In all cases, let π denote the net premium.

The first approach is an intuitive result. The key is that in addition to the pure endowment, there is a benefit equal in value to a temporary interest only annuity due with annual payment π. However, if the insured survives the 20 years, the value of the annuity is not received.

$$\pi \ddot{a}_{40:\overline{20}|} = 100,000\ {}_{20}E_{40} + \pi \ddot{a}_{40:\overline{20}|} - {}_{20}p_{40}\ \ddot{a}_{\overline{20}|5\%}\pi$$

Based on this equation,

$$\pi = \frac{100,000\,{}_{20}E_{40}}{{}_{20}p_{40}\ddot{a}_{\overline{20}|}} = \frac{100,000v^{20}}{\ddot{a}_{\overline{20}|}} = \frac{100,000}{\ddot{s}_{\overline{20}|}} = \frac{100,000}{34.71925} = 2880$$

The second approach is also intuitive. If you set an equation of value at the end of 20 years, the present value of benefits is 100,000 for all the people who are alive at that time. The people who have died have had their premiums returned with interest. Therefore, the premiums plus interest that the company has are only the premiums for those alive at the end of 20 years. The people who are alive have paid 20 premiums. Therefore $\pi \ddot{s}_{\overline{20}|} = 100,000$.

The third approach uses random variables to derive the expected present value of the return of premium benefit. Let *K* be the curtate future lifetime of (40). The present value random variable is then

$$Y = \begin{cases} \pi \ddot{s}_{\overline{K+1}|} v^{K+1}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$
$$= \begin{cases} \pi \ddot{a}_{\overline{K+1}|}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$
$$= \begin{cases} \pi \ddot{a}_{\overline{K+1}|} - 0, & K < 20 \\ \pi \ddot{a}_{\overline{20}|} - \pi \ddot{a}_{\overline{20}|}, & K \geq 20 \end{cases}.$$

The first term is the random variable that corresponds to a 20-year temporary annuity. The second term is the random variable that corresponds to a payment with a present value of $\pi \ddot{a}_{\overline{20}|}$ contingent on surviving 20 years. The expected present value is then $\pi \ddot{a}_{40:\overline{20}|} - {}_{20}p_{40}\ \ddot{a}_{\overline{20}|}\pi$.

The fourth approach takes the most steps.

$$\pi\ddot{a}_{40:\overline{20}|} = 100,000\ {}_{20}E_{40} + \pi\sum_{k=0}^{19} v^{k+1}\ \ddot{s}_{\overline{k+1}|}\ {}_{k|}q_{40} = 100,000\,{}_{20}E_{40} + \pi\sum_{k=0}^{19} v^{k+1}\ \frac{(1+i)^{k+1}-1}{d}\ {}_{k|}q_{40}$$

$$= 100,000\,{}_{20}E_{40} + \frac{\pi}{d}\left(\sum_{k=0}^{19} {}_{k|}q_{40} - v^{k+1}\ {}_{k|}q_{40}\right) = 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - A^{1}_{40:\overline{20}|}\right)$$

$$= 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - 1 + d\ddot{a}_{40:\overline{20}|} + v^{20}\ {}_{20}p_{40}\right)$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20}|} - \pi\ {}_{20}p_{40}\ \frac{1-v^{20}}{d}$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20}|} - {}_{20}p_{40}\ \ddot{a}_{\overline{20}|6\%}\pi.$$

Question # 6.8
Answer: B

$$\ddot{a}_{60:\overline{10}|} = 7.9555$$
$$\ddot{a}_{60:\overline{20}|} = 12.3816$$

Annual level amount $= \dfrac{40 + 5\ddot{a}_{60:\overline{10}|} + 5\ddot{a}_{60:\overline{20}|}}{\ddot{a}_{60}} = \dfrac{141.686}{14.9041} = 9.51$

Question # 6.9
Answer: D

$$\ddot{a}_{50:\overline{10}|} = 8.0550$$
$$A^{\ 1}_{50:\overline{20}|} = A_{50:\overline{20}|} - {}_{20}E_{50} = 0.38844 - 0.34824 = 0.04020$$
$$\ddot{a}_{50:\overline{20}|} = 12.8428$$

APV of Premiums = APV Death Benefit + APV Commission and Taxes + APV Maintenance

$$G\ddot{a}_{50:\overline{10}|} = 100,000A^{\ 1}_{50:\overline{20}|} + 0.12G\ddot{a}_{50:\overline{10}|} + 0.3G + 25\ddot{a}_{50:\overline{20}|} + 50$$
$$8.0550G = 4020 + 1.2666G + 371.07$$
$$6.7883G = 4391.07$$
$$\Rightarrow G = 646.86$$

Question # 6.10
Answer: D

$$\ddot{a}_{x:\overline{3}|} = \frac{\text{Actuarial PV of the benefit}}{\text{Level Annual Premium}} = \frac{152.85}{56.05} = 2.727$$

$$\ddot{a}_{x:\overline{3}|} = 1 + \frac{0.975}{1.06} + \frac{0.975\left(p_{x+1}\right)}{(1.06)^2} = 2.727$$

$$\Rightarrow p_{x+1} = 0.93$$

Actuarial PV of the benefit =

$$152.85 = 1,000\left[\frac{0.025}{1.06} + \frac{0.975(1-0.93)}{(1.06)^2} + \frac{0.975(0.93)\left(q_{x+2}\right)}{(1.06)^3}\right]$$

$$\Rightarrow q_{x+2} = 0.09 \Rightarrow p_{x+2} = 0.91$$

Question # 6.11
Answer: C

For calculating *P*

$$A_{50} = vq_{50} + vp_{50}A_{51} = v(0.0048) + v(1-0.0048)(0.39788) = 0.38536$$

$$\ddot{a}_{50} = \left(1 - A_{50}\right)/d = 15.981$$

$$P = A_{50}/\ddot{a}_{50} = 0.02411$$

For this particular life,

$$A'_{50} = vq'_{50} + vp'_{50}A_{51} = v(0.048) + (1-0.048)(0.39788) = 0.41037$$

$$\ddot{a}'_{50} = \left(1 - A'_{50}\right)/d = 15.330$$

Expected PV of loss = $A'_{50} - P\ddot{a}'_{50} = 0.41037 - 0.02411(15.330) = 0.0408$

Question # 6.12
Answer: E

1,020 in the solution is the 1,000 death benefit plus the 20 death benefit claim expense.

$$A_x = 1 - d\ddot{a}_x = 1 - d(12.0) = 0.320755$$
$$G\ddot{a}_x = 1{,}020A_x + 0.65G + 0.10G\ddot{a}_x + 8 + 2\ddot{a}_x$$
$$G = \frac{1{,}020A_x + 8 + 2\ddot{a}_x}{\ddot{a}_x - 0.65 - 0.10\ddot{a}_x} = \frac{1{,}020(0.320755) + 8 + 2(12.0)}{12.0 - 0.65 - 0.10(12.0)} = 35.38622$$

Let $Z = v^{K_x+1}$ denote the present value random variable for a whole life insurance of 1 on (*x*).
Let $Y = \ddot{a}_{\overline{K_x+1}|}$ denote the present value random variable for a life annuity-due of 1 on (*x*).

$$\begin{aligned} L &= 1{,}020Z + 0.65G + 0.10GY + 8 + 2Y - GY \\ &= 1{,}020Z + (2 - 0.9G)Y + 0.65G + 8 \\ &= 1{,}020v^{K_x+1} + (2 - 0.9G)\frac{1 - v^{K_x+1}}{d} + 0.65G + 8 \\ &= \left(1{,}020 + \frac{0.9G - 2}{d}\right)v^{K_x+1} + \frac{2 - 0.9G}{d} + 0.65G + 8 \end{aligned}$$

$$\begin{aligned} Var(L) &= \left[{}^2A_x - (A_x)^2\right]\left(1{,}020 + \frac{0.9G - 2}{d}\right)^2 \\ &= (0.14 - 0.320755^2)\left(1{,}020 + \frac{0.9(35.38622) - 2}{d}\right)^2 \\ &= 0.037116(2{,}394{,}161) \\ &= 88{,}861 \end{aligned}$$

Question # 6.13
Answer: D
If $T_{45} = 10.5$, then $K_{45} = 10$ and $K_{45} + 1 = 11.$

$${}_0L = 10{,}000v^{K_{45}+1} - G(1 - 0.10)\ddot{a}_{\overline{K_{45}+1}|} + G(0.80 - 0.10) = 10{,}000v^{11} - 0.9G\ddot{a}_{\overline{11}|} + 0.7G$$
$$4953 = 10{,}000(0.58468) - 0.9G(8.72173) + 0.7G$$
$$G = (5846.8 - 4953)/(7.14956) = 125.01$$
$$\begin{aligned} E({}_0L) &= 10{,}000A_{45} - (1 - 0.1)G\ddot{a}_{45} + (0.8 - 0.1)G \\ &= (10{,}000)(0.15161) - (0.9)(125.01)(17.8162) + (0.7)(125.01) \end{aligned}$$
$$E({}_0L) = -400.87$$

Question # 6.14
Answer: D

$$100,000A_{40} = P[\ddot{a}_{40:\overline{10|}} + 0.5\,{}_{10|}\ddot{a}_{40:\overline{10|}}]$$

$$P = \frac{100,000A_{40}}{\ddot{a}_{40:\overline{10|}} + 0.5\,{}_{10|}\ddot{a}_{40:\overline{10|}}} = \frac{100,000(0.12106)}{8.0863 + 0.5(4.9071)} = \frac{12,106}{10.53985} = 1148.59$$

where

$${}_{10|}\ddot{a}_{40:\overline{10|}} = {}_{10}E_{40}\left[\ddot{a}_{50:\overline{10|}}\right] = 0.60920[8.0550] = 4.9071$$

There are several other ways to write the right hand side of the first equation.

Question # 6.15
Answer: B

Woolhouse: $${}^{W}\ddot{a}_x^{(4)} = 3.4611 - \frac{3}{8} = 3.0861$$

UDD:
$$\begin{aligned} {}^{UDD}\ddot{a}_x^{(4)} &= \alpha(4)\ddot{a}_x - \beta(4) \\ &= 1.00019(3.4611) - 0.38272 \\ &= 3.0790 \end{aligned}$$

and $$A_x = 1 - d\,\ddot{a}_x = 1 - (0.04762)(3.4611) = 0.83518$$

$$P^{(W)} = \frac{1000(0.83518)}{3.0861} = 270.63$$

$$P^{(UDD)} = \frac{1000(0.83518)}{3.0790} = 271.25$$

$$\frac{P^{(UDD)}}{P^{(W)}} = \frac{271.25}{270.63} = 1.0023$$

Question # 6.16
Answer: A

$$P_{30:\overline{20|}} = \frac{1}{\ddot{a}_{30:\overline{20|}}} - d \Rightarrow \frac{2{,}143}{100{,}000} + 0.05 = \frac{1}{\ddot{a}_{30:\overline{20|}}} \Rightarrow \ddot{a}_{30:\overline{20|}} = 14$$

$$A_{30:\overline{20|}} = 1 - d\,\ddot{a}_{30:\overline{20|}} = 1 - 0.05(14) = 0.3$$

$$G\ddot{a}_{30:\overline{20|}} = 100{,}000 A_{30:\overline{20|}} + (200 + 50\ddot{a}_{30:\overline{20|}}) + (0.33G + 0.06G\,\ddot{a}_{30:\overline{20|}})$$

$$14G = 100{,}000(0.3) + [200 + 50(14)] + (0.33G + 0.84G)$$

$$12.83G = 30{,}900$$

$$G = 2408$$

Question # 6.17

Question 6.17 was misclassified and therefore was moved to Question 7.45.

Question # 6.18
Answer: D

$$P = 30{,}000\ {}_{20|}\ddot{a}_{40} + PA^{\ 1}_{40:\overline{20|}}$$

$$\Rightarrow \quad P = 30{,}000\ {}_{20|}\ddot{a}_{40} \Big/ \left(1 - A^{\ 1}_{40:\overline{20|}}\right)$$

$$= 30{,}000(5.46429) / (1 - 0.0146346) = 166{,}363$$

Question # 6.19
Answer: C

Let π be the annual premium, so that $\pi\ddot{a}_{50} = A_{50} + 0.01\ddot{a}_{50} + 0.19$

$$\Rightarrow \pi = \frac{A_{50} + 0.19}{\ddot{a}_{50}} + 0.01 = \frac{0.18931 + 0.19}{17.0245} + 0.01 = 0.03228$$

Loss at issue: $L_0 = v^{k+1} - (\pi - 0.01)\ddot{a}_{\overline{k+1}|}(1 - v^{k+1}) / d + 0.19$

$$\Rightarrow Var[L_0] = \left(1 + \frac{(\pi - 0.01)}{d}\right)^2 \left({}^2A_{50} - A_{50}^2\right)$$

$$= (2.15467)(0.05108 - 0.18931^2)$$
$$= (2.15467)(0.015242)$$
$$= 0.033$$

Question # 6.20
Answer: B

EPV(premiums) = EPV(benefits)

$$P\left(1 + vp_x + v^2\,{}_2p_x\right) = P\left(vq_x + 2v^2 p_x\, q_{x+1}\right) + 10000\left(v^3\,{}_2p_x\, q_{x+2}\right)$$

$$P\left(1 + \frac{0.9}{1.04} + \frac{0.9 \times 0.88}{1.04^2}\right) = P\left(\frac{0.1}{1.04} + \frac{2 \times 0.9 \times 0.12}{1.04^2}\right) + 10000\left(\frac{0.9 \times 0.88 \times 0.15}{1.04^3}\right)$$

$$2.5976P = 0.29588P + 1056.13$$
$$P = 459$$

Question # 6.21
Answer: C

$$P \times \ddot{a}_{75:\overline{15}|} = 1000\left(A^{1}_{75:\overline{15}|} + 15 \times P \times A_{75:\overline{15}|}^{\;\;\,\frac{1}{}}\right) \rightarrow P = \frac{1000A^{1}_{75:\overline{15}|}}{\ddot{a}_{75:\overline{15}|} - 15 \times A_{75:\overline{15}|}^{\;\;\,\frac{1}{}}}$$

$$A^{1}_{75:\overline{15}|} = A_{75:\overline{15}|} - A_{75:\overline{15}|}^{\;\;\,\frac{1}{}} = 0.7 - 0.11 = 0.59$$

$$\ddot{a}_{75:\overline{15}|} = \frac{1 - A_{75:\overline{15}|}}{d} = (1 - 0.7) / 0.04 = 7.5$$

$$\text{So } \; P = \frac{590}{7.5 - 15(0.11)} = 100.85$$

Question # 6.22
Answer: C

Let the monthly net premium $= \pi$

$$12\pi = \frac{100{,}000\bar{A}_{45}}{\ddot{a}^{(12)}_{45:\overline{20}|}}$$

$$\alpha(12) = 1.00020$$
$$\beta(12) = 0.46651$$
$$\frac{i}{\delta} = 1.02480$$

$$100{,}000\bar{A}_{45} = 100{,}000\frac{i}{\delta}A_{45} = (1.02480)(15{,}161) = 15{,}536.99$$

$$\begin{aligned}\ddot{a}^{(12)}_{45:\overline{20}|} &= \alpha(12)\ddot{a}_{45:\overline{20}|} - \beta(12)\left(1 - {}_{20}E_{45}\right)\\ &= 1.00020[12.9391] - 0.46651(1-0.35994)\\ &= 12.6431\end{aligned}$$

$$\begin{aligned}12\pi &= \frac{15{,}536.99}{12.6431}\\ 12\pi &= 1228.891\\ \pi &= 102.41\end{aligned}$$

Question # 6.23
Answer: D

$$\begin{aligned}G\ddot{a}_{x:\overline{30}|} &= \text{APV}\left[\text{gross premium}\right] = \text{APV}\left[\text{Benefits + expenses}\right]\\ &= FA_x + \left(30 + 30\ddot{a}_x\right) + G\left(0.6 + 0.10\ddot{a}_{x:\overline{30}|} + 0.10\ddot{a}_{x:\overline{15}|}\right)\end{aligned}$$

$$\begin{aligned}G &= \frac{FA_x + 30 + 30\ddot{a}_x}{\ddot{a}_{x:\overline{30}|} - 0.6 - 0.1\ddot{a}_{x:\overline{30}|} - 0.1\ddot{a}_{x:\overline{15}|}}\\ &= \frac{FA_x + 30 + 30(15.3926)}{14.0145 - 0.6 - 0.1(14.0145) - 0.1(10.1329)}\\ &= \frac{FA_x + 491.78}{10.9998}\\ &= \frac{FA_x}{10.9998} + \frac{491.78}{10.9998} = \frac{FA_x}{10.9998} + 44.71\end{aligned}$$

$$\Rightarrow h = 44.71$$

Question # 6.24
Answer: E

In general, the loss at issue random variable can be expressed as:

$$L = \bar{Z}_x - P \bullet \bar{Y}_x = \bar{Z}_x - P \bullet \left(\frac{1 - \bar{Z}_x}{\delta} \right) = \bar{Z}_x \bullet \left(1 + \frac{P}{\delta} \right) - \frac{P}{\delta}$$

Using actuarial equivalence to determine the premium rate:

$$P = \frac{\bar{A}_x}{\bar{a}_x} = \frac{0.3}{(1-0.3)/0.07} = 0.03$$

$$Var(L) = \left(1 + \frac{P}{\delta} \right)^2 \bullet Var\left(\bar{Z}_x \right) = \left(1 + \frac{0.03}{0.07} \right)^2 \bullet Var\left(\bar{Z}_x \right) = 0.18$$

$$Var\left(\bar{Z}_x \right) = \frac{0.18}{\left(1 + \frac{0.03}{0.07} \right)^2} = 0.088$$

$$Var(L^*) = \left(1 + \frac{P^*}{\delta} \right)^2 \bullet Var\left(\bar{Z}_x \right) = \left(1 + \frac{0.06}{0.07} \right)^2 (0.088) = 0.304$$

Question # 6.25
Answer: C

Need $\text{EPV(Ben + Exp)} - \text{EPV(Prem)} = -800$

$$\text{EPV(Prem)} = G\ddot{a}_{55:\overline{10}|} = 8.0192G$$

$$\begin{aligned}
\text{EPV(Ben + Exp)} &= 12{,}000\ {}_{10|}\ddot{a}_{55}^{(12)} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\ddot{a}_{65}^{(12)} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\left(\ddot{a}_{65} - \frac{m-1}{2m} \right) + 300\ddot{a}_{55} \\
&= 12{,}000(0.59342)\left(13.5498 - 11/24\right) + 300(16.0599) \\
&= 98{,}042.83
\end{aligned}$$

Therefore, $98{,}042.83 - 8.0192G = -800$

$G = 12{,}326$

Question # 6.26
Answer: D

EPV (Premiums) = $Pa_{90} = P(\ddot{a}_{90} - 1) = (4.1835)P$

EPV(Benefits) $= 1000A_{90} = 1000(0.75317) = 753.17$

Therefore,

$$P = \frac{753.17}{4.1835} = 180.03$$

Question # 6.27
Answer: D

$$
\begin{aligned}
\text{EPV(Premiums)} &= \text{EPV(Benefits)} \\
\text{EPV(Premiums)} &= 3P\,\bar{a}_x - 2P\,{}_{20}E_x\,\bar{a}_{x+20} \\
&= 3P\left(1/_{\mu+\delta}\right) - 2P\left(e^{-20(\mu+\delta)}\right)\left(1/_{\mu+\delta}\right) \\
&= 3P\left(1/_{0.09}\right) - 2Pe^{-1.8} - 1/_{0.09} \\
&= 29.66P \\
\text{EPV(Benefits)} &= 1{,}000{,}000\bar{A}_x - 500{,}000\,{}_{20}E_x\,\bar{A}_{x+20} \\
&= 1{,}000{,}000\left(\mu/_{\mu+\delta}\right) - 500{,}000e^{-20(\mu+\delta)}\;\;\mu/_{\mu+\delta} \\
&= 1{,}000{,}000\left(0.03/_{0.09}\right) - 500{,}000e^{-1.8}\;\;0.03/_{0.09} \\
&= 305{,}783.5
\end{aligned}
$$

$$29.66P = 305{,}783.5$$

$$P = \frac{305{,}783.5}{29.66}$$

$$P = 10{,}309.62$$

Question # 6.28
Answer: B

$$G\ddot{a}_{40:\overline{5}|} = 1000A_{40} + 0.15G + 0.05G\ddot{a}_{40:\overline{5}|} + 5 + 5\ddot{a}_{40:\overline{5}|}$$

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40} \bullet \ddot{a}_{45} = 18.4578 - (0.78113)(17.8162) = 4.5410$$

$$G = \frac{121.06 + 5 + 5(4.5410)}{-0.15 + 0.95(4.5410)} = 35.73$$

Question # 6.29
Answer: B

Per equivalence Principle:

$$G\ddot{a}_{35} = 100,000A_{35} + 0.4G + 150 + 0.1G\ddot{a}_{35} + 50\ddot{a}_{35}$$
$$1770\ddot{a}_{35} = 100,000\left(1 - d\ddot{a}_{35}\right) + 0.4(1770) + 150 + 0.1(1770)\ddot{a}_{35} + 50\ddot{a}_{35}$$
$$1770\ddot{a}_{35} = 100,000 + 708 + 150 + \ddot{a}_{35}\left(177 + 50 - 100,000\left(\frac{0.035}{1.035}\right)\right)$$

Solving for $\ddot{a}_{35}$, we have

$$\ddot{a} = \frac{100,858}{1770 + 3154.64} = \frac{100,858}{4924.64} = 20.48$$

Question # 6.30
Answer: A

The loss at issue is given by:

$$
\begin{aligned}
L_0 &= 100v^{K+1} + 0.05G + 0.05G\,\ddot{a}_{\overline{K+1|}} - G\,\ddot{a}_{\overline{K+1|}} \\
&= 100v^{K+1} + 0.05G - 0.95G\left(\frac{1-v^{K+1}}{d}\right) \\
&= \left(100 + \frac{0.95G}{d}\right)v^{K+1} + 0.05G - 0.95\frac{G}{d}
\end{aligned}
$$

Thus, the variance is

$$
\begin{aligned}
Var\left(L_0\right) &= \left[100 + \frac{0.95\left(2.338\right)}{0.04/1.04}\right]^2 \left({}^2A_x - \left(A_x\right)^2\right) \\
&= \left[100 + \frac{0.95\left(2.338\right)}{0.04/1.04}\right]^2 \left(0.17 - \left(1 - \frac{0.04}{1.04}(16.50)\right)^2\right) \\
&= 908.1414
\end{aligned}
$$

Question # 6.31
Answer: D

$$\overline{A}_{35} = \left(1 - e^{-35(\mu+\delta)}\right) \times \left(\frac{\mu}{\mu+\delta}\right) + e^{-35(\mu+\delta)}\overline{A}_{70} = 0.063421 + 0.146257 = 0.209679$$

$$\overline{a}_{35} = \frac{1-\overline{A}_{35}}{\delta} = \frac{1-0.209679}{0.05} = 15.80642$$

$$\overline{P}_{35} = \frac{\overline{A}_{35}}{\overline{a}_{35}} = \frac{0.209679}{15.80642} = 0.0132654$$

The annual net premium for this policy is therefore $100,000 \times 0.0132654 = 1,326.54$

Question # 6.32
Answer: C

Assuming UDD

Let P = monthly net premium

$$\text{EPV(premiums)} = 12P\ddot{a}_x^{(12)} \cong 12P\left[\alpha(12)\ddot{a}_x - \beta(12)\right]$$
$$= 12P\left[1.00020(9.19) - 0.46651\right]$$
$$= 104.7039P$$

$$\text{EPV(benefits)} = 100,000\bar{A}_x$$
$$= 100,000\frac{i}{\delta}A_x = 100,000\frac{i}{\delta}\left(1 - d\ddot{a}_x\right)$$
$$= 100,000\frac{0.05}{\ln(1.05)}\left(1 - \frac{0.05}{1.05}(9.19)\right)$$
$$= 57,632.62$$

$$P = \frac{57,632.62}{104.7039} = 550.43$$

Question # 6.33
Answer: B

The probability that the endowment payment will be made for a given contract is:

$$
\begin{aligned}
{}_{15}p_x &= \exp\left(-\int_0^{15} 0.02t\,dt\right) \\
&= \exp\left(-0.01t^2\Big|_0^{15}\right) \\
&= \exp\left(-0.01(15)^2\right) \\
&= 0.1054
\end{aligned}
$$

Because the premium is set by the equivalence principle, we have $E\left[{}_0L\right]=0.$ Further,

$$
\begin{aligned}
\operatorname{Var}\left({}_0L\right) &= 500\left[\left(10{,}000v^{15}\right)^2\left({}_{15}p_x\right)\left(1-{}_{15}p_x\right)\right] \\
&= 1{,}942{,}329{,}000
\end{aligned}
$$

Then, using the normal approximation, the approximate probability that the aggregate losses exceed 50,000 is

$$
P\left({}_0L>50{,}000\right)=P\left(Z>\frac{50{,}000-0}{\sqrt{1{,}942{,}329{,}000}}\right)=P(Z>1.13)=0.13
$$

Question # 6.34
Answer: A

Let B be the amount of death benefit.

$$
\begin{aligned}
&\text{EPV(Premiums)} = 500\ddot{a}_{61} = 500(14.6491) = 7324.55 \\
&\text{EPV(Benefits)} = \text{B}\cdot A_{61} = (0.30243)\text{B} \\
&\text{EPV(Expenses)} = (0.12)(500)+(0.03)(500)\ddot{a}_{61} = (0.12)(500)+(0.03)(7324.55) = 279.74 \\
&\text{EPV(Premiums)} = \text{EPV(Benefits)} + \text{EPV(Expenses)} \\
&7324.55 = (0.30243)\text{B} + 279.74 \\
&7044.81 = (0.30243)\text{B} \\
&\text{B} = 23{,}294
\end{aligned}
$$

Question # 6.35
Answer: D

Let *G* be the annual gross premium. By the equivalence principle, we have
$G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.09653)}{0.96(18.9728) - 0.15} = 534.38$$

Question # 6.36
Answer: B

By the equivalence principle,

$$4500\bar{a}_{x:\overline{20|}} = 100,000\bar{A}^{1}_{x:\overline{20|}} + R\bar{a}_{x:\overline{20|}}$$

where

$$\bar{A}^{1}_{x:\overline{20|}} = \frac{\mu}{\mu+\delta}\left(1-e^{-20(\mu+\delta)}\right) = \frac{0.04}{0.12}\left(1-e^{-20(0.12)}\right) = 0.3031$$
$$\bar{a}_{x:\overline{20|}} = \frac{1-e^{-20(\mu+\delta)}}{\mu+\delta} = \frac{1-e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for *R*, we have

$$R = 4500 - 100,000\left(\frac{0.3031}{7.5774}\right) = 500$$

Question # 6.37
Answer: D

By the equivalence principle, we have

$$G\ddot{a}_{35:\overline{10}|} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so

$$G = \frac{50,100A_{35} + 100\left(\ddot{a}_{35} - 1\right)}{\ddot{a}_{35:\overline{10}|}} = \frac{50,100\left(0.09653\right) + 100\left(17.9728\right)}{8.0926} = 819.69$$

Question # 6.38
Answer: B

Let *P* be the annual net premium

$$P = \frac{1000\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1000(0.192)}{\ddot{a}_{x:\overline{n}|}}$$

where

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{(1.05)}{(0.05)}\left(1 - A^{1}_{x:\overline{n}|} - A_{x:\overline{n}|}^{\ \ 1}\right)$$

$$\bar{A}_{x:\overline{n}|} = \frac{i}{\delta}\left(A^{1}_{x:\overline{n}|}\right) + {}_{n}E_{x}$$

$$\Rightarrow 0.192 = \frac{0.05}{0.04879}\left(A^{1}_{x:\overline{n}|}\right) + 0.172$$

$$\Rightarrow A^{1}_{x:\overline{n}|} = 0.019516$$

$$\Rightarrow \ddot{a}_{x:\overline{n}|} = \frac{1.05}{0.05}(1 - 0.019516 - 0.172) = 16.978$$

Therefore, we have

$$P = \frac{1000\left(0.192\right)}{16.978} = 11.31$$

Question # 6.39
Answer: A

Premium at issue for (40): $\dfrac{1000A_{40}}{\ddot{a}_{40}} = \dfrac{121.06}{18.4578} = 6.5587$

Premium at issue for (80): $\dfrac{1000A_{80}}{\ddot{a}_{80}} = \dfrac{592.93}{8.5484} = 69.3615$

Lives in force after ten years:

Issued at age 40: $10,000\,{}_{10}p_{40} = 10,000 \times \dfrac{98,576.4}{99,338.3} = 9923.30$

Issued at age 80: $10,000\,{}_{10}p_{80} = 10,000 \times \dfrac{41,841.1}{75,657.2} = 5530.35$

The total number of lives after ten years is therefore: $9923.30 + 5530.35 = 15,453.65$

The average premium after ten years is therefore:

$$\frac{(6.5587 \times 9923.30) + (69.3615 \times 5530.35)}{15,453.65} = 29.03$$

Question # 6.40
Answer: C

Let P be the annual net premium at x+1. Also, let A_y^* be the expected present value for the special insurance described in the problem issued to (y).

$$P\ddot{a}_{x+1} = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1}\ {}_{k|}q_{x+1} = 1000A_{x+1}^*$$

We are given

$$110\ddot{a}_x = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1}\ {}_{k|}q_x = 1000A_x^*$$

Which implies that

$$110\left(1+vp_x\ddot{a}_{x+1}\right) = 1000\left(1.03vq_x + 1.03vp_xA_{x+1}^*\right)$$

Solving for A_{x+1}^*, we get

$$A_{x+1}^* = \frac{\frac{110}{1000}\left[1+v(0.95)(7)\right]-1.03v(0.05)}{1.03v(0.95)} = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

Question # 6.41
Answer: B

Let P be the net premium for year 1.

Then:

$$P + 1.01Pvp_x = 100,000vq_x + (1.01)(100,000)v^2p_xq_{x+1}$$

$$P\left[1+\frac{1.01}{1.05}0.99\right] = 100,000\left(\frac{0.01}{1.05}+\frac{(1.01)(0.99)(0.02)}{(1.05)^2}\right) \Rightarrow P = 1416.93$$

Question # 6.42
Answer: D

The policy is fully discrete, so all cash flows occur at the start or end of a year.

$$\text{Die Year 1 ==> } L_0 = 1000v - 315.80 = 625.96$$

$$\text{Die Year 2 ==> } L_0 = 1000v^2 - 315.80(1+v) = 273.71$$

$$\text{Survive Year 2 ==> } L_0 = 1000v^3 - 315.80(1+v+v^2) = -58.03$$

There is a loss if death occurs in year 1 or year 2, otherwise the policy was profitable.

$$\Pr\left(\text{death in year 1 or 2}\right) = 1 - e^{-2\mu} = 0.113$$

Question # 6.43
Answer: C

$$\begin{aligned}
APV\left(\text{expenses}\right) &= 0.35G + 8 + 0.15G a_{30:\overline{4}|} + 4a_{30:\overline{9}|} \\
&= 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|}
\end{aligned}$$

$$G\,\ddot{a}_{30:\overline{5}|} = 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|} + 200{,}000 A^{1}_{30:\overline{10}|}$$

$$G = \frac{200{,}000 A^{1}_{30:\overline{10}|} + 4 + 4\ddot{a}_{30:\overline{10}|}}{0.85\ddot{a}_{30:\overline{5}|} - 0.20}$$

$$\begin{aligned}
200{,}000\, A^{1}_{30:\overline{10}|} &= 200{,}000\left[A_{30:\overline{10}|} - {}_{10}E_{30}\right] \\
&= 200{,}000(0.61447 - 0.61152) = 590
\end{aligned}$$

$$G = \frac{590 + 4 + 4\left(8.0961\right)}{0.85\left(4.5431\right) - 0.20} = 171.07$$

Question # 6.44
Answer: D

Let P be the premium per 1 of insurance.

$$P\ddot{a}_{50:\overline{10|}} = P(IA)^{1}_{50:\overline{10|}} + {}_{10}E_{50}A_{60}$$

$$\ddot{a}_{50:\overline{10|}} = \ddot{a}_{50} - {}_{10}E_{50}\ddot{a}_{60} = 17.0 - 0.60 \times 15.0 = 8$$

$$A_{60} = 1 - d\ddot{a}_{60} = 1 - \left(\frac{0.05}{1.05}\right)15 = 0.285714$$

$$P\left(\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}\right) = {}_{10}E_{50}A_{60}$$

$$P = \frac{{}_{10}E_{50}A_{60}}{\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}} = \frac{0.6 \times 0.285714}{8 - 0.15} = 0.021838$$

$$100P = 2.18$$

Question # 6.45
Answer: E

$$L_0 = 100{,}000v^T - 560\bar{a}_{\overline{T|}} = \left(100{,}000 + \frac{560}{\delta}\right)e^{-\delta T} - \frac{560}{\delta}$$

Since L_0 is a decreasing function of T, the 25th percentile of L_0 is $L_0(t)$ where t is such that $\Pr[T_{35} \le t] = 0.25$ or $\Pr[T_{35} > t] = 0.75$.

$$\frac{\ell_{35+t}}{\ell_{35}} = 0.75$$

$$\ell_{35+t} = 0.75\ell_{35} = 0.75 \times 99{,}556.7 = 74{,}667.5$$

$$\ell_{81} < 74{,}667.5 < \ell_{80}$$

$$t = (80-35) + s$$

$$\ell_{80+s} = s\ell_{81} + (1-s)\ell_{80}$$

$$74{,}667.5 = 73{,}186.3s + 75{,}657.2(1-s)$$

$$s = 0.40054$$

$$t = 45.40054$$

$$L_0(45.40054) = \left(100{,}000 + \tfrac{560}{\ln(1.05)}\right)e^{-45.40054\ln(1.05)} - \tfrac{560}{\ln(1.05)} = 689.25$$

Question # 6.46
Answer: E

Let P be the premium per 1 of insurance.

$$P\ddot{a}_{55:\overline{10|}} = 0.51213P + v^{10}{}_{10}p_{55}\,\ddot{a}_{65}$$

$$\ddot{a}_{55} = \ddot{a}_{55:\overline{10|}} + v^{10}{}_{10}p_{55}\,\ddot{a}_{65} \Rightarrow v^{10}{}_{10}p_{55}\,\ddot{a}_{65} = 12.2758 - 7.4575 = 4.8183$$

$$7.4575P = 0.51213P + 4.8183 \Rightarrow P = 0.693742738$$

$$300P = 208.12$$

Question # 6.47
Answer: D

$$G\ddot{a}_{70:\overline{10}|} = 100,000\,{}_{10}E_{70}\ddot{a}_{80} + 0.05G\ddot{a}_{70:\overline{10}|} + 0.7G$$

$$7.6491G = (100,000)(0.50994)(8.5484) + 0.05G(7.6491) + 0.7G$$

$$\Rightarrow G = 66,383.54$$

Question # 6.48
Answer: A

Actuarial present value of insured benefits:

$$100,000\left[\frac{0.95\times 0.02}{1.06^6} + \frac{0.95\times 0.98\times 0.03}{1.06^7} + \frac{0.95\times 0.98\times 0.97\times 0.04}{1.06^8}\right] = 5,463.32$$

$$\Rightarrow P\left(1 + \frac{0.95}{1.06^5}\right) = 5,463.32 \Rightarrow P = 3,195.12$$

Question # 6.49
Answer: C

$$G\ddot{a}^{(12)}_{40:\overline{20}|} = 100,000\left(\frac{i}{\delta}\right)A_{40} + 200 + 0.04G\ddot{a}^{(12)}_{40:\overline{20}|}$$

$$\begin{aligned}\ddot{a}^{(12)}_{40:\overline{20}|} &= \alpha(12)\ddot{a}_{40:\overline{20}|} - (1 - {}_{20}E_{40})\beta(12)\\ &= 1.00020\cdot 12.9935 - (1 - 0.36663)\cdot 0.46651 = 12.700625\end{aligned}$$

$$G = \frac{(100,000)(1.02480)(0.12106)+200}{0.96\times 12.700625} = 1033.92$$

$$\Rightarrow \text{G/12} = 86.16$$

Question # 6.50
Answer: A

$$1,000P = 1,000\frac{A_{35}}{\ddot{a}_{35}} = \frac{96.53}{18.9728} = 5.0878$$

Benefits paid during July 2018:

$$10{,}000 \times 1{,}000 \times q_{35} = 10,000 \times 0.391 = 3910$$

Premiums payable during July 2018:

$$10{,}000 \times (1 - q_{35}) \times 5.0878 = 9,996.09 \times 5.0878 = 50,858.10$$

Cash flow during July 2018:

$$3910 - 50,858 = -46,948$$

Question # 6.51
Answer: D

Under the Equivalence Principle

$$P\ddot{a}_{62:\overline{10|}} = 50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10|}}\right) + P\left((IA)^{1}_{62:\overline{10|}}\right)$$

where $(IA)^{1}_{62:\overline{10|}} = 11A^{1}_{62:\overline{10|}} - \sum_{k=1}^{10} A^{1}_{62:\overline{k|}} = 11(0.091) - 0.4891 = 0.5119$

So $$P = \frac{50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10|}}\right)}{\ddot{a}_{62:\overline{10|}} - (IA)^{1}_{62:\overline{10|}}} = \frac{50,000(12.2758 - 7.4574)}{7.4574 - 0.5119} = 34,687$$

Question # 7.1
Answer: C

$$\begin{aligned}{}_{10}V &= 50,000\left(A_{50} + {}_{10}E_{50}A_{60}\right) - (875)\left[\ddot{a}_{50:\overline{10|}}\right] \\ &= 50,000[0.18931 + (0.60182)(0.29028)] - 875[8.0550] \\ &= 11,152\end{aligned}$$

Question # 7.2
Answer: C

$_0V = 0$
$_2V = 2000$

Year 1: $\left({}_0V + P\right)\left(1+i\right) = q_x\left(2000 + {}_1V\right) + p_x\ {}_1V$

$$P(1.1) = 0.15(2000 + {}_1V) + 0.85({}_1V)$$
$$1.1P - 300 = {}_1V$$

Year 2: $\left({}_1V + P\right)\left(1+i\right) = q_{x+1}(2000 + {}_2V) + p_{x+1}(2000)$

$$(1.1P - 300 + P)(1.1) = 0.165(2000 + 2000) + 0.835(2000)$$
$$2.31P - 330 = 2330$$

$$P = \frac{2330 + 330}{2.31} = 1152$$

Question # 7.3
Answer: E

$i^{(4)} = 0.08$ means an interest rate of $j = 0.02$ per quarter. This problem can be done with two quarterly recursions or a single calculation.

Using two recursions:

$$_{10.75}V = \frac{[_{10.5}V + 60(1-0.1)](1.02) - \frac{800-706}{800}(1000)}{\frac{706}{800}}$$

$$753.72 = \frac{[_{10.5}V + 54](1.02) - 117.50}{0.8825} \Rightarrow {}_{10.5}V = 713.31$$

$$_{10.5}V = \frac{[_{10.25}V](1.02) - \frac{898-800}{898}(1000)}{\frac{800}{898}} \Rightarrow 713.31 = \frac{[_{10.25}V](1.02) - 109.13}{0.8909}$$

$$_{10.25}V = 730.02$$

Using a single step, $_{10.25}V$ is the EPV of cash flows through time 10.75 plus $_{0.5}E_{80.25}$ times the EPV of cash flows thereafter (that is, $_{10.75}V$).

$$_{10.25}V = (1000)\left[\frac{898-800}{898(1.02)} + \frac{800-706}{898(1.02)^2}\right] - (60)(1-0.1)\left[\frac{800}{898(1.02)}\right] + \left[\frac{706}{898(1.02)^2}\right](753.72) = 730$$

Question # 7.4
Answer: B

EPV of benefits at issue $=1000A_{40}+4\ {}_{11}E_{40}(1000A_{51})$

$=121.06+(4)(0.57949)(197.80)=579.55$

EPV of expenses at issue $=100+10(\ddot{a}_{40}-1)=100+10(17.4578)=274.58$

$\pi=(579.55+274.58)/\ddot{a}_{40}=854.13/18.4578=46.27$

$G=1.02\pi=47.20$

EPV of benefits at time $1=1000A_{41}+4\ {}_{10}E_{41}\times 1000A_{51}$

$=126.65+(4)(0.60879)(197.80)=608.32$

EPV of expenses at time $1=10(\ddot{a}_{41})=10(18.3403)=183.40$

Gross Prem Reserve $=608.32+183.40-G\ddot{a}_{41}=791.72-47.20(18.3403)=-73.94$

Question # 7.5
Answer: A

Net Amount at Risk = $1000-{}_3V=996.52$

Expected Deaths = $(10{,}000-10)q_{47}=9990(0.000916)=9.15$

Actual Deaths = 6

Mortality Gain/Loss = (Expected Deaths – Actual Deaths)(Net Amount At Risk)
$=(9.15-6)(996.52)=3139$

Question # 7.6
Answer: B

$$\left.\frac{d}{dt}({}_tV)\right|_{t=9.6} = G - E - S\mu + {}_{9.6}V(\mu+\delta) \text{ where } G, E, S \text{ and } \mu \text{ are evaluated at } t = 9.6 \text{ and}$$

where S includes claims-related expenses.

$$\left.\frac{d}{dt}({}_tV)\right|_{t=9.6} = 450 - (0.02)(450) - (106,000+200)(0.01) + {}_{9.6}V(0.01+0.05) = -621 + 0.06\,{}_{9.6}V$$

$${}_{9.6}V \approx {}_{9.8}V - 0.2\left[\left.\frac{d}{dt}({}_tV)\right|_{t=9.6}\right] = 126.68 - (0.2)[-621 + 0.06\,{}_{9.6}V] = 250.88 - 0.012\,{}_{9.6}V$$

$${}_{9.6}V \approx \frac{250.88}{1.012} = 247.91$$

Question # 7.7
Answer: E

$${}_{4.5}V = v^{0.5}\ {}_{0.5}p_{x+4.5}\ {}_5V + v^{0.5}\ {}_{0.5}q_{x+4.5}\ b, \text{ where } b = 10,000 \text{ is the death benefit during year 5}$$

$${}_{0.5}q_{x+4.5} = \frac{{}_{0.5}q_{x+4}}{1 - {}_{0.5}q_{x+4}} = \frac{0.5(0.04561)}{1 - 0.5(0.04561)} = 0.02334$$

$${}_{0.5}p_{x+4.5} = 0.97666$$

$${}_5V = \frac{({}_4V + P)(1.03) - q_{x+4}b}{p_{x+4}}$$

$${}_5V = \frac{(1,405.08 + 647.46)(1.03) - 0.04561(10,000)}{0.95439} = 1,737.25$$

$${}_{4.5}V = (1.03)^{-0.5}(0.97666)(1,737.25) + (1.03)^{-0.5}(0.02334)(10,000)$$
$$= 1,671.81 + 229.98 = 1,902$$

${}_{4.5}V$ can also be calculated recursively:

$${}_{0.5}q_{x+4} = 0.5(0.04561) = 0.02281$$

$${}_{4.5}V = \frac{(1,405.08 + 647.16)(1.03)^{0.5} - 0.02281(10,000)/(1.03)^{0.5}}{1 - 0.02281} = 1,902$$

The interest adjustment on the death benefit term is needed because the death benefit will not be paid for another one-half year.

Question # 7.8
Answer: B

$$\text{Net Premium} = 10,000A_{62} / \ddot{a}_{62} = 10,000(0.31495)/14.3861 = 218.93$$
$$G = 218.93(1.03) = 225.50$$

Let ${}_0L^*$ be the present value of future loss at issue for one policy.

$$\begin{aligned} {}_0L^* &= 10,000v^{K+1} - (G-5)\ddot{a}_{\overline{K+1|}} + 0.05G \\ &= 10,000v^{K+1} - (225.50-5)\frac{1-v^{K+1}}{d} + 0.05(225.50) \\ &= \left(10,000 + 4630.50\right)v^{K+1} - 4630.50 + 11.28 \\ &= 14,630.50v^{K+1} - 4619.22 \end{aligned}$$

$$E({}_0L^*) = 14,630.50A_{62} - 4619.22 = 14,630.50(0.31495) - 4619.22 = -11.34$$
$$Var({}_0L^*) = (14,630.50)^2\left({}^2A_{62} - A_{62}^2\right) = (14,630.50)^2\left(0.12506 - 0.31495^2\right) = 5,536,763$$

Let ${}_0L$ be the aggregate loss for 600 such policies.

$$E({}_0L) = 600E({}_0L^*) = 600(-11.34) = -6804$$
$$Var({}_0L) = 600Var({}_0L^*) = 600(5,536,763) = 3,322,057,800$$
$$StdDev({}_0L) = 3,322,057,800^{0.5} = 57,637$$

$$\Pr\left({}_0L < 40,000\right) = \Phi\left(\frac{40,000+6804}{57,637}\right) = \Phi\left(0.81\right) = 0.7910$$

Question # 7.9
Answer: E

Gross premium = *G*

$$G\ddot{a}_{45} = 2000A_{45} + \underbrace{\left(1\left(\frac{2000}{1000}\right)+20\right)}_{22} + \underbrace{\left(0.5\left(\frac{2000}{1000}\right)+10\right)}_{11}\ddot{a}_{45} + 0.20G + 0.05G\ddot{a}_{45}$$

$$\left(0.95\ddot{a}_{45} - 0.20\right)G = 2000A_{45} + 22 + 11\ddot{a}_{45}$$

$$G=\frac{2000A_{45}+22+11\ddot{a}_{45}}{0.95\ddot{a}_{45}-0.20}=\frac{2000(0.15161)+22+11(17.8162)}{0.95(17.8162)-0.20}=31.16$$

There are two ways to proceed. The first is to calculate the expense-augmented reserve and the net premium reserve and take the difference.

The net premium is $\frac{2000A_{45}}{\ddot{a}_{45}}=\frac{2000(0.15161)}{17.8162}=17.02$

The net premium reserve is $2000A_{55}-17.02\ddot{a}_{55}=2000(0.23524)-17.02(16.0599)=197.14$

The expense-augmented reserve, which is the gross premium reserve, is

$$2000A_{55}+[0.05(31.16)+0.5(2000/1000)+10]\ddot{a}_{55}-31.16\ddot{a}_{55}$$
$$=2000(0.23524)+(12.56-31.16)(16.0599)=171.77$$

Expense reserve is 171.77 – 197.14 = –25

The second is to calculate the expense reserve directly based on the pattern of expenses. The first step is to determine the expense premium.

The present value of expenses is

$$[0.05G+0.5(2000/1000)+10]\ddot{a}_{45}+0.20G+1.0(2000/1000)+20$$
$$=12.558(17.8162)+28.232=251.97$$

The expense premium is 251.97/17.8162=14.14

The expense reserve is the expected present value of future expenses less future expense premiums, that is,

$$[0.05G+0.5(2000/1000)+10]\ddot{a}_{55}-14.14\ddot{a}_{55}=-1.582(16.0599)=-25$$

There is a shortcut with the second approach based on recognizing that expenses that are level throughout create no expense reserve (the level expense premium equals the actual expenses). Therefore, the expense reserve in this case is created entirely from the extra first year expenses. They occur only at issue so the expected present value is 0.20(31.16)+1.0(2000/1000)+20 = 28.232. The expense premium for those expenses is then 28.232/17.8162 = 1.585 and the expense reserve is the present value of future non-level expenses (0) less the present value of those future expense premiums, which is 1.585(16.0599) = 25 for a reserve of –25.

Question # 7.10
Answer: A

$$q_x^{\mathrm{NS}} = q_{x+1}^{\mathrm{NS}} = 1 - e^{-0.1} = 0.095$$

Then the annual premium for the non-smoker policies is P^{NS}, where

$$P^{\mathrm{NS}}\left(1 + vp_x^{\mathrm{NS}}\right) = 100,000vq_x^{\mathrm{NS}} + 100,000v^2 p_x^{\mathrm{NS}} q_{x+1}^{\mathrm{NS}} + 30,000v^2 p_x^{\mathrm{NS}} p_{x+1}^{\mathrm{NS}}$$

$$P^{\mathrm{NS}} = \frac{100,000(0.926)(0.095) + 100,000(0.857)(0.905)(0.095) + 30,000(0.857)(0.905)^2}{1 + (0.926)(0.905)}$$

$$P^{\mathrm{NS}} = 20,251$$

And then $P^{\mathrm{S}} = 40,502.$

$$q_x^S = q_{x+1}^S = 1.5(1 - e^{-0.1}) = 0.143$$

$$\begin{aligned} EPV(L^{\mathrm{S}}) &= 100,000vq_x^{\mathrm{S}} + 100,000v^2 p_x^{\mathrm{S}} q_{x+1}^{\mathrm{S}} + 30,000v^2 p_x^{\mathrm{S}} p_{x+1}^{\mathrm{S}} - P^{\mathrm{S}} - P^{\mathrm{S}} vp_x^{\mathrm{S}} \\ &= 100,000(0.926)(0.143) + 100,000(0.857)(0.857)(0.143) \\ &\quad + 30,000(0.857)(0.857)^2 - 40,502 - 40,502(0.926)(0.857) \\ &= -30,017 \end{aligned}$$

Question # 7.11
Answer: D

$$\ddot{a}_{x+10} = (1 - A_{x+10}) / d = (1 - 0.4) / (0.05 / 1.05) = 12.6$$

$$\ddot{a}_{x+10}^{(12)} \approx 12.6 - 11/24 = 12.142$$

$${}_{10}V = 10,000A_{x+10} + 100\ddot{a}_{x+10} - 12\ddot{a}_{x+10}^{(12)}(30)(1 - 0.05)$$

$${}_{10}V = 10,000(0.4) + 100(12.6) - 12(12.142)(28.50)$$

$${}_{10}V = 1107$$

Question # 7.12
Answer: C

Simplest solution is recursive:

${}_0V = 0$ since the reserves are net premium reserves.

$$q_{[70]} = (0.7)(0.010413) = 0.007289$$

$${}_1V = \frac{(0+35.168)(1.05)-(1000)(0.007289)}{1-0.007289} = 29.86$$

Prospectively, $q_{[70]+1} = (0.8)(0.011670) = 0.009336; \quad q_{[70]+2} = (0.9)(0.013081) = 0.011773$

$$\begin{aligned} A_{[70]+1} &= (0.009336)v + (1-0.009336)(0.011773)v^2 \\ &\quad + (1-0.009336)(1-0.011773)(0.47580)v^2 = 0.44197 \\ \ddot{a}_{[70]+1} &= \left(1-A_{[70]+1}\right)/d = (1-0.44197)/(0.05/1.05) = 11.7186 \\ {}_1V &= (1000)(0.44197)-(11.7186)(35.168) = 29.85 \end{aligned}$$

Question # 7.13
Answer: A

Let P = 0.00253 be the monthly net premium per 1 of insurance.

$$\begin{aligned} {}_{10}V &= 100{,}000\left[\frac{i}{\delta}A^{1}_{55:\overline{10}|} + A_{55:\overset{\ \ 1}{\overline{10}|}} - 12P\ddot{a}^{(12)}_{55:\overline{10}|}\right] \\ &= 100{,}000\left[1.02480(0.02471)+0.59342-(12)(0.00253)(7.8311)\right] \\ &\approx 38{,}100 \end{aligned}$$

Where

$$\begin{aligned} A^{1}_{55:\overline{10}|} &= A_{55:\overline{10}|} - {}_{10}E_{55} = 0.61813-0.59342 = 0.02471 \\ A_{55:\overset{\ \ 1}{\overline{10}|}} &= {}_{10}E_{55} = 0.59342 \\ \ddot{a}_{55:\overline{10}|} &= 8.0192 \\ \ddot{a}^{(12)}_{55:\overline{10}|} &= \alpha(12)\ddot{a}_{55:\overline{10}|} - \beta(12)\left[1-{}_{10}E_{55}\right] \\ &= 1.00020(8.0192)-0.46651(1-0.59342) = 7.8311 \end{aligned}$$

Question # 7.14
Answer: C

Use superscript *g* for gross premiums and gross premium reserves.
Use superscript *n* (representing "net") for net premiums and net premium reserves.
Use superscript *e* for expense premiums and expense reserves.

$$P^g = 977.60 \text{ (given)}$$

$$P^e = \frac{0.58P^g + 450 + \left(0.02P^g + 50\right)\ddot{a}_{45}}{\ddot{a}_{45}}$$
$$= \frac{0.58(977.60) + 450 + [0.02(977.60) + 50]17.8162}{17.8162} = 126.64$$

Alternatively,

$$P^n = \frac{100,000A_{45}}{\ddot{a}_{45}} = 850.97 \qquad P^e = P^g - P^n = 126.63$$

$${}_5V^e = \left(0.02P^g + 50\right)\ddot{a}_{50} - P^e\ddot{a}_{50} = [0.02(977.60) + 50](17.0245) - 126.64(17.0245) = -972$$

Alternatively,

$${}_5V^n = 100,000A_{50} - P^n\ddot{a}_{50}$$
$$= 100,000(0.18931) - 850.97(17.0245) = 4443.66$$
$${}_5V^g = 100,000A_{50} + \left(50 + 0.02P^g - P^g\right)\ddot{a}_{50}$$
$$= 100,000(0.18931) + [50 + 0.02(977.60) - 977.60](17.0245) = 3471.93$$
$${}_5V^e = {}_5V^g - {}_5V^n = -972$$

Question # 7.15
Answer: B

$$L = 10,000v^{K_{45}+1} - P\ddot{a}_{\overline{K_{45}+1|}} = 10,000v^{11} - P\ddot{a}_{\overline{11|}}$$
$$4450 = 10,000(0.58468) - 8.7217P$$
$$P = (5,846.8 - 4,450)/8.7217 = 160.15$$
$$A_{55} = 1 - d\ddot{a}_{55} = 1 - (0.05/1.05)(13.4205) = 0.36093$$
$${}_{10}V = 10,000A_{55} - P\ddot{a}_{55} = (10,000)(0.36093) - (160.15)(13.4205) = 1,460$$

Question # 7.16
Answer: B

$$L_A = v^T - 0.10\bar{a}_{\overline{T|}} = \left(1+\frac{10}{6}\right)v^T - \frac{10}{6}$$

$$Var[L_A] = \left(1+\frac{10}{6}\right)^2 Var[v^T] = 0.455 \Rightarrow Var[v^T] = 0.06398$$

$$L_B = 2v^T - 0.16\bar{a}_{\overline{T|}} = \left(2+\frac{16}{6}\right)v^T - \frac{16}{6}$$

$$Var[L_B] = \left(2+\frac{16}{6}\right)^2 Var[v^T] = \left(2+\frac{16}{6}\right)^2 (0.06398) = 1.39$$

Question # 7.17
Answer: E

In the final year: $({}_{24}V + P)(1+\mathrm{i}) = b_{25}(q_{68}) + 1(p_{68})$

Since $b_{25} = 1,$ this reduces to $({}_{24}V + P)(1+i) = 1 \Rightarrow (0.6+P)(1.04) = 1 \Rightarrow P = 0.36154$

Looking back to the 12th year: $({}_{11}V + P)(1+i) = b_{12}(q_{55}) + {}_{12}V(p_{55})$

$$\Rightarrow (5.36154)(1.04) = 14(0.15) + {}_{12}V(0.85) \Rightarrow {}_{12}V = 4.089$$

Question # 7.18
Answer: A

This first solution recognizes that the full preliminary term reserve at the end of year 10 for a 30 year endowment insurance on (40) is the same as the net premium reserve at the end of year 9 for a 29 year endowment insurance on (41). Then, using superscripts of FPT for full preliminary term reserve and NLP for net premium reserve to distinguish the reserves, we have

$$\begin{aligned} 1000\,{}_{10}V^{FPT} &= 1000\,{}_{9}V^{NLP} = 1000(A_{50:\overline{20|}} - P_{41:\overline{29|}}\ddot{a}_{50:\overline{20|}}) \\ &= 1000[0.38844 - 0.01622(12.8428)] = 180 \end{aligned}$$

or $\quad = 1000\left(1 - \dfrac{\ddot{a}_{50:\overline{20|}}}{\ddot{a}_{41:\overline{29|}}}\right) = 1000\left(1 - \dfrac{12.8428}{15.6640}\right) = 180$

where

$$\ddot{a}_{41:\overline{29}|} = \ddot{a}_{41} - {}_{29}E_{41}\,\ddot{a}_{70}$$
$$= 18.3403 - (0.2228726)(12.0083)$$
$$= 15.6640$$
$$A_{41:\overline{29}|} = 1 - d(15.6640) = 0.254095$$
$${}_{29}E_{41} = v^{29}\left(\frac{l_{70}}{l_{41}}\right) = (0.242946)\left(\frac{91,082.4}{99,285.9}\right) = 0.2228726$$
$$P_{41:\overline{29}|} = \frac{0.254095}{15.6640} = 0.01622$$

Alternatively, working from the definition of full preliminary term reserves as having ${}_1V^{FPT} = 0$ and the discussion of modified reserves in the Notation and Terminology Study Note, let α be the valuation premium in year 1 and β be the valuation premium thereafter. Then (with some of the values taken from above),

$$\alpha = 1000vq_{40} = 0.5019$$
$$\text{APV (valuation premiums)} = \text{APV (benefits)}$$
$$\alpha + {}_1E_{40}(\ddot{a}_{41:\overline{29}|})\beta = 1000A_{40:\overline{30}|}$$
$$0.5019 + 0.95188(15.6640)\beta = 242.37$$
$$\beta = \frac{242.37 - 0.5019}{14.9102} = 16.22$$

Where

$${}_1E_{40} = (1 - 0.000527)v = 0.95188$$
$$A_{40:\overline{30}|} = A_{40} + {}_{20}E_{40}({}_{10}E_{60})(1 - A_{70})$$
$$= 0.12106 + 0.36663(0.57864)(1 - 0.42818) = 0.24237$$
$${}_{10}V^{FPT} = 1000A_{50:\overline{20}|} - \beta\ddot{a}_{50:\overline{20}|} = 1000(0.38844) - 16.22(12.8427) = 180$$

Question # 7.19
Answer: E

No cash flow beginning of year, the one item earning interest is the reserve at the end of the previous year

Gain due to interest = (reserves at the beginning of year)(actual interest – anticipated interest) $= 1000(8929.18)(0.04 - 0.03) = 89292$.

Question # 7.20
Answer: A

$$
\begin{aligned}
\left({}_5V + 0.96G - 50\right)(1.05) &= q_{50}(100,200) + p_{50}\ {}_6V \\
\left(5500 + 0.96G - 50\right)(1.05) &= (0.009)(100,200) + (1 - 0.009)(7100) \\
(1.05)(0.96)G + 5722.5 &= 7937.9 \\
(1.05)(0.96)G &= 2215.4 \\
G &= 2197.8
\end{aligned}
$$

Question # 7.21
Answer: E

$$
\begin{aligned}
&{}_{15.6}V\left(1+i\right)^{0.4} = {}_{0.4}p_{x+15.6}\ \ {}_{16}V + {}_{0.4}q_{x+15.6}\ \ 100 \\
&{}_{15.6}V(1.05)^{0.4} = 0.957447(49.78) + 0.042553(100) \\
&{}_{15.6}V = 50.91
\end{aligned}
$$

Question # 7.22
Answer: D

APV future benefits $= 1000\left[0.04v + 0.05 \times 0.96v^2 + 0.96 \times 0.95 \times (0.06 + 0.94 \times 0.683)v^3\right] = 630.25$

APV future premiums $= 130(1 + 0.96v) = 248.56$

$E[{}_3L] = 630.25 - 248.56 = 381.69$

Question # 7.23
Answer: D

$$\frac{V\left[{}_{10}L\right]}{V\left[{}_{11}L\right]}=\frac{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+10}-A_{x+10}^2\right)}{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+11}-A_{x+11}^2\right)}$$

$$A_{x+10}=vq_{x+10}+vp_{x+10}\ A_{x+11}$$

$$=(0.90703)^{1/2}(0.02067)+(0.90703)^{1/2}(1-0.02067)(0.52536)=0.50969$$

$${}^2A_{x+10}=v^2q_{x+10}+v^2p_{x+10}\ {}^2A_{x+11}$$

$$=(0.90703)(0.02067)+(0.90703)(1-0.02067)(0.30783)=0.29219$$

$$\Rightarrow\frac{\operatorname{Var}\left({}_kL\right)}{\operatorname{Var}\left({}_{k+1}L\right)}=\frac{(0.29219)-(0.50969)^2}{(0.30783)-(0.52536)^2}=\frac{0.03241}{0.03183}=1.018$$

Question # 7.24
Answer: A

$${}_1V_x=A_{x+1}-P_x\,\ddot{a}_{x+1}=1-d\ddot{a}_{x+1}-P_x\,\ddot{a}_{x+1}$$

$$=1-\underbrace{\left(P_x+d\right)}\ddot{a}_{x+1}=1-\ddot{a}_{x+1}/\ddot{a}_x$$

$$\Rightarrow\ddot{a}_x\left(1-{}_1V_x\right)=\ddot{a}_{x+1}$$

Since $\ddot{a}_x=1+vp_x\ddot{a}_{x+1}$ substituting we get

$$\ddot{a}_x\left(1-{}_1V_x\right)=\frac{\ddot{a}_x-1}{vp_x}\Rightarrow\ddot{a}_x\left(1-{}_1V_x\right)vp_x=\ddot{a}_x-1$$

Solving for $\ddot{a}_x$, we get $\ddot{a}_x=\dfrac{1}{1-\left(1-{}_1V_x\right)vp_x}=\dfrac{1}{1-(1-0.012)\left(\frac{1}{1.04}\right)\left(1-0.009\right)}$

$$=17.07942$$

Question # 7.25
Answer: D

Let *G* be the annual gross premium.

Using the equivalence principle, $0.90G\ddot{a}_{40} - 0.40G = 100,000A_{40} + 300$

So $G = \dfrac{100,000(0.12106) + 300}{0.90(18.4578) - 0.40} = 765.2347$

The gross premium reserve after the first year and immediately after the second premium and associated expenses are paid is

$$100,000A_{41} - 0.90G\left(\ddot{a}_{41} - 1\right)$$

$$= 12,665 - 0.90(765.2347)(17.3403)$$

$$= 723$$

Question # 7.26
Answer: C

The expected end of the year profit B $= 722 = \left({}_{20}V + G - WG - 60\right)\left(1.08\right)$
$-(0.004736)(150000)$
$-(1 - 0.004736)({}_{21}V)$

Plug in given values, we have

$$722 = \left(24,496 + \left(1 - W\right)\left(1212\right) - 60\right)\left(1.08\right) - 0.004736\left(150,000\right) - 0.995264\left(26,261\right)$$
$$722 = 852.8121 - 1308.96W$$

Solving *W*, $W = \dfrac{852.8121 - 722}{1308.96} = 10\%$

Question # 7.27
Answer: E

We assume that we have assets equal to reserves at the beginning of the year. We collect premiums, earn interest, and pay the claims which gives us the assets at the end of the year:

$$(980)(19.9+8.8)(1+j)-(500)(7)=24,626+28,126j$$

Actual total reserve at the end of the year = $(980-7)(27.77)=27,020.21$

Since there is no gain or loss that means that the assets must equal the reserves, so:

$$j=\frac{27,218.27-24,626}{28,126}=8.512\%$$

Question # 7.28
Answer: A

If *G* denotes the gross premium, then

$$G=\frac{1000A_{35}+30\ddot{a}_{35}+270}{0.96\ddot{a}_{35}-0.26}=\frac{1000(0.09653)+30(18.9728)+270}{0.96(18.9728)-0.26}=52.12$$

So that,

$$R=1000A_{36}+(30-0.96G)\ddot{a}_{36}$$
$$=1000(0.10101)+(30-0.96\times 52.12)(18.8788)=-277.23$$

Note that $S=0$ as per definition of FPT reserve.

Question # 7.29
Answer: D

$$\pi = \frac{1000 \; {}_{10|}\ddot{a}_{55}}{\ddot{a}_{55:\overline{10|}} - (IA)^{1}_{55:\overline{10|}}} = \frac{1000(0.59342)(13.5498)}{8.0192 - 0.14743} = 1021.46$$

$${}_{9}V = 1000 \; {}_{1|}\ddot{a}_{64} + 10\pi A^{1}_{64:\overline{1|}} - \pi \ddot{a}_{64:\overline{1|}}$$

$$= 1000\frac{1}{1.05}\left(\frac{94,579.7}{95,082.5}\right)13.5498 + 10(1021.46)\frac{1}{1.05}(0.005288) - 1021.46$$

$$= 11,866$$

Question # 7.30
Answer: C

$$V\left[L_0 \# 1\right] = \left(B_1 + \frac{P_1}{d}\right)^2 \left({}^2A_x - A_x^2\right) = 20.55 \qquad ==> \left(8 + \frac{1.25(1.06)}{0.06}\right)^2 \left({}^2A_x - A_x^2\right) = 20.55$$

$${}^2A_x - A_x^2 = \frac{20.55}{\left(8 + \frac{1.25(1.06)}{0.06}\right)^2} = 0.0227$$

$$V\left[L_0 \# 2\right] = \left(12 + \frac{1.875(1.06)}{0.06}\right)^2 \left({}^2A_x - A_x^2\right) = \left(12 + \frac{1.875(1.06)}{0.06}\right)^2 (0.0227) = 46.24$$

Question # 7.31
Answer: D

We have Present Value of Modified Premiums = Present Value of level net premiums

$$vq_x + \beta\left(\ddot{a}_{25:\overline{20|}} - 1\right) + P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20|}} = P\ddot{a}_{25:\overline{40|}}$$

$$\Rightarrow \beta = \frac{P\left(\ddot{a}_{25:\overline{40|}}\right) - P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20|}} - vq_x}{\ddot{a}_{25:\overline{20|}} - 1} = \frac{P\,\ddot{a}_{25:\overline{20|}} - vq_x}{\ddot{a}_{25:\overline{20|}} - 1}$$

We are given that $P = 0.0216$

$$\Rightarrow \beta = \frac{0.0216(11.087) - (1.04)^{-1}(0.005)}{11.087 - 1} = 0.023265$$

For insurance of 10,000, $\beta = 233.$

Question # 7.32
Answer: C

$P^g = P^n + P^e$ where P^e is the expense loading

$$P^n = 1,000,000\frac{A_{50}}{\ddot{a}_{50}} = 1,000,000\left(\frac{0.18931}{17.0245}\right) = 11,119.86$$

$$P^e = P^g - P^n = 11,800 - 11,120 = 680$$

Question # 7.33
Answer: B

$$
\begin{aligned}
{}_3V^{FPT} &= 100,000\,A_{[55]+3} - 100,000\,P_{[55]+1}\,\ddot{a}_{[55]+3} \\
&= 100,000\,A_{58} - 100,000\frac{A_{[55]+1}}{\ddot{a}_{[55]+1}}\ddot{a}_{58} \\
&= 100,000\left(0.27 - \frac{0.24}{\dfrac{1-0.24}{d}}\cdot\frac{1-0.27}{d}\right) \\
&= 3947.37
\end{aligned}
$$

Question # 7.34
Answer: D

$$
\begin{aligned}
&{}_1V = \left({}_0V + P\right)(1+i) - \left(25,000 + {}_1V - {}_1V\right)q_x = P(1+i) - (25,000)q_x \\
&{}_2V = \left({}_1V + P\right)(1+i) - \left(50,000 + {}_2V - {}_2V\right)q_{x+1} = 50,000 \\
&\Big(\big(P(1+i) - 25,000q_x\big) + P\Big)(1+i) - 50,000q_{x+1} = 50,000 \\
&\Big(\big(P(1.05) - 25,000(0.15)\big) + P\Big)(1.05) - 50,000(0.15) = 50,000
\end{aligned}
$$

Solving for *P*, we get

$$P = \frac{61,437.50}{2.1525} = 28,542.39$$

Question # 7.35
Answer: C

$$ {}_4V = \frac{(505+220-30)(1.05)-10{,}000q_{53}}{1-q_{53}} = 666.2807 $$

The easiest way to calculate the profit is to calculate the assets at the end of the period using actual experience and the reserves at the end of the period using the reserve assumptions. The difference is the profit.

Ending assets $= 4885(505+220-34)(1.06)-42(10{,}000) = 3{,}158{,}067.10$

Ending reserves $= (4885-42)(666.2807) = 3{,}226{,}797.43$

Profit = $3{,}158{,}067.10 - 3{,}226{,}797.43 = -68{,}730.33$

Alternatively, we can calculate the profit by source of profit. For example, if we calculated the gain by source calculation, done in the order of interest, expense, and mortality, the profit for policy year 4 is

$$ (4885)[(505+220-30)(0.01)+(30-34)(1.06)+(10{,}000-666.2807)(0.0068-42/4885)] $$

$$ = -68{,}730.37 $$

Question # 7.36
Answer: B

Since G is determined using the equivalence principle, ${}_0V = 0$

$$ \text{Then, } {}_1V^e = \frac{\left(0+\overbrace{G-187}^{P^e}-0.25G-10\right)(1.03)}{0.992} = -38.7 $$

$$ \Rightarrow 0.75G = \frac{-38.7(0.992)}{1.03}+187+10 = 159.72 $$

$$ \Rightarrow G = 212.97 $$

Question # 7.37
Answer: D

$$\frac{d}{dt}{}_tV = \delta_t {}_tV + P_t - e_t - (S_t + E_t - {}_tV)\mu_{x+t}$$

At $t = 30.5$,

$$292 = 0.05\,{}_{30.5}V + 100 - 0 - (10,000 + 0 - {}_{30.5}V) \times 0.038$$

$$572 = {}_{30.5}V(0.05 + 0.038)$$

$${}_{30.5}V = 6500$$

Question # 7.38
Answer: E

$$G\ddot{a}_{45:\overline{10}|} = HA_{45} + G + 0.05G\ddot{a}_{45:\overline{10}|} + 80 + 10\ddot{a}_{45} + 10\ddot{a}_{45:\overline{10}|}$$

$$G = \frac{HA_{45} + 80 + 10\left(\ddot{a}_{45} + \ddot{a}_{45:\overline{10}|}\right)}{0.95\ddot{a}_{45:\overline{10}|} - 1}$$

$$G = \frac{HA_{45} + 80 + 10\left(17.8162 + 8.0751\right)}{(0.95 \times 8.0751) - 1}$$

$$G = \frac{A_{45}}{(0.95 \times 8.0751) - 1}H + \frac{80 + 10\left(17.8162 + 8.0751\right)}{(0.95 \times 8.0751) - 1}$$

$$g = \frac{A_{45}}{(0.95 \times 8.0751) - 1}$$

$$f = \frac{80 + 10\left(17.8162 + 8.0751\right)}{(0.95 \times 8.0751) - 1} = 50.80$$

Question # 7.39

Answer: D

$$G = 0.35G + 2000\left(\frac{0.1}{1.08} + \frac{0.9 \times 0.1}{1.08^2} + \frac{0.9 \times 0.9 \times 0.1}{1.08^3}\right)$$

$$0.65G = 468.107$$

$$G = 720.16$$

Question # 7.40
Answer: C

Expected expense financial impact = Expected expenses + Foregone interest

$$100 + 0.07 \times 100 = 107$$

Actual expense financial impact = Actual expenses + Foregone interest

$$75 + 0.07 \times 75 = 80.25$$

Gain from expenses:

$$107 - 80.25 = 26.75$$

Question # 7.41
Answer: A

On a unit basis, $Var(L_0) = \left(1+\frac{P}{d}\right)^2 \left[{}^2A_{45} - (A_{45})^2\right] = \left(1+\frac{A_{45}}{d\ddot{a}_{45}}\right)^2 \left[{}^2A_{45} - (A_{45})^2\right]$

$$= \left(\frac{d\ddot{a}_{45} + 1 - d\ddot{a}_{45}}{d\ddot{a}_{45}}\right)^2 \left[{}^2A_{45} - (A_{45})^2\right] = \frac{{}^2A_{45} - (A_{45})^2}{(d\ddot{a})^2}$$

$$= \frac{0.03463 - 0.15161^2}{\left(\frac{0.05}{1.05} \times 17.8162\right)^2} = 0.016178038$$

The standard deviation of $L_0 = 0.127193$

$(200,000)$(The standard deviation of L_0) $= 25,439$

Question # 7.42
Answer: D

${}_{20}V = 0 ==> 1000A_{65} = (P+W) \times \ddot{a}_{65}$

At issue, present value of benefits must equal present value of premium, so:
$1000A_{45} = P\ddot{a}_{45} + W\, {}_{20}E_{45} \times \ddot{a}_{65}$

$354.77 = (P+W)(13.5498) \Rightarrow P+W = 26.182674 \Rightarrow P = 26.182674 - W$

$151.61 = 17.8162P + W(0.35994)(13.5498)$

$151.61 = 17.8162(26.182674 - W) + W(0.35994)(13.5498)$

$\Rightarrow W = 24.33447$

Question # 7.43
Answer: E

$$V_{10} = 2,290 = B\left(1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}\right) = B\left(1 - \frac{11.4}{14.8}\right) \Rightarrow B = 9,968.24$$
$$G\ddot{a}_x = 25 + 5\ddot{a}_x + B \times A_x$$
$$A_x = 1 - d\ddot{a}_x = 1 - \left(\frac{0.04}{1.04} \times 14.8\right) = 0.430769231$$
$$G \times 14.8 = 25 + 5 \times 14.8 + 9,968.24 \times 0.430769231$$
$$\Rightarrow G = 296.82$$

$${}_{10}V^g = 9,968.24A_{x+10} + 5\ddot{a}_{x+10} - 296.82\ddot{a}_{x+10}$$
$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - \left(\frac{0.04}{1.04} \times 11.4\right) = 0.561538462$$
$${}_{10}V^g = 9,968.24 \times 0.561538462 + 5 \times 11.4 - 296.82 \times 11.4$$
$$\Rightarrow {}_{10}V^g = 2,270.80$$

Alternatively, the expense net premium is based on the extra expenses in year 1, so

$$P^e = (30 - 5) / 14.8 = 1.68919$$

$${}_{10}V^e = 0 - 1.68919(11.4) = -19.26$$

$${}_{10}V^g = {}_{10}V^n + {}_{10}V^e = 2290 - 19.26 = 2270.74$$

Question # 7.44
Answer: E

$$L_{10} = 10,000A_{35} = 965.30$$
$$L_{10}^* = 10,000$$
$$L_{10}^* - L_{10} = 10,000 - 965.30 = 9034.70$$

Question # 7.45
Answer: E

Future expenses at $x+2=0.08G+5$

Expense load at $x+2=P^e$

$-23.64=(0.08G+5)-P^e$

$\Rightarrow P^e=58.08$

$1000P_{x:\overline{3}|}=368.05-58.08=309.97$

Question # 8.1
Answer: B

Because it is impossible to return to state 0, ${}_tp_0^{\overline{00}}$ and ${}_tp_0^{00}$ are the same, so

$${}_1p_0^{00}={}_1p_0^{\overline{00}}=e^{\left\{-\int_0^1\sum_{j=1}^{2}\mu_{0+s}^{0j}\,ds\right\}}=e^{\left\{-\int_0^1\left[0.015+0.03(2^t)\right]dt\right\}}=e^{\left\{-\left[0.015t+\frac{0.03(2^t)}{\ln 2}\right]_0^1\right\}}=$$

$$\exp[-(0.015+0.08656-0-0.04328)]=\exp(-0.05828)=0.943$$

Note: The sum of $\mu_{x+t}^{01}+\mu_{x+t}^{02}=\left[0.015+0.03(2^t)\right]$ is a form of Makeham’s law and ${}_1p_0^{00}$ could be calculated using the formula provided in the tables instead of integrating.

Question # 8.2
Answer: D

For $t = 0$ and $h = 0.5$,

$$ {}_{0.5}p^{10} = {}_{0}p^{10} - 0.5\left[{}_{0}p^{10}\left(\mu^{01} + \mu^{02}\right) - {}_{0}p^{11}\mu^{10} - {}_{0}p^{12}\mu^{20} \right] $$
$$ = 0 - 0.5\left(0 - 1\mu^{10} - 0\right) = 0.5\mu^{10} = 0.03 $$

Similarly ${}_{0.5}p^{12} = 0.5\mu^{12} = 0.05$

Then, ${}_{0.5}p^{11} = 1 - 0.03 - 0.05 = 0.92$

For $t = 0.5$ and $h = 0.5$,

$$ {}_{1}p^{10} = {}_{0.5}p^{10} - 0.5\left({}_{0.5}p^{10}\left(\mu^{01} + \mu^{02}\right) - {}_{0.5}p^{11}\mu^{10} - {}_{0.5}p^{12}\mu^{20} \right) $$
$$ = 0.03 - 0.5[0.03(0.02) - 0.92(0.06) - 0] = 0.0573 $$

Question # 8.3
Answer: D

possible transitions	probability	discounted benefits	APV
$H \to Z$	0.05	$250v$	11.904762
$H \to L$	0.04	$250v$	9.523810
$H \to Z \to D$	$0.05(0.7) = 0.035$	$1000v^2$	31.746032
$H \to L \to D$	$0.04(0.6) = 0.024$	$1000v^2$	21.768707
$H \to H \to Z$	$0.90(0.05) = 0.045$	$250v^2$	10.204082
$H \to H \to L$	$0.90(0.04) = 0.036$	$250v^2$	8.163265

Sum of these gives total APV =93.31066

Question # 8.4
Answer: D

$$p_{50}^{00} = \frac{l_{51}^{(\tau)}}{l_{50}^{(\tau)}} = \frac{90,365}{100,000} = 0.90365$$

$$q_{51}'^{(3)} = q_{50}'^{(3)} = 1 - p_{50}'^{(3)} = 1 - p_{50}^{00\;\frac{p_{50}^{03}}{p_{50}^{0\bullet}}} = 1 - p_{50}^{00\;\frac{d_{50}^{(3)}/l_{50}^{(\tau)}}{1-p_{50}^{00}}} = 1 - 0.90365^{\frac{1100/100,000}{1-0.90365}} = 0.0115$$

$$d_{51}^{30} = l_{51}^{(\tau)}\,p_{51}^{03} = l_{51}^{(\tau)}\,\frac{\ln\ p_{51}'^{(3)}}{\ln\ p_{51}^{00}}\ p_{51}^{0\bullet} = l_{51}^{(\tau)} \times \frac{\ln(1-0.0115)}{\ln\left[\frac{l_{52}^{(\tau)}}{l_{51}^{(\tau)}}\right]} \times \left(1 - \frac{l_{52}^{(\tau)}}{l_{51}^{(\tau)}}\right)$$

$$= 90,365 \times \frac{\ln(1-0.0115)}{\ln(80,000/90,365)} \times (1 - 80,000/90,365) = 984$$

$$d_{51}^{(1)} = l_{51}^{(\tau)} - l_{52}^{(\tau)} - d_{51}^{(2)} - d_{51}^{(3)} = 90,365 - 80,000 - 8200 - 984 = 1181$$

$$q_{51}'^{(1)} = 1 - p_{51}'^{(1)} = 1 - p_{51}^{00\;\frac{p_{51}^{01}}{p_{51}^{0\bullet}}} = 1 - p_{51}^{00\;\frac{d_{51}^{(1)}/l_{51}^{(\tau)}}{1-p_{51}^{00}}} = 1 - 80,000/90,365^{\frac{1181/90,365}{1-80,000/90,365}} = 0.0138$$

$$10,000 \times q_{51}'^{(1)} = 10,000 \times 0.0138 = 138$$

Note: This solution uses multi-state notation for dependent probabilities. There is alternative notation for these when the context is strictly multiple decrement, as it is here.

Question # 8.5
Answer: C

There are four career paths Joe could follow. Each has probability of the form:

$p_{35}(p_{36})(p_{37})(\text{transition probability 1})(\text{transition probability 2})$

where the survival probabilities depend on each year's employment type. For example, the first entry below corresponds to Joe being an actuary for all three years.

The probability that Joe is alive on January 1, 2016 is:

$$\begin{aligned}&(0.9)(0.85)(0.8)(0.4)(0.4)\\&+(0.9)(0.85)(0.65)(0.4)(0.6)\\&+(0.9)(0.7)(0.8)(0.6)(0.8)\\&+(0.9)(0.7)(0.65)(0.6)(0.2)=0.50832\end{aligned}$$

The expected present value of the endowment is $(100,000)0.50832/(1.08)^3 = 40,352$

Question # 8.6
Answer: E

This is an application of Thiele's differential equation for a multi-state model.

The components of $\frac{d}{dt}\,{}_tV^{(s)}$ are

Interest on the current reserve: $\delta\,{}_tV^{(s)}$

Rate of premiums received while in state *s*: 0

Rate of benefits paid while in state *s*: $-B$

Transition intensity for transition to state *h*, times the change in reserve upon transition (hold reserve for *h* and release reserve for *s*): $-\mu_{x+t}^{sh}({}_tV^{(h)} - {}_tV^{(s)})$

Similar to previous, noting that the reserve if dead is 0: $-\mu_{x+t}^{sd}(0 - {}_tV^{(s)})$

Adding these terms yields the solution.

Question # 8.7
Answer: A

$$Q=\begin{bmatrix}0.7 & 0.3 & 0.0\\ 0.2 & 0.4 & 0.4\\ 0.0 & 0.0 & 1.0\end{bmatrix} \qquad Q^2=\begin{bmatrix}0.55 & 0.33 & 0.12\\ 0.22 & 0.22 & 0.56\\ 0.00 & 0.00 & 1.00\end{bmatrix}$$

Let p = 0.75 be the probability of renewing.

PV costs for Medium Risk:

$$=300v^{0.5}+\left[100(0.2)+300(0.4)+600(0.4)\right]pv^{1.5}+\left[100(0.22)+300(0.22)+600(0.56)\right]p^2v^{2.5}$$
$$=291.4+261.1+206.2=759$$

The present value of costs for the new portfolio is (0.9)317 + (0.1)759 = 361.2. The increase is (361.2/317) – 1 = 0.14, or 14%.

Question # 8.8
Answer: B

$$\text{Prob}(H\to D\text{ in 2 months})=(0.75\ \ 0.2\ \ 0.05)\begin{pmatrix}0.05\\ 0.20\\ 1\end{pmatrix}=0.1275$$

You could do more extensive matrix multiplication and also obtain the probability that it is *H* after 2 or it is *S* after 2, but those aren't needed.

Let *D* be the number of deaths within 2 months out of 10 lives

Then *D*~binomial with $n=10, \quad p=0.1275$

$$P(D=4)=\binom{10}{4}(0.1275)^4(1-0.1275)^6=0.0245$$

Question # 8.9
Answer: E

Let *A* denote Alive, which is equivalent to not Dead. It is also equivalent to Healthy or Disabled. Let *H* denote Healthy. The conditional probability is:

$$P(H|A)=\frac{P(H \text{ and } A)}{P(A)}=\frac{P(H)}{P(H)+P(\text{Disabled})},$$

Where

$$P(H)={}_{10}p^{00}=e^{-\int_0^{10}\left(\mu^{01}+\mu^{02}\right)ds}=e^{-\int_0^{10}(0.05)ds}=e^{-0.5}=0.607$$

And

$$\begin{aligned}P(\text{Disabled})={}_{10}p^{01}&=\int_0^{10}e^{-\int_0^u\left(\mu^{01}+\mu^{02}\right)ds}\mu^{01}e^{-\int_u^{10}\mu^{12}ds}du\\&=\int_0^{10}e^{-0.05u}(0.02)e^{0.05u-0.5}du\\&=\int_0^{10}(0.02)e^{-0.5}du\\&=(0.2)e^{-0.5}=0.121\end{aligned}$$

Then

$$P(H|A)=\frac{P(H)}{P(H)+P(\text{Disabled})}=\frac{0.607}{0.607+0.121}=0.83$$

Question # 8.10
Answer: A

$$\begin{aligned}{}_tp_x^{00}&=\exp\left[-\int_0^t\left(\mu_{x+s}^{01}+\mu_{x+s}^{02}\right)ds\right]=\exp\left[-\int_0^t\left(0.20+0.10s+0.05+0.05s\right)ds\right]\\&=\exp\left[-\left(0.25s+0.075s^2\right)\Big|_0^t\right]=\exp\left(-0.25t+0.075t^2\right)\end{aligned}$$

$${}_3p_x^{00}=\exp\left(-0.25\times3+0.075\times9\right)=\exp(-1.425)=0.2405$$

EPV, *through time n*, $=\int_0^n g(t)dt$, where $g(t)=10,000\times\left({}_tp_x^{00}\mu_{x+t}^{02}+{}_tp_x^{01}\mu_{x+t}^{12}\right)e^{-\delta t}dt$

$$g(3)=10,000\times\left[0.2405\times\left(0.05+0.05\times3\right)+0.4174\times\left(0.15+0.01\times3^2\right)\right]\times e^{-3\times0.02}=1,400$$

Question # 8.11
Answer: B

Let x be the person's age.

$$
\begin{aligned}
{}_{15}p_x^{\overline{11}} &= \exp\left[-\int_0^{15}\left(\mu_{x+s}^{10}+\mu_{x+s}^{12}\right)ds\right] \\
&= \exp\left[-\int_0^{15}(0.10+0.05)ds\right] \\
&= \exp[-15(0.15)] \\
&= \exp[-2.25] = 0.1054
\end{aligned}
$$

Question # 8.12
Answer: D

Let P be the annual premium

APV(premium) $= P\left(1+v\dfrac{945}{1000}+v^2\dfrac{895}{1000}\right)=2.711791P$

APV(benefits) $=100\left(v\dfrac{20}{1000}+v^2\dfrac{25}{1000}+v^3\dfrac{895}{1000}\right)=81.4858$

$$P=\frac{81.4858}{2.711791}=30.05$$

Note: The term $v^3\dfrac{895}{1000}$ is the sum of $v^3\dfrac{30}{1000}$ for death benefits and $v^3\dfrac{895-30}{1000}$ for maturity benefits.

Question # 8.13
Answer: A

By U.D.D.:

$$0.2 = q_x^{(2)} = q_x'^{(2)}\left(1 - \frac{1}{2}q_x'^{(1)}\right)$$

$$= q_x'^{(2)}\left(1 - \frac{1}{2}(0.1)\right) = 0.95\, q_x'^{(2)}$$

$$\Rightarrow q_x'^{(2)} = 0.2105$$

$$p_x^{(\tau)} = p_x'^{(1)} p_x'^{(2)} = (0.90)(1 - 0.2105) = 0.71055$$

$$q_x^{(1)} = q_x^{(\tau)} - q_x^{(2)} = 1 - p_x^{(\tau)} - q^{(2)}$$

$$= 1 - 0.71055 - 0.2 = 0.08945$$

Question # 8.14
Answer: C

Path	Probability
$S \to S \to S \to S$	0.216
$S \to S \to C \to S$	0.012
$S \to C \to S \to S$	0.012
$S \to C \to C \to S$	0.010
Total across all paths:	0.250

Question # 8.15
Answer: D

Intuitively:

(A) A lower interest rate increases premium, but a higher recovery rate decreases premium, because there are lower projected benefits and more policyholders paying premiums.

(B) A lower death rate of healthy lives $\to$ more pay premium $\to$ lower premium

(C) A higher death rate of sick lives $\to$ fewer benefits $\to$ lower premium

(D) A lower recovery rate $\to$ higher sickness benefits $\to$ higher premium
A lower death rate of sick lives $\to$ higher sickness benefits $\to$ higher premium

(E) A higher rate of interest $\to$ lower premium
A lower mortality rate for healthy lives may result in lower premium because more healthy lives are paying premium.

Question # 8.16
Answer: C

The desired probability is:

$$
\begin{aligned}
&= \int_0^{14} \exp\left\{-\int_0^u \left(\mu^{01} + \mu^{02}\right) ds\right\} \cdot \mu^{01} \cdot \exp\left\{-\int_0^1 \mu^{12} ds\right\} du \\
&= \int_0^{14} e^{-0.05u} \cdot (0.02) \cdot e^{-0.11} du \\
&= (0.02) \cdot e^{-0.11} \int_0^{14} e^{-0.05u} du \\
&= \frac{0.02}{0.05} \cdot e^{-0.11} \left(1 - e^{-0.7}\right) \\
&= 0.18
\end{aligned}
$$

The limits of the outer integral are 0 to 14 because you must be disabled by 64 if you are to have been disabled for at least one year by 65.

Question # 8.17
Answer: C

The probabilities are:

Sick $t = 1 \Rightarrow 0.025$
Sick $t = 2 \Rightarrow (0.95)(0.025) + (0.025)(0.6) = 0.03875$
Sick $t = 3 \Rightarrow (0.95)(0.95)(0.025) + (0.95)(0.025)(0.6) + (0.025)(0.6)(0.6)$
$+ (0.025)(0.3)(0.025) = 0.046$

$$EPV = 20{,}000\left(0.025v + 0.03875v^2 + 0.046v^3\right) = 1934$$

Question # 8.18
Answer: A

The probability that Johnny will not have any accidents in the next year is:

$$p_{\overline{(1)}}^{\overline{00}} = e^{-\int_0^1 \left(\mu_s^{01}+\mu_s^{02}\right)ds}$$, where

$$\mu_t^{01} + \mu_t^{02} = 3.718\mu_t^{01} = 3.718\left(0.03 + 0.06 \times 2^t\right)$$

So that

$$\int_0^1 3.718\left(0.03 + 0.06 \times e^{t(ln2)}\right)dt = 3.718\left(0.03\right) + \frac{3.718\left(0.06\right)}{ln\left(2\right)} = 0.4333764$$

and

$$p_{\overline{(1)}}^{\overline{00}} = e^{-0.4333764} = 0.6483164$$

The probability Johnny will have at least one accident is therefore

$$1 - 0.6483164 = 0.3516836\,.$$

Note: The sum of $\mu_{x+t}^{01} + \mu_{x+t}^{02}$ is a form of Makeham's law and $p_{\overline{(1)}}^{\overline{00}}$ could be calculated using the formula provided in the tables instead of integrating.

Question # 8.19
Answer: A

The probability of being fully functional after two years for a single television is:

$$\begin{pmatrix} 0.82 & 0.10 & 0.08 \end{pmatrix}\begin{pmatrix} 0.82 \\ 0.60 \\ 0.00 \end{pmatrix} = (0.82)(0.82) + (0.10)(0.60) + (0.08)(0.00) = 0.7324$$

The number of the five televisions being fully functional has a binomial distribution with parameters of $n = 5$ and $p = 0.7324$. The probability that there will be exactly two televisions that are fully functioning is therefore:

$$\binom{5}{2}(0.7324)^2\left(1 - 0.7324\right)^3 = (10)(0.53641)(0.019163) = 0.10279$$

Question # 8.20
Answer: E

Probability $= \int_0^5 {}_t p^{\overline{00}} \mu^{01} {}_{5-t} p^{\overline{11}} dt$

$$= \int_0^5 e^{-0.06t} 0.05 e^{-0.08(5-t)} dt = \int_0^5 e^{-0.06t} 0.05 e^{-0.40} e^{0.08t} dt =$$

$$= e^{-0.40}(0.05) \int_0^5 e^{+0.02t} dt = e^{-0.40}\left(\frac{0.05}{0.02}\right)\left(e^{0.10} - 1\right) = 0.1762$$

Question # 8.21
Answer: C

$${}_5 p_x^{01} = \int_0^5 {}_t p_x^{\overline{00}} \mu_{x+t} \, {}_{5-t} p_x^{\overline{1}\overline{1}} dt = \int_0^5 e^{-(\mu^{01}+\mu^{03})t} \mu^{01} e^{-(5-t)(\mu^{12}+\mu^{13})} dt$$

$$= \int_0^5 e^{-(0.01+0.02)t} (0.01) e^{-(5-t)(0.30+0.40)} dt = \int_0^5 e^{-(0.03)t} (0.01) e^{-(5-t)(0.7)} dt$$

$$= (0.01) e^{-3.5} \int_0^5 e^{0.67t} dt = (0.01) e^{-3.5} \left(\frac{e^{0.67(5)} - 1}{0.67}\right) = 0.01239568$$

Question # 8.22
Answer: D

Healthy lives' probabilities:

Probability of a Healthy life at time 0 being Healthy at:

Time 1: 0.9

Time 2: (0.9)(0.9) = 0.81

Probability of a Healthy life at time 0 is Sick at time 1 and then Healthy at:

Time 2: (0.05)(0.30) = 0.015

Probability of being Healthy at time 1: 0.9

Probability of being Healthy at time 2: 0.81 + 0.015 = 0.825

Sick lives' probabilities:

Probability of a Healthy life at time 0 being Sick at:

Time 1: 0.05

Probability of a Healthy life at time zero is Sick at time 1 and then Sick at:

Time 2: (0.05)(0.60) = 0.03

Probability of a Healthy life at time 0 is Healthy at time 1 and thenSick at:

Time 2: (0.90)(0.05) = 0.045

Probability of being Sick at time 1: 0.05

Probability of being Sick at time 2: 0.03 + 0.045 = 0.075

APV (Healthy): $500(e^{-0.5\times 0.04} + 0.9e^{-1.5\times 0.04} + 0.825e^{-2.5\times 0.04}) = 1287.138812$

APV (Sick):

$$5000(0.05e^{-1.5\times 0.04} + 0.075e^{-2.5\times 0.04}) = 574.755165$$

Total APV: 1287.14 + 574.76 = 1861.90

Question # 8.23
Answer: C

$$ {}_2q_{53}^{(1)} = q_{53}^{(1)} + p_{53}^{(\tau)} \cdot q_{54}^{(1)} $$

$$ q_{54}^{(1)} = q_{54}^{(\tau)} - q_{54}^{(2)} = (1 - p_{54}^{(\tau)}) - q_{54}^{(2)} = \left(1 - \frac{4625}{5000}\right) - 0.040 = 0.035 $$

$$ p_{53}^{(\tau)} = 1 - q_{53}^{(1)} - q_{53}^{(2)} = 1 - 0.025 - 0.030 = 0.945 $$

$$ {}_2q_{53}^{(1)} = 0.025 + 0.945 \cdot 0.035 = 0.058 $$

Question # 8.24
Answer: A

The probability of death by year 3:

$$ 0.2 + 0.64 \times 0.20 + 0.16 \times 0.4 = 0.392 $$

Expected number of deaths = $1000 \times 0.392 = 392$

Variance of the number of deaths = $1000 \times 0.392 \times 0.608 = 238.336$

$$ \Pr(X < 375) = \Pr\left(\frac{X - 392}{\sqrt{238.336}} < \frac{375 - 392}{\sqrt{238.336}} \right) = \Pr(Z < -1.10) = \Phi(-1.10) = 0.1357 $$

Question # 8.25
Answer: B

$$ d_{41}^{(3)} = \ell_{41}^{(\tau)} q_{41}^{(3)} $$
$$ \ell_{41}^{(\tau)} = \ell_{40}^{(\tau)} (1 - q_{40}^{(1)} - q_{40}^{(2)} - q_{40}^{(3)}) $$

If $q_{40}^{(1)}$ increases by 0.01, then the change in $\ell_{41}^{(\tau)} = -0.01\ell_{40}^{(\tau)}$.
The change in $d_{41}^{(3)} = -0.01\ell_{40}^{(\tau)} q_{41}^{(3)} = -0.01 \times 15,000 \times 0.10 = -15$

Question # 8.26
Answer: B

Outcome (z)	Prob (p)	$p \times z$	$p \times z^2$
$1000v = 943.40$	0.04	37.74	35,603.92
Fv	0.20	$0.20Fv$	0.20 $(Fv)^2$
0	0.76	0	0

$$E(Z) = 37.74 + 0.1887F$$

$$E\left(Z^2\right) = 35,603.92 + 0.1780F^2$$

$$Var(Z) = 35,603.92 + 0.1780F^2 - (37.74 + 0.1887F)^2$$

$$= 34,174.61 - 14.243F + 0.1424F^2$$

Take derivative with respect to *F*.
Derivative $= 0.2848F - 14.243$
Set = 0 and solve; get *F* = 14.243 / 0.2848 = 50

It should be obvious that this is a minimum rather than a maximum; you could prove it by noting that the second derivative = 0.2848 > 0.

Note: the result does not depend on *v*. If you carry *v* symbolically through all steps, all instances cancel.

Question # 8.27
Answer: A

Expected Present Value of Benefits:

$$5000(10/1000)/1.06 + 7500(15/1000)/1.06^2 + 10,000(18/1000)/1.06^3$$

$$= 47.17 + 100.12 + 151.13 = 298.42.$$

Expected Present Value of Premiums:

$$[1 + (870/1000)/1.06 + (701/1000)/1.06^2]P = 2.4446P.$$

The annual premium is *P* = 298.42/2.4446 = 122

Question # 9.1
Answer: E

Pr(last survivor dies in the third year) =

$$ {}_2p_{\overline{80:90}} - {}_3p_{\overline{80:90}} $$
$$ = \left({}_2p_{80} + {}_2p_{90} - {}_2p_{80:90} \right) - \left({}_3p_{80} + {}_3p_{90} - {}_3p_{80:90} \right) $$
$$ = [(0.9)(0.8) + (0.6)(0.5) - (0.9)(0.8)(0.6)(0.5)] $$
$$ \quad - [(0.9)(0.8)(0.7) + (0.6)(0.5)(0.4) - (0.9)(0.8)(0.7)(0.6)(0.5)(0.4)] $$
$$ = (0.72 + 0.30 - 0.216) - (0.504 + 0.12 - 0.06048) $$
$$ = 0.804 - 0.56352 $$
$$ = 0.24048 $$

Question # 9.2
Answer: E

$$ 1{,}000{,}000 = \text{APV(benefits)} = 100{,}000 A_{65} + R\ddot{a}_{65:65} + 0.60R\ddot{a}_{65|65} + 0.70R\ddot{a}_{65|65} $$
$$ \left(\text{where } \ddot{a}_{65|65} = \ddot{a}_{65} - \ddot{a}_{65:65} \right) $$
$$ = 100{,}000 A_{65} + R\left(1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65} \right) $$

$$ R = \frac{1{,}000{,}000 - 100{,}000 A_{65}}{1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65}} $$

$$ A_{65} = 0.35477 $$
$$ \ddot{a}_{65} = 13.5498 $$
$$ \ddot{a}_{65:65} = 11.6831 $$

$$ R = \frac{964{,}523}{14.10981} = 68{,}358 $$

Question # 9.3
Answer: A

$$ {}_{10}p_{65:60}^{02} = \int_0^{10} {}_tp_{65:60}^{00} \mu^{02} {}_{10-t}p_{65+t:60+t}^{22} \, dt $$
$$ = \int_0^{10} e^{-0.01t}(0.005)e^{-0.008(10-t)} dt $$
$$ = 0.005e^{-0.08} \int_0^{10} e^{-0.002t} dt $$
$$ = 0.005e^{-0.08} \frac{1 - e^{-0.02}}{0.002} = 0.0457 $$

Question # 9.4
Answer: A

$$\begin{aligned}\text{APV(insurance)} &= 1000\int_0^\infty e^{-0.05t}\,{}_tp_{xy}^{00}\,\mu^{03}\,dt \\ &= 1000(0.005)\int_0^\infty e^{-0.05t}\,e^{-0.045t}\,dt \\ &= 1000\frac{(0.005)}{0.095} = 52.63158\end{aligned}$$

Question # 9.5
Answer: D

$$P = \frac{10,000\left(vq_{\overline{30:30}} + v^2{}_{1|}q_{\overline{30:30}}\right)}{1+vp_{\overline{30:30}}}$$

$$q_{\overline{30:30}} = (q_{30})^2 = 0.0016$$

$$p_{\overline{30:30}} = 1 - q_{\overline{30:30}} = 1 - 0.0016 = 0.9984$$

$${}_{1|}q_{\overline{30:30}} = {}_2q_{\overline{30:30}} - q_{\overline{30:30}} = ({}_2q_{30})^2 - 0.0016 = [0.04+0.96(0.06)]^2 - 0.0016 = 0.00793$$

$$\text{APV of Benefits} = 10,000\left(\frac{0.0016}{1.05}+\frac{0.00793}{1.05^2}\right) = 87.17$$

$$P = \frac{87.17}{1+\dfrac{0.9984}{1.05}} = 44.68$$

Question # 9.6
Answer: C

$$1000\,{}_{5|}q_{\overline{60:70}} = 1000\left[{}_5p_{60}\,{}_5p_{70}\,q_{65}\,q_{75} + {}_5q_{60}\,{}_5p_{70}\,q_{75} + {}_5q_{70}\,{}_5p_{60}\,q_{65}\right]$$

$$1000[(0.92)(0.88)(0.02132)(0.05169)+(1-0.92)(0.88)(0.05169)$$
$$+(1-0.88)(0.92)(0.02132)]$$
$$=[0.0008922+0.00363898+0.00235373]1000 = 6.8849$$

Question # 9.7
Answer: B

$$
\begin{aligned}
{}_{10}p^{00}_{30:30} &= e^{-\int_0^{10}\left(\mu^{01}_{30+t:30+t}+\mu^{02}_{30+t:30+t}\right)dt} \\
&= e^{-\int_0^{10}\left(0.006+0.014+0.0007\times 1.075^{30+t}\right)dt} \\
&= e^{-\int_0^{10}\left(0.02+0.0007\times 1.075^{30+t}\right)dt} \\
&= {}_{10}p_{30} \text{ under Makeham's law with } A=0.02;\ B=0.0007;\text{ and } c=1.075 \\
&= \exp\left(-0.02(10)+\left[\frac{-0.0007}{\ln(1.075)}\right](1.075^{30})\left(1.075^{10}-1\right)\right)=0.748
\end{aligned}
$$

Note: The above is what candidates would have needed to do in Spring 2015. Candidates could have solved the integral even without recognizing the distribution as Makeham. In LTAM, solving the integral without recognition is still valid, but if candidates recognize it as Makeham they could plug into the formula given in the tables.

Question # 9.8
Answer: B

$$
\begin{aligned}
&\text{APV(Premiums)} = \text{APV(Benefits)} \\
&\text{APV(Benefits)} = 60,000\ddot{a}_{45|45} + 3P\ddot{a}_{45|45} \\
&\text{where } \ddot{a}_{45|45} = \ddot{a}_{45} - \ddot{a}_{45:45} \\
&= 17.8162 - 16.8122 \\
&= 1.0040 \\
&\text{APV(Premiums)} = P\ddot{a}_{45:45} \\
&P(16.8122) = 60,000(1.0040) + 3P(1.004) \\
&P = 4365
\end{aligned}
$$

Question # 9.9
Answer: B

$$
\begin{aligned}
APV &= 30,000A_{50:50} + 70,000A_{\overline{50:50}} \\
&= 30,000A_{50:50} + 70,000\left(A_{50}+A_{50}-A_{50:50}\right) \\
&= 70,000(2)A_{50} - 40,000A_{50:50} \\
&= 140,000(0.18931) - 40,000(0.24669) \\
&= 16,635.80
\end{aligned}
$$

Question # 9.10
Answer: B

$$ {}_t p_{50} = \left(1-\frac{t}{50}\right), 0 \le t \le 50 $$

$$ {}_t p_{55} = e^{-0.04t}, t \ge 0 $$

$$ {}_t p_{50:55} = \Pr(T_{50:55} > t) = \begin{cases} \left(1-\frac{t}{50}\right)e^{-0.04t}, & 0 \le t \le 50 \\ 0, & t > 50 \end{cases} $$

where $T_{50:55} = \min(T_{50}, T_{55})$ is the time until the first death.

$$ L = 100e^{-0.05T_{50:55}} - 60 > 0 \Rightarrow e^{-0.05T_{50:55}} > 0.6 \Rightarrow T_{50:55} < -20\ln(0.6) $$

$$ \begin{aligned} \Pr(L > 0) &= \Pr\left[T_{50:55} < -20\ln(0.60)\right] \\ &= 1-\Pr\left[T_{50:55} \ge -20\ln(0.60)\right] \\ &= 1-\Pr\left[T_{50} \ge -20\ln(0.60)\right]\Pr\left[T_{55} \ge -20\ln(0.60)\right] \\ &= 1-\left(1-\frac{-20\ln(0.60)}{50}\right)e^{-0.04[-20\ln(0.60)]} \\ &= 0.4712 \end{aligned} $$

Question # 9.11
Answer: A

$$ \begin{aligned} 100,000A^{\ 1}_{\overline{50:60}:\overline{10|}} &= 100,000\left[A^{1}_{50:\overline{10|}} + A^{1}_{60:\overline{10|}} - A_{\overset{1}{50:60}:\overline{10|}}\right] \\ &= 100,000\left[0.01461 + 0.04252 - 0.05636\right] = 77.00 \end{aligned} $$

where

$$ \begin{aligned} A^{1}_{50:\overline{10|}} &= A_{50:\overline{10|}} - {}_{10}E_{50} = 0.61643 - 0.60182 = 0.01461 \\ A^{1}_{60:\overline{10|}} &= A_{60:\overline{10|}} - {}_{10}E_{60} = 0.62116 - 0.57864 = 0.04252 \\ A_{\overset{1}{50:60}:\overline{10|}} &= A_{50:60} - (1.05)^{10}\ {}_{10}E_{50}\ {}_{10}E_{60} A_{60:70} \\ &= 0.32048 - (1.628895)(0.60182)(0.57864)(0.46562) = 0.05636 \end{aligned} $$

Question # 9.12
Answer: B

$$P\ddot{a}_{40:40:\overline{10}|} = 1{,}000{,}000\,{}_{35}E_{40:40}\ddot{a}_{75:75}$$

$${}_{35}E_{40:40} = \left(\frac{l_{75}}{l_{40}}\right)^2 (1.05)^{-35} = 0.13337$$

$$8.0649P = (1{,}000{,}000)(0.13337)(8.2085)$$

$$P = 135{,}745$$

Question # 9.13
Answer: B

The expected present value of the premiums is:
$P\ddot{a}_{55:55:\overline{10}|} = 7.9321P$

The benefit is 6,000 per year to each of them while they are alive, but while they are both alive they must, between them, return 2,000 since the benefit is only 10,000.

The expected present value of benefits is therefore:

$$= 2\times 6{,}000\times {}_{10}E_{55}\times \ddot{a}_{65} - 2{,}000\left({}_{10}E_{55}\right)^2 \left(1.05\right)^{10} \ddot{a}_{65:65}$$
$$= 2\times 6{,}000\times 0.59342\times 13.5498 - 2{,}000\times 0.59342^2 \times 1.05^{10} \times 11.6831$$
$$= 83{,}085.56$$

Using the equivalence principle, we get

$$P = \frac{83{,}085.56}{7.9321} = 10{,}474.60$$

Question # 9.14
Answer: C

$$\bar{a}_{xy} = \bar{a}_x + \bar{a}_y - \bar{a}_{\overline{xy}} = 10.06 + 11.95 - 12.59 = 9.42$$

$$\bar{a}_{xy} = \frac{1-\bar{A}_{xy}}{\delta}$$

$$9.42 = \frac{1-\bar{A}_{xy}}{0.07} \Rightarrow \bar{A}_{xy} = 0.34$$

$$\bar{A}_{xy} = \bar{A}^{1}_{xy} + \bar{A}_{xy}^{\;1}$$

$$0.34 = \bar{A}^{1}_{xy} + 0.09 \Rightarrow \bar{A}^{1}_{xy} = 0.25$$

Question # 9.15
Answer: B

$$\begin{aligned}
a_{50:60:\overline{20}|} &= a_{50:60} - v^{20}\,{}_{20}p_{50:60}\,a_{70:80} \\
&= a_{50:60} - v^{20}\,{}_{20}p_{50}\,{}_{20}p_{60}\,a_{70:80} \\
&= \left(\ddot{a}_{50:60} - 1\right) - {}_{20}E_{50}\frac{\ell_{80}}{\ell_{60}}\left(\ddot{a}_{70:80} - 1\right) \\
&= 13.2699 - 0.34824\left(\frac{75,657.2}{96,634.1}\right)(6.7208) \\
&= 11.4375
\end{aligned}$$

Question # 10.1
Answer: B

Final average salary

$$\frac{50,000}{3}\left[(1.04)^{26}+(1.04)^{25}+(1.04)^{24}\right]=133,360.2$$

Annual retirement benefit

$$=0.017(27)(\text{final average salary})(0.85)=52,030$$

Note that the factor of 0.85 is based on an interpretation of the 5% reduction as producing a factor of 1 – 3(0.05) = 0.85. The Notation and Terminology Study Note states that this is the method to be used.

Question # 10.2
Answer: E

Defined Benefit:

$$0.015\times\text{Final Average Earnings}\times\text{Years of Service}$$

$=0.015\times\left(50,000\times\left(1.05^{19}+1.05^{18}+1.05^{17}\right)/3\right)\times 20=36,128$ per year

APV at 65 of Defined Benefit = $36,128\ddot{a}_{65}=36,1268(10.0)=361,280.$

Defined Contribution accumulated value at 65:

$$X\%\times 50,000\times 1.05^{20}+X\%\times(50,000\times 1.05)\times 1.05^{19}+\ldots+X\%\times\left(50,000\times 1.05^{19}\right)\times 1.05$$

$$=X\%\times 50,000\times 1.05^{20}\times 20=X\%(2,653,298)$$

Therefore,

$$361,280=X\%(2,653,298)$$

$$X\%=0.136$$

$$X=13.6$$

Question # 10.3
Answer: E

Defined benefit plan projected benefit

$$= 50,000\left(\frac{1.02^{22}+1.02^{23}+1.02^{24}}{3}\right)(30)(0.005)$$
$$= 50,000(1.5771054)(30)(0.005)$$
$$= 11,828$$

The additional income desired = 42,000 – 11,828 = 30,172.
The necessary defined contribution accumulation at age 65 is
$30{,}172\ddot{a}_{65} = 30{,}172(9.9) = 298,703.$

Question # 10.4
Answer: C

Annual pension at age 65 $\quad = 45,000\left[\frac{1+(1.04)+\cdots+(1.04)^{29}}{30}\right](0.02)(30)$

$$= 45,000(0.02)\frac{(1.04)^{30}-1}{0.04} = 50,476.44$$

Question # 10.5
Answer: E

Annual Retirement Benefit
$$(0.0175)\left[525,000+\sum_{K=0}^{9}(50,000)(1.03)^{K}\right] = 19,218.39$$

APV at age 65
$$(19,218.39)\ {}_{10|}\ddot{a}_{55}^{(12)} = 19,218.39(1.04)^{-10}\left({}_{10}p_{55}\right)\ddot{a}_{65}^{(12)}$$
$$= 19,218.39(0.6756)(0.925)(12.60) = 151,328$$

Question # 10.6
Answer: E

Final average salary before retirement $= 40,000\left(\frac{1.035^{32}+1.035^{33}+1.035^{34}}{3}\right)$

$= 124,526.80$

Retirement Pension $= 35 \times 0.016 \times 124,526.80$

$= 69,735.01$

Salary in final year $= 40,000 \times 1.035^{34}$

$= 128,834.41$

Replacement Ratio $= \frac{69,735.01}{128,834.41} = 54.13\%$

Question # 10.7
Answer: C

Fred gets: $(120,000)(1-5\times 0.04)(0.02)(35) = 67,200$

Glenn gets: $\left(120,000 + 5(4800)\right)\times 0.02 \times 40 = 115,200$

Fred gets his for 5 years more, so he is 336,000 ahead of Glenn.

Once Glenn starts drawing he gets 48,000 more per year. It takes him $336,000/48,000 = 7$ years to catch up to Fred.

Question # 10.8
Answer: E

Retirement Age	63	64	65
Years of Service (K)	33	34	35
v^{K-20}	0.414964	0.387817	0.362446
Probability of Retirement	0.4	(0.6)(0.2)	(0.6)(0.8)(1.0)
Benefit	(33)(12)(25)	(34)(12)(25)	(35)(12)(25)
Annuity Factor	12	11.5	11
Benefit Reduction Factor	0.856	0.928	1.0
Contribution to Actuarial Present Value of Retirement Benefit	16,879.56	5,065.87	20,094.01

The actuarial present value of the retirement benefit is therefore:

$$16,879.56 + 5,065.87 + 20,094.01 = 42,039.44$$

Question # 10.9
Answer: C

Let S_{35} be Colton's starting salary which is the annual salary from age 35 to age 36.

$$\overbrace{\frac{S_{35}}{30}\left[1+1.025+1.025^{2}+\ldots+1.025^{29}\right]}^{\text{career average salary}} \times 0.02 \times 30 = \overbrace{\frac{S_{35}}{5}\left[1.025^{29}+1.025^{28}+\ldots+1.025^{25}\right]}^{\text{find average salary}} \times R\% \times 30$$

$$0.02\frac{1.025^{30}-1}{0.025} = 6(R\%)\left(1.025^{25}\right)\frac{1.025^{5}-1}{0.025}$$

$$R\% = \frac{0.02}{6}1.025^{-25}\frac{1.025^{30}-1}{1.025^{5}-1} = 0.1501727 = 1.5\%$$

Question # 10.10
Answer: D

Let S_{35} be the employee's starting salary which is the annual salary from age 35 to age 36.

For Plan I, the accumulated contributions are:

$$(0.15)S_{35}(1.03)^{30}+(0.15)S_{35}(1.03)(1.03)^{29}$$
$$+(0.15)S_{35}(1.03)^{2}(1.03)^{28}+\ldots$$
$$=(0.15)S_{35}(1.03)^{30}(30)=10.923S_{35}$$
$$=(12B)\ddot{a}_{65}^{(12)}$$
$$\Rightarrow B=\frac{10.923S_{35}}{12(9.44)}=0.096S_{35}$$

For Plan II, we have:

$$\frac{1}{2}\left(S_{35}(1.03)^{28}+S_{35}(1.03)^{29}\right)=2.322S_{35}$$
$$B=\frac{1}{12}\left(30\times 0.015\right)S_{35}\left(2.322\right)=0.087S_{35}$$
$$\Rightarrow\frac{0.096}{0.087}=1.107$$

Question # 10.11
Answer: B

Under the Traditional Unit Credit cost method the actuarial accrued liability (AAL) is the actuarial present value of the accrued benefit on the valuation date.

The formula for the accrued benefit, *B*, is

$$B = (0.02)(FAS)(SVC)$$

Where *FAS* is the final average salary and *SVC* is years of service.

FAS is the average of the salaries in the years 2013, 2014, and 2015, which is 35,000 x $(1.03^2 + 1.03^3 + 1.03^4)/3$ = 38,257. Therefore

$$B = (0.02)(38,257)(5.0) = 3826\,.$$

The AAL is the actuarial present value (as of the valuation date) of the accrued benefit and is given by

$$\begin{aligned}\text{AAL} &= B \cdot \ddot{a}_{65} \cdot q_{65}^{(r)} \cdot {}_{65-35}p_{35}^{(\tau)} \cdot v^{30} \\ &= (3826)(11.0)(1.00)(0.95)^{30}\left(1.04\right)^{-30} \\ &= 2785\end{aligned}$$

Question # 10.12
Answer: D

We know that:

$${}_tV + C_t = EPV \text{ of benefits for mid-year exits} + v \cdot {}_1p_x^{(\tau)} \cdot {}_{t+1}V \quad \text{where:}$$

C_t = Normal Cost for year t to $t+1$ and ${}_tV$ is the Actuarial Accrued Liability at time t

Average Salary at 12-31-2015	35,000 x ($1.03^2 + 1.03^3 + 1.03^4$)/3 = 38,257
Accrued Benefit at 12-31-2015	$(0.02)(38,257)(5.0) = 3826$
Actuarial Accrued Liability 12-31-2015, ${}_tV$	$(3826)(11.0)(1.00)(0.95)^{30}(1.04)^{-30} = 2785$

If you do not understand the above numbers, you can look at the solution to Number 10.11 for more details.

Average Salary at 12-31-2016	35,000 x ($1.03^3 + 1.03^4 + 1.03^5$)/3 = 39,404
Accrued Benefit at 12-31-2016	$(0.02)(39,404)(6.0) = 4728$
Actuarial Accrued Liability 12-31-2016, ${}_{t+1}V$	$(4728)(11.0)(1.00)(0.95)^{29}(1.04)^{-29} = 3768$

Note that EPV of benefits for mid-year exit is zero. Then:

$${}_tV + C_t = EPV \text{ of benefits for mid-year exits} + v \cdot {}_1p_x^{(\tau)} \cdot {}_{t+1}V$$

$$2785 + C_t = 0 + (1.04)^{-1}(0.95)(3768)$$

$$C_t = 657$$

Question # 10.13
Answer: B

Under the Projected Unit Credit cost method, the actuarial liability is the actuarial present value of the accrued benefit. The accrued benefit is equal to the projected benefit at the decrement date multiplied by service as of the valuation date and by the accrual rate.

We have the following information.

Projected Final Average Salary at 65	$(35{,}000)(1.03^{32} + 1.03^{33} + 1.03^{34})/3 = 92{,}859$
Service at valuation date	5
Accrual Rate	0.02
Projected Benefit	(92,859)(0.02)(5) = 9286

The actuarial liability is the actuarial present value (as of the valuation date) of the projected benefit and is given by

$$\begin{aligned}\text{Actuarial Liability} &= (\text{Projected Benefit})\cdot \ddot{a}_{65}\cdot q_{65}^{(r)}\cdot {}_{65-35}p_{35}^{(\tau)}\cdot v^{30}\\ &= (9286)(11.0)(1.00)(0.95)^{30}(1.04)^{-30}\\ &= 6760\end{aligned}$$

Question # 10.14
Answer: D

We know that:

$${}_tV + C_t = EPV \text{ of benefits for mid-year exits} + v \cdot {}_1p_x^{(\tau)} \cdot {}_{t+1}V \quad \text{where:}$$

C_t = Normal Cost for year t to $t+1$ and ${}_tV$ is the Actuarial Liability at time t

We have the following information.

Projected Final Average Salary at 65	$(35,000)(1.03^{32} + 1.03^{33} + 1.03^{34})/3 = 92,859$
Projected Benefit at 12-31-2015	(92,859)(0.02)(5) = 9286
Accrued Liability 12-31-2015, ${}_tV$	$(9286)(11.0)(1.00)(0.95)^{30}(1.04)^{-30} = 6760$

If you do not understand the above numbers, you can look at the solution to Number 10.13 for more details.

Projected Final Average Salary at 65	$(35,000)(1.03^{32} + 1.03^{33} + 1.03^{34})/3 = 92,859$
Projected Benefit at 12-31-2016	(92,859)(0.02)(6) = 11,143
Accrued Liability 12-31-2016, ${}_{t+1}V$	$(11,143)(11.0)(1.00)(0.95)^{29}(1.04)^{-29} = 8880$

Note that EPV of benefits for mid-year exit is zero. Then:

$${}_tV + C_t = EPV \text{ of benefits for mid-year exits} + v \cdot {}_1p_x^{(\tau)} \cdot {}_{t+1}V$$

$$6760 + C_t = 0 + (1.04)^{-1}(0.95)(8880)$$

$$C_t = 1352$$

Question # 10.15
Answer: C

$${}_tV + C_t = EPV \text{ of benefits for mid-year exits} + v_1 p_x^{00} \cdot {}_{t+1}V$$

For this problem, EPV of benefits for mid-year exits $= 0$

$$\therefore C_{35} = v_1 p_{60}^{00} \cdot {}_{36}V - {}_{35}V$$

Since the pension plan has only a retirement benefit at Normal Retirement, under PUC:

${}_tV =$ accrual rate × years of past service × probability of survival to retirement × discount to retirement × APV at time of retirement of the retirement benefit

$${}_{35}V = accrual\ rate \times 35 \times v^R \times {}_R\, p_{60}^{00} \times APV\ of\ retirement\ benefit$$

$${}_{36}V = accrual\ rate \times 36 \times v^{R-1} \times {}_{R-1}\, p_{61}^{00} \times APV\ of\ retirement\ benefit$$

$$==> v_1 p_{60}^{00} \cdot {}_{36}V = accrual\ rate \times 36 \times v^R \times {}_R\, p_{60}^{00} \times APV\ of\ retirement\ benefit = \frac{36}{35} \cdot {}_{35}V$$

$$C_{35} = v_1 p_{60}^{00} \cdot {}_{36}V - {}_{35}V = \frac{36}{35} \cdot {}_{35}V - {}_{35}V = \frac{{}_{35}V}{35}$$

Question # 10.16
Answer: A

By age 65, member would have served total of 35 years in which case, benefit would be $35 \times 0.02 = 70\%$. Thus, set it at 60%.

$$\text{EPV(benefits)} = (0.60)(50{,}000)(1.03)^{19}\left(\frac{1}{1.05^{20}}\right)\left(\frac{l_{65}^{(\tau)}}{l_{45}^{(\tau)}}\right)\ddot{a}_{65}^{(12)}$$

$$= (0.60)(50{,}000)(1.03)^{19}\left(\frac{1}{1.05}\right)^{20}\left(\frac{3000}{5000}\right)(7.8) = 92{,}787.29$$

Question # 10.17
Answer: E

Replacement ratios

Plan 1: $R = \dfrac{\text{(Pension Benefit)(Years of Service)}}{\text{Salary at Retirement}} = \dfrac{(1250)(25)}{S_0(1.04)^{24}}$

Plan 2:

$$R = \frac{\text{(Career Average Salary)(Benefit Rate Per Year)(Years of Service)}}{\text{Salary at Retirement}}$$

$$= \frac{S_0\left(\dfrac{1.04^{25}-1}{0.04}\right)\left(\dfrac{1}{25}\right)(0.02)(25)}{S_0(1.04)^{24}} = \frac{\left(\dfrac{1.04^{25}-1}{0.04}\right)(0.02)}{(1.04)^{24}} = 0.324934$$

The two are equal, so that

$$\frac{(1250)(25)}{S_0(1.04)^{24}} = 0.32494$$

$$\Rightarrow S_0 = \frac{(1250)(25)}{0.32494(1.04)^{24}} = 37{,}519$$

Question # 10.18
Answer: E

At 66, the total retirement fund is $1{,}500{,}000(1.08)$ and is to be used to purchase a quarterly annuity equal to $X\ddot{a}_{66}^{(4)}$ so that:

$$X = \frac{1{,}500{,}000(1.08)}{\ddot{a}_{66} - \dfrac{3}{8}} = \frac{1{,}620{,}000}{13.2557 - \dfrac{3}{8}} = 125{,}769.56$$

Replacement Ratio $= \dfrac{X}{250{,}000} = \dfrac{125{,}769.56}{250{,}000} = 0.50$

Question # 10.19
Answer: A

There is no projected salary for traditional unit credit method:

$$ {}_0V = (0.02)(10)(150,000)v^{20}\ {}_{20}p_{45}\ddot{a}_{65}^{(12)} = 127,157.50 $$

$$ {}_1V = (0.02)(11)(150,000)v^{19}\ {}_{19}p_{46}\ddot{a}_{65}^{(12)} $$

Let *C* = normal contribution

$$ {}_0V + C = \underbrace{vp_{45}\ {}_1V}_{\frac{11}{10}\bullet {}_0V} \Rightarrow C = \frac{1}{10}\ {}_0V = 12,715.75 $$

Question # 10.20
Answer: D

Kaitlyn's annual retirement benefit is

(Final Average Salary)(Number of Years)(Accrual Rate)(Reduction Factor)

$$ = \frac{50,000(1.025^{26} + 1.025^{27} + 1.025^{28} + 1.025^{29} + 1.025^{30})}{5}(31)(0.02)\left(1 - 0.07\left(3\right)\right) = 48,923.98 $$

Question # 10.21
Answer: A

$$ APV = \sum_{x=62}^{64} B_x \frac{d_x^{(r)}}{\ell_{50}} v^{x+0.5-50} $$

$$ = 20,000 \times \frac{5017.5}{117,145.5} \times 1.05^{-12.5} + 25,000 \times \frac{4515.2}{117,145.5} \times 1.05^{-13.5} + 30,000 \times \frac{4061.0}{117,145.5} \times 1.05^{-14.5} $$

$$ = 1477 $$

Question # 10.22
Answer: A

$$\text{AAL}_{2017} = AB_{2017} \times {}_{15}E_{45} \times \ddot{a}_{65}^{(12)}$$
$$AB_{2017} = 15 \times 0.02 \times 63{,}000 \times 1.05^{14} = 37{,}420.71$$
$$\text{AAL}_{2017} = 37{,}420.71 \times 0.15 \times 11 = 61{,}744.17$$

Question # 10.23
Answer: A

$$V_0 = 0.02 \times YOS_0 \times S_0 \times \frac{\ell_{65}}{\ell_{51}} \times (1+i)^{-14} \times \ddot{a}_{65}^{(12)}$$
$$= 0.02 \times 10 \times 68{,}700 \times \frac{94{,}579.7}{98{,}457.2} \times 1.05^{-14} \times 13.0915 = 87{,}272.30$$

The normal contribution is:

$$NC = V_0\left(\frac{S_1}{S_0} \times \frac{YOS_1}{YOS_0} - 1\right) = 87{,}272.30\left(\frac{70{,}400}{68{,}700} \times \frac{11}{10} - 1\right)$$
$$= 11{,}103$$

Question # 10.24
Answer: E

Years of service at age 65: $15+(65-45)=35$

Final one-year salary: $(120{,}000)(1.04^{20})=262{,}935$

Projected pension: $(262{,}935)(35)(0.015)=138{,}041$

Actuarial present value of projected pension:

$$\frac{(138{,}041)(0.552)(10.60)}{1.05^{20}}=304{,}415.7$$

Actuarial liability: $\left(\frac{15}{35}\right)\left(304{,}415.7\right)=130{,}464$

Normal cost under projected unit credit with no benefits paid on next year's terminations is:

$$\frac{130{,}464}{15}=8{,}697.6$$

Question # 12.1
Answer: E

$${}_1V_{70}=(10{,}000)\left(1-\frac{\ddot{a}_{71}}{\ddot{a}_{70}}\right)=(10{,}000)\left(1-\frac{11.6803}{12.0083}\right)=273.14$$

$${}_2V_{70}=(10{,}000)\left(1-\frac{\ddot{a}_{72}}{\ddot{a}_{70}}\right)=(10{,}000)\left(1-\frac{11.3468}{12.0083}\right)=550.87$$

$$\text{Expected Profit}=\left[273.14+800\left(1-0.10\right)\right]1.07-10{,}000(0.03)-550.87\left(1-0.03-0.04\right)=250.35$$

Question # 12.2
Answer: B

$$\begin{aligned}\Pr_2 &= {}_1V + P - E + I - EDB - E_2V\\ &= (400+1500-100)1.072-(100,000)(0.012)-(0.988)(700)\\ &= 38.00\end{aligned}$$

Question # 12.3
Answer: E

EPV of Premium $= 250\left(1+vp_{50}\right)$

EPV of Profit $= -165+100v+125v^2 p_{50}$

Profit Margin $= \dfrac{-165+100v+125v^2 p_{50}}{250\left(1+vp_{50}\right)} = 0.06$

Solving for p_{50}, we get:

$$p_{50} = \frac{-165+100v-0.06(250)}{0.06(250)v-125v^2} = \frac{-89.09091}{-89.66942}$$

$$= 0.9935484$$

$$\left(\text{where } v = 1.10^{-1}\right)$$

Question # 12.4
Answer: C

$245 = p_{40}274$ and $300 = {}_2p_{40}395$

Present value of expected premiums:

$1000[1+(245/274)(1/1.12)+(300/395)(1/1.12^2)] = 2403.821.$

Present value of expected profits:

$-400+150/1.12+245/1.12^2+300/1.12^3 = 142.775.$

PV Profit / PV premium = 5.94%

Question # 12.5
Answer: D

$$DPP = \min\{t : NPV(t) \geq 0\}$$

$$NPV(0) = \pi_0 = -550$$

$$NPV(1) = \pi_0 + \pi_1 v = -550 + \frac{300}{1.12} = -282.14$$

$$NPV(2) = NPV(1) + \pi_2 v = -282.14 + \frac{275}{1.12^2} = -62.91$$

$$NPV(3) = NPV(2) + \pi_3 v^3 = -62.91 + \frac{75}{1.12^3} = -9.53$$

$$NPV(4) = NPV(3) + \pi_4 v^4 = -9.53 + \frac{150}{1.12^4} = 85.80$$

$$NPV(4) \geq 0 \Rightarrow DPP = 4$$

Question # 12.6
Answer: B

$$\text{NPV} = \text{PreContractExp} + \sum_{k=1}^{3}(\text{Pr}_k)v^k \,{}_{k-1}p_{55}$$

$$\text{NPV} = \text{PreContractExp} +$$

$$\sum_{k=1}^{3}(\text{StartingRes}_k + GP_k - E_k + \text{InvEarn}_k - \text{ExpDeathBenefits}_k - \text{ExpReserveCosts}_k)v^k \,{}_{k-1}p_{55}$$

$$\text{PreContractExp} = -100$$

$$\text{Pr}_1 = 75 - 20 + 2.80 - 10.0 - 64.35 = -16.55$$

$$\text{Pr}_2 = 65 + 75 - 20 + 6.00 - 15.0 - 123.13 = -12.13$$

$$\text{Pr}_3 = 125 + 75 - 20 + 9.00 - 21.0 = 168.00$$

$$NPV = -100 - 16.55 \times 1.1^{-1} \times 1 - 12.13 \times 1.1^{-2} \times 0.99 + 168 \times 1.1^{-3} \times 0.99 \times 0.985$$

$$= -100 - 15.05 - 9.92 + 123.08$$

$$= -1.89$$

Question # LM.1
Answer: E

$$\hat{S}(12) = \hat{S}(10)\left(\frac{600 - 200 - 100}{600 - 200}\right) = 0.6$$

Question # LM.2
Answer: D

$$\hat{S}(1) = 0.8$$

$$V[S(1)] = \frac{S(1)(1-S(1)}{n} \approx \frac{(0.8)(0.2)}{1000} = 0.01265^2$$

$$\Rightarrow 95\% \text{ CI is approx } (0.8 \pm 1.96(0.01265)) = (0.775, 0.825)$$

Question # LM.3
Answer: D

$$\hat{H}(1.5) = \frac{1}{90} + \frac{3}{81} = 0.048148$$

$$\hat{S}(1.5) = e^{-\hat{H}(1.5)} = 0.9530$$

Question # LM.4
Answer: B

$$\hat{S}(21) = \frac{55}{60} \times \frac{42}{48} \times \frac{28}{35} \times \frac{15}{21} \times \frac{4}{10} = 0.1833$$

$$V[S(21)] \approx 0.1833^2 \left(\frac{5}{60 \times 55} + \frac{6}{48 \times 42} + \frac{7}{35 \times 28} + \frac{6}{21 \times 15} + \frac{6}{10 \times 4} \right)$$

$$\approx 0.00607 = 0.0779^2$$

$$\Rightarrow \text{Upper 80\% Confidence limit is } 0.1833 + 1.282 \times 0.0779$$

$$= 0.283$$

Question # LM.5
Answer: B

$$\hat{F}(10)=\frac{28+19}{100}=0.47 \quad \hat{F}(20)=\frac{28+19+15}{100}=0.62$$

Use linear interpolation to find $\hat{F}(12)$

$$\hat{F}(12)=\left(\frac{20-12}{20-10}\right)\hat{F}(10)+\left(\frac{12-10}{20-10}\right)\hat{F}(20)+=0.8(0.47)+0.2(0.62)=0.50$$

Question # LM.6
Answer: C

Exposure is $T_0=3+0.35+0.25+0.45=4.05$

Number of 0-1 transitions is $d^{01}=2$

Variance of the estimator for $\hat{\mu}_x^{01}$ is $\dfrac{d^{01}}{(T_0)^2}=\dfrac{2}{(4.05)^2}=0.12193=0.35^2$.

Standard Deviation $=0.35$

Question # S1.1
Answer: C

Alice is sick for 5 months from July-November 2018; of this 2 months is eliminated through the waiting period, giving three months benefit. She is not sick for three months and then sick again for 8 months. Because the recovery period is less than the off period of the benefit, the payments start again as soon as she becomes ill the second time, with 8 months of benefit payable. That gives a total of 11 months of sickness benefit during the two years 2018-2019.

Question # S2.1
Answer: C

$$a^{01}_{50:\overline{10|}} = a^{01}_{50} - v^{10}\,{}_{10}p^{00}_{50}\,a^{01}_{60} - v^{10}\,{}_{10}p^{01}_{50}\,a^{11}_{60}$$

Note that

$a^{ij}_x = \ddot{a}^{ij}_x$ for $i \neq j$ Because under either annuity, there is no payment at $t = 0$

$a^{ii}_x = \ddot{a}^{ii}_x - 1$ Because $\ddot{a}^{ii}_x$ includes a payment at $t = 0$ but a^{ii}_x does not

So

$$\begin{aligned} a^{01}_{50:\overline{10|}} &= \ddot{a}^{01}_{50} - v^{10}\,{}_{10}p^{00}_{50}\,\ddot{a}^{01}_{60} - v^{10}\,{}_{10}p^{01}_{50}\left(\ddot{a}^{11}_{60} - 1\right) \\ &= 1.9618 - (1.05)^{-10}(0.83936)(2.6283) - (1.05)^{-10}(0.06554)(10.7144 - 1) \\ &= 0.2166 \end{aligned}$$

Question # S3.1
Answer: B

$$\left({}_{10}V^{(0)} + 0.95P\right)(1.06) = p^{00}_{60}\,{}_{11}V^{(0)} + p^{01}_{60}\left(30{,}000(1.05) + {}_{11}V^{(1)}\right) =$$

$$\left(5946 + 0.95(2360)\right)(1.06) = (0.97026)_{11}V^{(0)} + (0.01467)\left(30{,}000(1.05) + {}_{11}V^{(1)}\right)$$

$$\Rightarrow 8217.175 = 0.97026\,{}_{11}V^{(0)} + 0.01467\,{}_{11}V^{(1)} \qquad \text{(A)}$$

Also ${}_{10}V^{(1)}(1.06) = p^{10}_{60}\,{}_{11}V^{(0)} + p^{11}_{60}\left(30000(1.05) + {}_{11}V^{(1)}\right)$

$$(200{,}640)(1.06) = (0.00313)_{11}V^{(0)} + (0.97590)\left(30000(1.05) + {}_{11}V^{(1)}\right)$$

$$\Rightarrow 181{,}937.55 = 0.00313\,{}_{11}V^{(0)} + 0.97590\,{}_{11}V^{(1)}$$

$$181{,}937.55 = 0.00313\,{}_{11}V^{(0)} + 0.97590\left(\frac{8217.175 - 0.97026\,{}_{11}V^{(0)}}{0.01467}\right)$$

$$\Rightarrow {}_{11}V^{(0)} = 5650.5$$

Question # S4.1
Answer: C

$${}_3p_{0,0} = [1-q(0,0)][1-q(1,1)][1-q(2,2)]$$

$$= [1-q(0,0)][1-\{1-\varphi(1,1)\}q(1,0)][1-\{1-\varphi(2,1)\}\{1-\varphi(2,2)\}q(2,0)]$$

$$= [1-0.4][1-\{1-0.08\}(0.5)][1-\{1-0.06\}\{1-0.04\}(0.6)]$$

$$(0.6)(1-0.92\times 0.5)(1-0.94\times 0.96\times 0.6) = 0.149$$

Question # S4.2
Answer: D

Set 2017 to be $t=0$. The spline function is $C(x,t) = at^3 + bt^2 + ct + d$ and the first derivative is $C'(x,t) = 3at^2 + 2bt + c$. Then

$$C(35,0) = d = 0.037 \quad C'(35,0) = (0.037-0.035) = 0.002 = c$$
$$C(35,10) = 1000a + 100b + 10c + d = 0.015 \Rightarrow -0.042 = 1000a + 100b$$
$$C'(35,10) = 300a + 20b + c = 0 \Rightarrow -0.002 = 300a + 20b$$
$$\Rightarrow a = 0.000064 \quad b = -0.00106$$
$$\Rightarrow C(35,5) = 125a + 25b + 5c + d = 0.0285$$

Question # S4.3
Answer: D

$$lm(60,2020) = \log(m(60,2020) = \alpha_{60} + \beta_{60}K_{2020}$$
$$K_{2020} = K_{2018} + c + \sigma_K Z_{2018} + c + \sigma_K Z_{2019} = K_{2018} + 2c + \sigma_K\left(Z_{2018} + Z_{2019}\right)$$
where $(\mathrm{Z}_{2018}, \mathrm{Z}_{2019})$ are i.i.d. $\mathrm{N}(0,1) \Rightarrow (Z_{2018} + Z_{2019}) \sim N(0,2)$
$$\Rightarrow lm(60,2020) = -4.0 + 0.25\left(-3.0 - 0.1 + 0.9(Z_{2018} + Z_{2019})\right) = -4.775 + 0.225(Z_{2018} + Z_{2019})$$
$$\Rightarrow lm(60,2020) \sim N(-4.775, [(0.225)^2 V(Z_{2018} + Z_{2019})])$$
$$\Rightarrow lm(60,2020) \sim N(-4.775,\ 0.10125)$$
$$\Rightarrow m(60,2020) \sim \log N(-4.775, \sqrt{0.10125})$$
$$\Rightarrow E[m(60,2020)] = e^{-4.775+0.10125/2} = 0.00888$$

Question # S4.4
Answer: A

$$lm(60,2020) = \log(m(60,2020) = \alpha_{60} + \beta_{60} K_{2020}$$
$$K_{2020} = K_{2018} + c + \sigma_K Z_{2018} + c + \sigma_K Z_{2019} = K_{2018} + 2c + \sigma_K \left(Z_{2018} + Z_{2019} \right)$$
where $(\mathrm{Z}_{2018}, Z_{2019})$ are i.i.d. $\mathrm{N}(0,1) \Rightarrow (Z_{2018} + Z_{2019}) \sim N(0,2)$
$$\Rightarrow lm(60,2020) = -4.0 + 0.25\left(-3.0 - 0.1 + 0.9(Z_{2018} + Z_{2019})\right) = -4.775 + 0.225(Z_{2018} + Z_{2019})$$
$$\Rightarrow lm(60,2020) \sim N(-4.775, [(0.225)^2 V(Z_{2018} + Z_{2019})])$$
$$\Rightarrow lm(60,2020) \sim N(-4.775,\ 0.10125)$$
$$\Rightarrow m(60,2020) \sim \log N(-4.775, \sqrt{0.10125})$$
$$\Rightarrow E[m(60,2020)] = e^{-4.775 + 0.10125/2} = 0.00888$$
$$\Rightarrow V[m(60,2020)] = \left(e^{-4.775+0.10125/2}\right)^2 \left(e^{0.10125} - 1\right) = 0.002897^2$$

$$\Rightarrow \text{Standard Deviation} = 0.002897$$

Question # S4.5
Answer: B

Let $Q_\alpha(X)$ denote the $\alpha -$ quantile of a random variable X .

$$lm(60,2020) = \log(m(60,2020) = \alpha_{60} + \beta_{60} K_{2020}$$
$$K_{2020} = K_{2018} + c + \sigma_K Z_{2018} + c + \sigma_K Z_{2019} = K_{2018} + 2c + \sigma_K \left(Z_{2018} + Z_{2019} \right)$$
where $(\mathrm{Z}_{2018}, Z_{2019})$ are i.i.d. $\mathrm{N}(0,1) \Rightarrow (Z_{2018} + Z_{2019}) \sim N(0,2)$
$$\Rightarrow lm(60,2020) = -4.0 + 0.25\left(-3.0 - 0.1 + 0.9(Z_{2018} + Z_{2019})\right) = -4.775 + 0.225(Z_{2018} + Z_{2019})$$
$$\Rightarrow lm(60,2020) \sim N(-4.775, [(0.225)^2 V(Z_{2018} + Z_{2019})])$$
$$\Rightarrow lm(60,2020) \sim N(-4.775,\ 0.10125)$$
$$\Rightarrow m(60,2020) \sim \log N(-4.775, \sqrt{0.10125})$$
$$\Rightarrow Q_{0.95}(m(60,2020)) = e^{-4.775 + 1.645\sqrt{0.10125}} = 0.01424$$

Question # S4.6
Answer: E

$lm(60,2020) \sim N(-4.775, 0.10125)$

$\Rightarrow m(60,2020) \sim \log N(-4.775, \sqrt{0.10125})$

Under UDD we have $p_x = \dfrac{1-m_x/2}{1+m_x/2}$ which is a decreasing function of m_x .

Let $Q_\alpha(X)$ denote the $\alpha-$ quantile of X . Then

$$Q_{0.95}(p(60,2020)) = \frac{1-Q_{0.05}(m(60,2020))/2}{1+Q_{0.05}(m(60,2020))/2}$$

$$Q_{0.05}(m(60,2020)) = e^{-4.775-1.645(\sqrt{0.10125})} = 0.0050$$

$$\Rightarrow Q_{0.95}(p(60,2020)) = 0.99501$$

Note: Since p_x is a decreasing function, to find the 95th quantile of p_x,

we must use the 5th quantile of m_x.

Question # S5.1
Answer: A

$$\bar{a}_x^{00} + \bar{a}_x^{01} + \bar{a}_x^{02} = \bar{a}_x = 0.52 + 3.24 + 5.60 = 9.36$$

$$\bar{A}_x^{03} = \bar{A}_x = 1 - \delta \bar{a}_x = 1 - (0.04)(9.36) = 0.6256$$

$$10,000\bar{A}_x^{03} = 6256$$

Alternative Solution:

First note that $\bar{a}_x^{00} + \bar{a}_x^{01} + \bar{a}_x^{02} + \bar{a}_x^{03} = \bar{a}_{\overline{\infty}|} = \dfrac{1}{\delta} = 25.00 \Rightarrow \bar{a}_x^{03} = 15.64$

Also, we have that

$$\bar{a}_x^{03} = \int_0^\infty \left({}_t p_x^{00} \mu_{x+t}^{03} \bar{a}_{x+t}^{33} e^{-\delta t} + {}_t p_x^{01} \mu_{x+t}^{13} \bar{a}_{x+t}^{33} e^{-\delta t} + {}_t p_x^{02} \mu_{x+t}^{23} \bar{a}_{x+t}^{33} e^{-\delta t} \right) dt$$

Note that $\bar{a}_{x+t}^{33} = \dfrac{1}{\delta}$ since state 3 is an absorbing state – the annuity payable while the life is in state 3 given that they are already in state 3 is a perpetuity.

So

$$\begin{aligned}\bar{a}_x^{03} &= \frac{1}{\delta}\int_0^\infty \left({}_t p_x^{00} \mu_{x+t}^{03} e^{-\delta t} + {}_t p_x^{01} \mu_{x+t}^{13} e^{-\delta t} + {}_t p_x^{02} \mu_{x+t}^{23} e^{-\delta t} \right) dt \\ &= \frac{1}{\delta}\bar{A}_x^{03} \Rightarrow \bar{A}_x^{03} = \delta \times 15.64 = 0.6256\end{aligned}$$

Question # S6.1
Answer: E

Note that as the premium inflation rate is constant,

$$\ddot{a}_B(65,t) = 1 + v_{i^*}\, p_{65} + v_{i^*}^2\ {}_2p_{65} + v_{i^*}^3\ {}_3p_{65} + \cdots \quad \text{which is not a function of } t$$

$$\text{where } i^* = \frac{1+i}{(1.04)(1.03)} - 1 = -0.010456$$

$$\Rightarrow \ddot{a}_B(65,4) = \ddot{a}_B(65,2) = 26.708$$

$$\Rightarrow \quad \ddot{a}_B(63,2) = 1 + v_{i^*}\, p_{63} + v_{i^*}^2\ {}_2p_{63} + v_{i^*}^3\ {}_3p_{63} + \cdots = 1 + v_{i^*}\, p_{63} + v_{i^*}^2\ {}_2p_{63}\, \ddot{a}_B(65,4)$$

$$= 1 + (1-0.010456)^{-1}(1-0.004730) + (1-0.010456)^{-2}(1-0.004730)(1-0.005288)(26.708)$$

$$= 29.01$$

Question # S6.2
Answer: B

Normal cost (NC) assuming retirement at age 60 is

$$\frac{AVTHB}{\text{Projected Years of Service}} = \frac{{}_{10}p_{50}\, v^{10}\, 5000(1+j)^{10}\, \ddot{a}_B(60)}{30}$$

And for age 61 is, similarly, $\dfrac{{}_{11}p_{50}\, v^{11}\, 5000(1+j)^{11}\, c\, \ddot{a}_B(61)}{31}$

$$\text{Where} \quad \ddot{a}_B(x) = \ddot{a}_x\big|_{i^*} \quad i^* = \frac{1+i}{(1+j)c} - 1 = \frac{1.05}{(1.03)(1.0194)} = 0.0$$

$$\Rightarrow \ddot{a}_B(x) = e_x + 1$$

So the NC assuming age 60 retirement is $\dfrac{\left(\dfrac{96,634.1}{98,576.4}\right)(1.05)^{-10}(5000)(1.03)^{10}(26.71+1)}{30} = 3735.3$

And for age 61 retirement is, $\dfrac{\left(\dfrac{96,305.8}{98,576.4}\right)(1.05)^{-11}(5000)(1.03)^{11}(1.0194)(25.80+1)}{31} = 3484.1$

So the NC is $0.5 \times 3735.3 + 0.5 \times 3484.1 = 3609.7$