RATE MAKING HOMEWORK SOLUTIONS

1. You are setting rates for the time period of November 1, 2020 to November 1, 2021.

   You are given the following data:

Rate Making Data			
Accident Year	Earned Exposure Units	Ultimate Losses (Fully Developed)	Number of Incurred Claims
2015	500,000	96,030,000	30,000
2016	550,000	114,296,875	34,375
2017	600,000	122,276,400	37,200
2018	650,000	142,233,650	41,275
2019	700,000	157,525,200	44,100

   You want to use this data to project loss costs with trend to the midpoint of the of the rate making period.

   a. Calculate the values in the following table:

Average Frequency, Average Severity, and Loss Cost				
Accident Year	Average Claim Frequency	Average Loss Severity	Loss Cost Per Unit Exposure	Ln(Loss Cost)
2015	30,000/500,000 = 0.060	96,030,000/30,000 = 3201	96,030,000/500,000 = 192.06	LN(192.06) = 5.2578
2016	0.0625	3325	207.81	5.3366
2017	0.0620	3287	203.79	5.3171
2018	0.0635	3446	218.82	5.3883
2019	0.0630	3572	225.04	5.4163

   You determine that the least squares line fitting the natural log of the loss cost is:

$$Y = 5.2695 + 0.03685X$$

   b. Use this equation to project the loss cost to the midpoint of the rate making period.

   Solution:

   The midpoint for the rate period of November 1, 2020 to November 1, 2021 is May 1, 2021. The midpoint for accident year 2015 is July 1, 2015. The difference is 5 10/12.

$$Y = 5.2695 + 0.03685(5\ 10/12) = 5.48446$$

$$\text{Trended Loss Cost} = e^{5.48446} = 240.92$$

You know that the above formula indicates that the loss costs are growing at an exponential growth rate of 3.685%.

c. Use the lost cost for 2018 and the exponential growth rate to project the lost cost to the midpoint of the rate making period.

Solution:

The midpoint for the rate period of November 1, 2020 to November 1, 2021 is May 1, 2021. The midpoint for accident year 2018 is July 1, 2018. The difference is 2 10/12.

$$\text{Trended Loss Cost} = (\text{Loss Cost for 2018})e^{0.036852(2\ 10/12)} = (218.82)e^{0.036852(2\ 10/12)} = 242.90$$

d. Use the lost cost for 2019 and the exponential growth rate to project the lost cost to the midpoint of the rate making period.

Solution:

The midpoint for the rate period of November 1, 2020 to November 1, 2021 is May 1, 2021. The midpoint for accident year 2019 is July 1, 2019. The difference is 1 10/12.

$$\text{Trended Loss Cost} = (\text{Loss Cost for 2019})e^{0.036852(1\ 10/12)} = (225.04)e^{0.036852(1\ 10/12)} = 240.77$$

2. You are setting rates for a short term insurance product. You are given the following data:

Calendar Year	Earned Premium
2017	4000
2018	5000
2019	6000

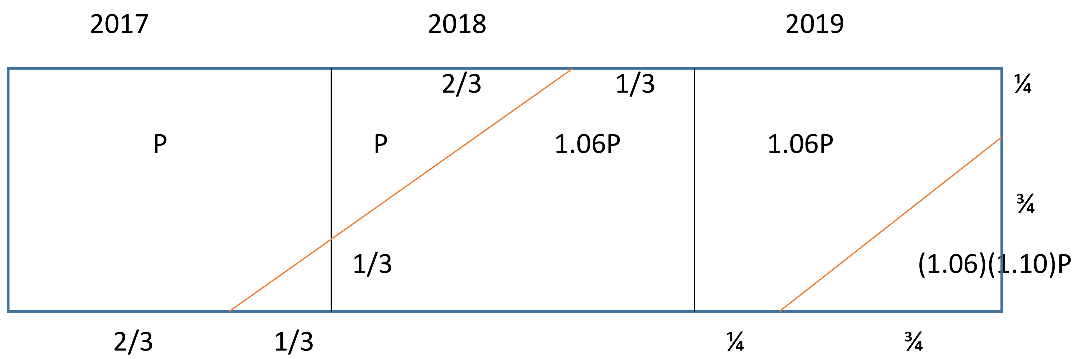
Assume that all policies are one year policies and the policies are issued uniformly throughout the year.

The following rate changes have occurred:

Date	Rate Change
September 1, 2017	6%
April 1, 2019	10%

Using the parallelogram method, calculate the earned premium for 2017, 2018, and 2019 based on current rates.

Solution:



2017

$$\text{Weighted Premium} = \left[\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}{2}\right]1.06P + \left\{1 - \left[\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}{2}\right]\right\}P = 1.0033333P$$

$$\text{Current Rate Earned Premium} = (4000)\left(\frac{(1.06)(1.10)}{1.0033333}\right) = 4648.50$$

2018

$$\text{Weighted Premium} = \left[\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)}{2}\right]P + \left\{1 - \left[\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)}{2}\right]\right\}1.06P = 1.046667P$$

$$\text{Current Rate Earned Premium} = (5000)\left(\frac{(1.06)(1.10)}{1.046667}\right) = 5570.06$$

2019

$$\text{Weighted Premium} = \left[\frac{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}{2}\right](1.06)(1.10)P + \left\{1 - \left[\frac{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}{2}\right]\right\}1.06P = 1.0898125$$

$$\text{Current Rate Earned Premium} = (6000)\left(\frac{(1.06)(1.10)}{1.0898125}\right) = 6419.45$$

3. You are determining the new average gross premium rate based on the following data:

Expected Effective Period Incurred Losses	10,000,000
Earned Exposure Units	250,000
Earned Premium at Current Rates	16,000,000
Fixed Expenses	1,000,000
Permissible Loss Ratio	0.625

a. Use the loss cost method to calculate the new average gross premium rate.

Solution:

$$\text{Present Average Manual Rate} = \frac{16,000,000}{250,000} = 64.00$$

$$\text{Fixed Expense Per Exposure Unit} = \frac{1,000,000}{250,000} = 4.00$$

$$\text{Expected Effective Loss Cost} = \frac{10,000,000}{250,000} = 40.00$$

$$\text{New Average Gross Rate} = \frac{40.00 + 4.00}{0.625} = 70.40$$

b. Use the loss ratio method to calculate the new average gross premium rate.

$$\text{Expected Effective Loss Ratio} = \frac{10,000,000}{16,000,000} = 0.625$$

$$\text{Indicated Rate Change Factor} = \frac{0.625 + 4/64}{0.625} - 1 = 0.10$$

$$\text{New Average Gross Rate} = (64)(1.10) = 70.40$$

4. During the rate making process, you have been asked to use the following data to determine revised differentials for Class B and Class C.

Class	Existing Differential	Experience Period Loss Ratio at Current Rates	Experience Period Loss Cost
A	1.00	0.60	40.00
B	0.80	0.66	35.20
C	1.25	0.55	45.83

a. Use the loss ratio method to determine the indicated differentials for Class B and Class C.

Solution:

$$\text{Indicated Differential for Class B} = (0.80)\left(\frac{0.66}{0.60}\right) = 0.88$$

$$\text{Indicated Differential for Class C} = (1.25)\left(\frac{0.55}{0.60}\right) = 1.15$$

b. Use the loss cost method to determine the indicated differentials for Class B and Class C.

Solution:

$$\text{Indicated Differential for Class B} = \left(\frac{35.20}{40.00}\right) = 0.88$$

$$\text{Indicated Differential for Class C} = \left(\frac{45.83}{40}\right) = 1.15$$

5. You are setting rates and you need to balance the indicated rate increase with the indicated changes in differentials. You have following information:

Class Differentials			
Class	**Existing Differential**	**Proposed Differential**	**Earned Exposure Units**
A	1.00	1.00	150
B	0.80	0.88	70
C	1.25	1.15	30
			250

Assume that the current premium is 64 and that the indicated base rate change is an increase of 10%.

a. Calculate the total premium before any changes.

Solution:

$$64\left[(1.00)(150)+(0.80)(70)+(1.25)(30)\right]=15,584$$

b. Calculate the total premium after changes.

Solution:

$$70.40\left[(1.00)(150)+(0.88)(70)+(1.15)(30)\right]=17,325.44$$

$$\frac{17,325.44}{15,584.00}=1.1117 \text{ ==> } 11.17\% \text{ Increase in rates}$$

c. Calculate the off balance factor.

$$\text{Off Balance Factor} = \frac{\left[(1.00)(150)+(0.80)(70)+(1.25)(30)\right]}{\left[(1.00)(150)+(0.88)(70)+(1.15)(30)\right]}=1.010678$$

Or

$$\text{Off Balance Factor} = \frac{1.1117}{1.1}=1.01064 \quad \text{The difference is rounding of 1.1117}$$

d. Calculate the balance back factor.

Solution:

$$\text{Balance Back Factor} = \frac{1}{\text{Off Balance Factor}} = \frac{1}{1.010678} = 0.989435$$

e. Calculate the increase in base rates that would bring the rates into balance.

Solution:

$$\text{Increase} = (1.10)(0.989435) = 1.08838$$

6. You are to calculate rates effective for the two years beginning July 1, 2020 for one year policies. You use the loss cost method and the following data:

Accident Year	**Earned Exposure Units**	**Ultimate Losses**
2017	2000	1,600,000
2018	2200	1,815,000

You are also given:

i. Rates are based on a weighted average of 25% of the 2017 data and 75% of the 2018 data.

ii. Trend is 5%

iii. Fixed Expenses per exposure is 60

iv. The permissible loss ratio is 75%.

Calculate the new indicated gross rate.

Solution:

Loss Cost 2017 = $\frac{1,600,000}{2000} = 800$

Midpoint for policies issued in 2017 is 7-1-2017. Policies under the new rates will be issued 7-1-2020 to 7-1-2022 and will run from 7-1-2020 to 7-1-2023 so the midpoint is 1/1/2022.

Time from 7-1-2017 to 1/1/2022 is 4.5 years. Trend is $(1.05)^{4.5}$.

Loss Cost 2018 = $\frac{1,815,000}{2200} = 825$

Midpoint for policies issued in 2018 is 7-1-208. Policies under the new rates will be issued 7-1-2020 to 7-1-2022 and will run from 7-1-2020 to 7-1-2023 so the midpoint is 1/1/2022.

Time from 7-1-2018 to 1/1/2022 is 3.5 years. Trend is $(1.05)^{3.5}$.

Loss Cost = $(0.25)(800)(1.05)^{4.5} + (0.75)(825)(1.05)^{3.5} = 983.07$

Indicated Gross Rate = $\frac{983.07 + 60}{0.75} = 1390.76$

7. *You are given the following information to determine a rate change:

Accident Year	Earned Premiums at Current Rates	Incurred Losses	Weight Given to Accident Year
AY8	4252	2260	40%
AY9	5765	2610	60%

You are also given:

i. Trend factor is 7% per year

ii. Loss development factor to Ultimate is 1.08 for AY8 and 1.18 for AY9.

iii. Permissible Loss Ratio is 65.7%

iv. All policies are one year policies and assumed to be issued uniformly throughout the year. Rate are effective for one year.

v. Proposed rate effective date is July 1, CY10.

Calculate the indicated rate change as a percentage.

Solution:

Developed Claims for AY8 $= (2260)(1.08) = 2440.80$

Midpoint for AY8 is 7-1-AY8. Policies under the new rates will be issued 7-1-CY10 to 7-1-CY11 and will run from 7-1-CY10 to 7-1-CY12 so the midpoint is 7-1-CY11. There are 3 years of trend.

Trended and Developed Claims for AY8 $= (2260)(1.08)(1.07)^3 = 2990.08$.

Loss Ratio for AY8 = $\dfrac{2990.08}{4252} = 0.7032$

Developed Claims for AY9 $= (2610)(1.18) = 3079.80$

Midpoint for AY9 is 7-1-AY9. Policies under the new rates will be issued 7-1-CY10 to 7-1-CY11 and will run from 7-1-CY10 to 7-1-CY12 so the midpoint is 7-1-CY11. There are 2 years of trend.

Trended and Developed Claims for AY9 $= (2610)(1.18)(1.07)^2 = 3526.06$

Loss Ratio for AY9 = $\dfrac{3526.06}{5765} = 0.6116$

Weighted Average Loss Ratio = $(0.4)(0.7032) + (0.6)(0.6116) = 0.6483$

Indicated Rate Change = $\dfrac{0.6483}{0.657} - 1 = -0.0133$

8. *You are given data for three territories as follows:

Territory	Earned Premium at Current Rates	Incurred Loss & ALEA	Claim Count	Current Relativity
1	520,000	420,000	600	0.60
2	1,680,000	1,250,000	1320	1.00
3	450,000	360,000	390	0.52
Total	2,650,000	2,030,000	2310	

The full credibility standard is 1082 claims and partial credibility is calculated using the square root rule. The compliment of credibility is applied to the existing relativity factor.

Calculate the indicated territorial relativity for Territory 3.

Solution:

Using the loss ratio method, the loss ratio for Territory 2 is $\frac{1250}{1680} = 0.744048$ and for Territory 3 is $\frac{360}{450} = 0.80.$

Indicated relativity for Territory 3 before credibility is $(0.52)\left(\frac{0.8}{0.744048}\right) = 0.5591$

Credibility factor = $\sqrt{\frac{390}{1082}} = 0.60.$

Indicated relativity for Territory 3 with credibility is $(0.60)(0.5591) + (0.40)(0.52) = 0.5435$

9.