

1. The Freebourn Stop Loss Insurance Company uses the Expected Loss Ratio Method to set reserves for its stop loss business. During 2020, Freebourn had earned premium of 1,400,000 and paid claims of 250,000. Freebourn expects a loss ratio of 55%.

Determine the reserve that Freebourn should hold at December 31, 2020.

**Solution:**

$$\begin{aligned}\text{Expected claims incurred} &= (\text{Earned Premium})(\text{Expected Loss Ratio}) \\ &= (1,400,000)(0.55) = 770,000\end{aligned}$$

$$\text{Reserve} = (\text{Expected Claims Incurred}) - \text{Losses Already Paid}$$

$$= 770,000 - 250,000 = 520,000$$

2. During the first quarter of 2020, The Miller Insurance Company collect the following premium amounts:

Month	January	February	March
Premium Collected	120,000	150,000	180,000

All premiums are paid on the first day of the month and all premiums are annual premiums.

Calculate the earned premium by Miller during the first quarter of 2020.

**Solution:**

Since the premium for January was collected on January 1 and was for a whole year (12 months), we have earned 3 months of the premium as 3 months have passed so for January 1.

$$\text{Earned Premium} = (120,000) \left( \frac{3}{12} \right) = 30,000.$$

Since the premium for February was collected on February 1 and was for a whole year (12 months), we have earned 2 months of the premium as 2 months have passed so for February

$$\text{Earned Premium} = (150,000) \left( \frac{2}{12} \right) = 25,000.$$

Since the premium for March was collected on March 1 and was for a whole year (12 months), we have earned 1 month of the premium as 1 month has passed so for March

$$\text{Earned Premium} = (180,000) \left( \frac{1}{12} \right) = 15,000.$$

$$\text{Total Earned Premium} = 30,000 + 25,000 + 15,000 = 70,000$$

3. You are given the following Paid Claims triangle:

Incremental Loss Payments by Development Year				
Accident Year	Development Year			
	0	1	2	3
2016	15,000	10,000	4,000	2,000
2017	20,000	18,000	6,000	
2018	25,000	12,000		
2019	30,000			

There is no further development after year 3.

- Calculate the loss reserve on December 31, 2019 using the chain ladder method with arithmetic average loss development factors.
- Calculate the loss reserve on December 31, 2019 using the chain ladder method with volume weighted average loss development factors.

**Solutions to a. and b.**

There is a spreadsheet on the website with all the formulas if you cannot follow these tables.

Cumulative Loss Payments by Development Year				
Accident Year	Development Year			
	0	1	2	3
2016	15,000	25,000	29,000	31,000
2017	20,000	38,000	44,000	
2018	25,000	37,000		
2019	30,000			

Age-to-Age Paid Loss Development Factors Based on Cumulative Payments			
Ratio of Successive Development Years			
Accident Year	1/0	2/1	3/2
2016	1.667	1.160	1.069
2017	1.900	1.158	
2018	1.480		
Average	1.682	1.159	1.069
Vol Wgted	1.667	1.159	1.069

Estimated Paid Losses and Loss Reserves by Accident Year						
Based on Average Paid Loss Development Factors Derived Above						
	Development Year					
Accident Year	1	2	3	Estimated Ultimate Losses	Paid to Date	Estimated Loss Reserve
2016				31,000	31,000	0
2017			47,034	47,034	44,000	3,034
2018		42,881	45,838	45,838	37,000	8,838
2019	50,467	58,488	62,522	62,522	30,000	32,522
Total				186,395	142,000	44,395

Estimated Paid Losses and Loss Reserves by Accident Year						
Based on Volume Weighted Paid Loss Development Factors Derived Above						
	Development Year					
Accident Year	1	2	3	Estimated Ultimate Losses	Paid to Date	Estimated Loss Reserve
2016				31,000	31,000	0
2017			47,034	47,034	44,000	3,034
2018		42,873	45,830	45,830	37,000	8,830
2019	50,000	57,937	61,932	61,932	30,000	31,932
Total				185,796	142,000	43,796

- c. What is the total amount of claims paid in 2019?

**Solution:**

The total claims paid in 2019 is the bottom diagonal.

Total claims = 30,000 + 12,000 + 6,000 + 2,000 = 50,000

- d. If the earned premium for 2016 was 50,000, calculate the loss ratio for 2016.

**Solution:**

$$\text{Loss Ratio} = \frac{\text{Expected total loss paid}}{\text{Earned Premium}} = \frac{15,000 + 10,000 + 4,000 + 2,000}{50,000}$$

$$= \frac{31,000}{50,000} = 0.62$$

4. The following table shows the link ratios for cumulative payments based on the chain ladder method:

Development Years	Link Ratio
1/0	2.00
2/1	1.25
3/2	1.10
4/3	1.05

There is no further development after four years.

The following table shows the total amount of claims paid through the end of December 2019:

Accident Year	Cumulative Claim Payment Through 12/31/2019
2016	10,000
2017	8000
2018	6000
2019	4000

Calculate the reserve using the chain ladder method for December 31, 2019.

**Solution:**

Accident Year	Cumulative Claim Payment Through 12/31/2019	Cumulative Lag Factor	Estimated Total Loss
2016	10,000	1.05	10,500.00
2017	8000	(1.05)(1.10)	9,240.00
2018	6000	(1.05)(1.10)(1.25)	8,662.50
2019	4000	(1.05)(1.10)(1.25)(2.00)	11,550.00
Total			39,952.50

**Loss Reserve = Estimated Total Loss – Claims Already Paid = 39,952.50 – 28,000 = 11,952.50**

5. The following table shows the link ratios for cumulative payments based on the chain ladder method:

Development Years	Link Ratio
1/0	2.00
2/1	1.25
3/2	1.10
4/3	1.05

There is no further development after four years.

For accident year 2018, the earned premium was 100,000. The expected loss ratio was 0.70. The claims paid during 2018 and 2019 totaled 50,000.

For the claims from accident year 2018, determine the reserves using:

- a. The Loss Ratio method

**Solution:**

Expected Total Losses = (Earned Premium)(Expected Loss Ratio)

$$= (100,000)(0.70) = 70,000$$

Reserve = Expected Total Losses – Paid Losses to Date =

$$70,000 - 50,000 = 20,000$$

- b. The claim ladder method

**Solution:**

Expected Total Losses = (Paid Losses to Date)( $f_{ult}$ )

$$= (50,000)[(1.25)(1.10)(1.05)] = 72,187.50$$

Reserve = Expected Total Losses – Paid Losses to Date =

$$72,187.50 - 50,000 = 22,187.50$$

- c. The Bornhuetter-Ferguson method

**Solution:**

$$\text{Reserve} = (\text{Expected Total Losses Under the Loss Ratio Method}) \left( 1 - \frac{1}{f_{ult}} \right)$$

$$f_{ult} = [(1.25)(1.10)(1.05)] = 1.44375$$

(Expected Total Losses Under the Loss Ratio Method) = 70,000 from Part a.

$$\text{Reserve} = (70,000) \left( 1 - \frac{1}{1.44375} \right) = 21,515.15$$

6. You are given the following information:

Accident Year	Earned Premium	Expected Loss Ratio	Cumulative Loss Payments through Development Month			
			12	24	36	48
AY5	19,000	0.90	4,850	9,700	14,100	16,200
AY6	20,000	0.85	5,150	10,300	14,900	
AY7	21,000	0.91	5,400	10,800		
AY8	22,000	0.88	7,200			

There is no development past 48 months.

Calculate the actuarial reserve using the Bornhuetter-Ferguson method and volume weighted average loss development factors.

**Solution:**

$$f(\infty / 4) = 1 \implies f_{Ult}^{AY5} = 1$$

$$f(4 / 3) = \frac{16,200}{14,100} = 1.14894 \implies f_{Ult}^{AY6} = (1.14894)(1) = 1.14894$$

$$f(3 / 2) = \frac{14,100 + 14,900}{9,700 + 10,300} = 1.45 \implies f_{Ult}^{AY7} = (1.45)(1.14894)(1) = 1.66596$$

$$f(2 / 1) = \frac{9,700 + 10,300 + 10,800}{4,850 + 5,150 + 5,400} = 2 \implies f_{Ult}^{AY8} = (2)(1.45)(1.14894)(1) = 3.33123$$

Expected Claims Based On Loss Ratio Method = (Earned Premium)(Expected Loss Ratio)

$$\text{Expected Claims - AY5} = (19,000)(0.90) = 17,100$$

$$\text{Expected Claims - AY6} = (20,000)(0.85) = 17,000$$

$$\text{Expected Claims - AY7} = (21,000)(0.91) = 19,110$$

$$\text{Expected Claims - AY8} = (22,000)(0.88) = 19,360$$

$$\text{Res - AY5} = (\text{Expected Claims}) \left( 1 - \frac{1}{f_{Ult}^{AY5}} \right) = (17,100) \left( 1 - \frac{1}{1} \right) = 0$$

$$\text{Res - AY6} = (\text{Expected Claims}) \left( 1 - \frac{1}{f_{Ult}^{AY6}} \right) = (17,000) \left( 1 - \frac{1}{1.14894} \right) = 2203.75$$

$$\text{Res - AY7} = (\text{Expected Claims}) \left( 1 - \frac{1}{f_{Ult}^{AY7}} \right) = (19,110) \left( 1 - \frac{1}{1.66596} \right) = 7639.14$$

$$\text{Res - AY8} = (\text{Expected Claims}) \left( 1 - \frac{1}{f_{Ult}^{AY8}} \right) = (19,360) \left( 1 - \frac{1}{3.33123} \right) = 13,548.33$$

$$\text{Total Reserve} = 0 + 2203.75 + 7639.14 + 13,548.33 = 23,391.22$$



7. You are given the following information for a given accident year for a block of business with case reserves:

- i. Earned Premium = 1,000,000
- ii. Expected Loss Ratio = 75%
- iii. Claims paid to date = 450,000
- iv. Case Reserves = 250,000
- v.  $f_{Ult} = 1.15$

Find the Total Actuarial Reserve using:

- a. The Loss Ratio Method

**Solution:**

Expected Total Losses = (Earned Premium)(Expected Loss Ratio)

$$= (1,000,000)(0.75) = 750,000$$

Reserve = Expected Total Losses – Paid Losses to Date =

$750,000 - 450,000 = 300,000$  is the Total Actuarial Reserves which are split as Case reserves of 250,000 and IBNR reserves of 50,000

- b. The Chain Ladder Method

**Solution:**

Expected Total Losses = (Paid Losses to Date + Case Reserves)( $f_{Ult}$ )

$$= (700,000)(1.15) = 805,000$$

Reserve = Expected Total Losses – Paid Losses to Date =

$805,000 - 450,000 = 355,000$  is the Total Actuarial Reserves which are split as Case reserves of 250,000 and IBNR reserves of 105,000

c. The Bornhuetter-Ferguson Method

**Solution:**

Expected Total Losses Under Loss Ratio Method = (Earned Premium)(Expected Loss Ratio)

$$= (1,000,000)(0.75) = 750,000$$

$$\text{IBNR Reserve} = (\text{Expected Total Losses}) \left( 1 - \frac{1}{f_{ult}} \right) = (750,000) \left( 1 - \frac{1}{1.15} \right) = 97,826$$

$$\text{Total Reserve} = \text{IBNR Reserve} + \text{Case Reserve} = 97,826 + 250,000 = 347,826$$

8. Below is the reserve development from Table 3.5:

**Table 3.5**

<b>Estimated Paid Losses and Loss Reserves by Accident Year, Based on 5-Year Average Paid Loss Development Factors Derived in Table 3.3</b>										
Accident Year	Development Year							Estimated Ultimate Losses	Paid- To- Date	Estimated Loss Reserve
	1	2	3	4	5	6	7			
AY1								14,032	14,032	0
AY2							14,197	14,197	14,015	182
AY3						18,031	18,266	18,266	17,506	760
AY4					22,614	23,293	23,595	23,595	21,599	1,996
AY5				25,948	27,167	27,982	28,346	28,346	23,827	4,519
AY6			24,979	27,202	28,481	29,335	29,716	29,716	21,478	8,238
AY7		27,282	31,729	34,553	36,177	37,262	37,747	37,747	22,253	15,494
AY8	24,851	30,467	35,433	38,586	40,400	41,612	42,153	<u>42,153</u>	<u>15,162</u>	<u>26,991</u>
<b>TOTAL</b>								208,052	149,872	58,180

Using an annual effective interest rate of 6% and assuming future loss payments are made in the middle of the year, calculate the discounted reserve using the values in Table 3.5.

**Solution:**

See spreadsheet for detailed formulas if you do not match these numbers.

[illegible]