Homework Problems
Stat 479

Chapter 13

80. You are given the following random sample of 3 data points from a population with a Pareto distribution with $\theta = 70$:

    X: 15 27 43

    Calculate the maximum likelihood estimate for α.

81. * You are given:

    a. Losses follow an exponential distribution with mean θ.

    b. A random sample of 20 losses is distributed as follows:

Range	**Frequency**
[0,1000]	7
(1000, 2000)	6
(2000,∞)	7

    Calculate the maximum likelihood estimate of θ.

82. * You are given the following:

    i. The random variable X has the density function $f(x) = \{2(\theta - x)\}/\theta^2$, $0 < x < \theta$

    ii. A random sample of two observations of X yields values of 0.50 and 0.90.

    Determine the maximum likelihood estimate for θ.

83. * You are given the following:
    a. The random variable X follows the exponential distribution with parameter θ.
    b. A random sample of three observations of X yields values of 0.30, 0.55, and 0.80

    Determine the maximum likelihood estimate of θ.

84. * Ten laboratory mice are observed for a period of five days. Seven mice die during the observation period, with the following distribution of deaths:

Time of Death in Days	Number of Deaths
2	1
3	2
4	1
5	3

The lives in the study are subject to an exponential survival function with mean of θ.

Calculate the maximum likelihood estimate of θ.

85. * A policy has an ordinary deductible of 100 and a policy limit of 1000. You observe the following 10 payments:

15 50 170 216 400 620 750 900 900 900

An exponential distribution is fitted to the ground up distribution function, using the maximum likelihood estimate.

Determine the estimated parameter θ.

86. * Four lives are observed from time $t = 0$ until death. Deaths occur at $t = 1, 2, 3$, and 4. The lives are assumed to follow a Weibull distribution with $\tau = 2$.

Determine the maximum likelihood estimator for θ.

87. * The random variable X has a uniform distribution on the interval $[0,\theta]$. A random sample of three observations of X are recorded and grouped as follows:

Interval	Number of Observations
$[0,k)$	1
$[k,5)$	1
$[5,\theta]$	1

Calculate the maximum likelihood estimate of θ

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88. * A random sample of three claims from a dental insurance plan is given below:

    225 525 950

    Claims are assumed to follow a Pareto distribution with parameters θ = 150 and α.

    Determine the maximum likelihood estimate of α.

89. * The following claim sizes are experienced on an insurance coverages:

    100 500 1,000 5,000 10,000

    You fit a normal distribution to this experience using maximum likelihood.

    Determine the resulting estimate of σ.

90. You have the following four claims:

    10 20 30 40

    You believe that the claims for this insurance are distributed as an exponential distribution with a mean of θ.

    You use the maximum likelihood estimate to determine $\hat{\theta}$.

    Determine the variance of $\hat{\theta}$.

91. * Losses from a group health policy follow an exponential distribution with unknown mean. A sample of losses is:

    100 200 400 800 1400 3100

    Use the delta method to

    a. Approximate the variance of the maximum likelihood estimator of $S(1500)$.
    b. Determine the approximate 95% confidence interval for $S(1500)$.

92. You fit an inverse exponential sample of claims using maximum likelihood to estimate θ. The resulting estimate of θ is 2.07. The variance of the estimate is 0.00264.

    Let $Y = E[X^{-1}]$.

    Use the delta method to estimate the variance of Y.

93. You are given the following data from a sample:

k	n_k
0	20
1	25
2	30
3	15
4	8
5	2

Use this data for the next four problems.

Assuming a Poisson distribution, approximate the 90% confidence interval for the true value of λ.

94. You are given the following 20 claims:

10, 40, 60, 65, 75, 80, 120, 150, 170, 190, 230, 340, 430,
440, 980, 600, 675, 950, 1250, 1700

The data is being modeled using an exponential distribution with $\theta = 427.5$.

Calculate D(200).

95. You are given the following 20 claims:

10, 40, 60, 65, 75, 80, 120, 150, 170, 190, 230, 340, 430,
440, 980, 600, 675, 950, 1250, 1700

The data is being modeled using an exponential distribution with $\theta = 427.5$.

You are developing a *p-p plot* for this data.

What are the coordinates for $x_7 = 120$.

96. Mark the following statements True or False with regard to the Kolmogorov-Smirnov test:

The Kolmogorov-Smirnov test may be used on grouped data as well as individual data.

If the parameters of the distribution being tested are estimated, the critical values do not need to be adjusted.

If the upper limit is less than ∞, the critical values need to be larger.

97. Balog's Bakery has workers' compensation claims during a month of:

    100, 350, 550, 1000

    Balog's owner, a retired actuary, believes that the claims are distributed exponentially with $\theta = 500$.

    He decides to test his hypothesis at a 10% significance level.

    Calculate the Kolmogorov-Smirnov test statistic.

    State the critical value for his test and state his conclusion.

98. * The observations of 1.7, 1.6, 1.6, and 1.9 are taken from a random sample. You wish to test the goodness of fit of a distribution with probability density function given by $f(x) = 0.5x$ for $0 \le x \le 2$.

    Using the Kolmogorov-Smirnov statistic, which of the following should you do?

    a. Accept at both levels
    b. Accept at the 0.01 level but reject at the 0.10 level
    c. Accept at the 0.10 level but reject at the 0.01 level
    d. Reject at both levels
    e. Cannot be determined.

99. * Two lives are observed beginning at time $t = 0$. One dies at time 5 and the other dies at time 9. The survival function $S(t) = 1 - 0.1t$ is hypothesized.

    Calculate the Kolmogorov-Smirnov statistic.

100. * From a laboratory study of nine lives, you are given:
    a. The times of death are 1, 2, 4, 5, 5, 7, 8, 9, 9
    b. It has been hypothesized that the underlying distribution is uniform with $\omega = 11$.

    Calculate the Kolmogorov-Smirnov statistic for the hypothesis.

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101. You are given the following data:

Claim Range	**Count**
0-100	30
100-200	25
200-500	20
500-1000	15
1000+	10

H_0: The data is from a Pareto distribution.
H_1: The data is not from a Pareto distribution.

Your boss has used the data to estimate the parameters as $\alpha = 4$ and $\theta = 1200$.

Calculate the chi-square test statistic.

Calculate the critical value at a 10% significance level.

State whether you would reject the Pareto at a 10% significance level.

102. During a one-year period, the number of accidents per day in the parking lot of the Steenman Steel Factory is distributed:

Number of Accidents	**Days**
0	220
1	100
2	30
3	10
4+	5

H_0: The distribution of the number of accidents is distributed as Poison with a mean of 0.625.

H_1: The distribution of the number of accidents is not distributed as Poison with a mean of 0.625.

Calculate the chi-square statistic.

Calculate the critical value at a 10% significance level.

State whether you would reject the H_0 at a 10% significance level.

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103. * You are given the following random sample of automobile claims:

54	140	230	560	600	1,100	1,500	1,800	1,920	2,000
2,450	2,500	2,580	2,910	3,800	3,800	3,810	3,870	4,000	4,800
7,200	7,390	11,750	12,000	15,000	25,000	30,000	32,200	35,000	55,000

You test the hypothesis that automobile claims follow a continuous distribution F(x) with the following percentiles:

x	**310**	**500**	**2,498**	**4,876**	**7,498**	**12,930**
F(x)	0.16	0.27	0.55	0.81	0.90	0.95

You group the data using the largest number of groups such that the expected number of claims in each group is at least 5.

Calculate the Chi-Square goodness-of-fit statistic.

104. Based on a random sample, you are testing the following hypothesis:

H_0: The data is from a population distributed binomial with m = 6 and q = 0.3.

H_1: The data is from a population distributed binomial.

You are also given:

$L(\boldsymbol{\theta}_0) = 0.1$ and $L(\boldsymbol{\theta}_1) = 0.3$

Calculate the test statistic for the Likelihood Ratio Test

State the critical value at the 10% significance level

105. State whether the following are true or false
    i. The principle of parsimony states that a more complex model is better because it will always match the data better.
    ii. In judgment-based approaches to determining a model, a modeler's experience is critical.
    iii. In most cases, judgment is required in using a score-based approach to selecting a model.

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106. A random number generated from a uniform distribution on (0, 1) is 0.6. Using the inverse transformation method, calculate the simulated value of X assuming X is distributed Pareto with $\alpha = 3$ and $\theta = 2000$.

107. * You are given that $f(x) = \dfrac{x^2}{9}$ for $0 \le x \le 3$.

You are to simulate three observations from the distribution using the inversion method. The follow three random numbers were generated from the uniform distribution on [0,1]:

0.008 0.729 0.125

Using the three simulated observations, estimate the mean of the distribution.

108. * You are to simulate four observations from a binomial distribution with two trials and probability of success of 0.30. The following random numbers are generated from the uniform distribution on [0,1]:

0.91 0.21 0.72 0.48

Determine the number of simulated observations for which the number of successes equals zero.

109. Kyle has an automobile insurance policy. The policy has a deductible of 500 for each claim. Kyle is responsible for payment of the deductible.

The number of claims follows a Poison distribution with a mean of 2. Automobile claims are distributed exponentially with a mean of 1000.

Kyle uses simulation to estimate the claims. A random number is first used to calculate the number of claims. Then each claim is estimated using random numbers using the inverse transformation method.

The random numbers generated from a uniform distribution on (0, 1) are 0.7, 0.1, 0.5, 0.8, 0.3, 0.7, 0.2.

Calculate the simulated amount that Kyle would have to pay in the first year.

110. * Insurance for a city's snow removal costs covers four winter months.

You are given:

- There is a deductible of 10,000 per month.
- The insurer assumes that the city's monthly costs are independent and normally distributed with mean of 15,000 and standard deviation of 2000.
- To simulate four months of claim costs, the insurer uses the inversion method (where small random numbers correspond to low costs).
- The four numbers drawn from the uniform distribution on [0,1] are:

0.5398 0.1151 0.0013 0.7881

Calculate the insurer's simulated claim cost.

111. * Annual dental claims are modeled as a compound Poisson process where the number of claims has mean of 2 and the loss amounts have a two-parameter Pareto distribution with $\theta = 500$ and $\alpha = 2$.

An insurance pays 80% of the first 750 and 100% of annual losses in excess of 750.

You simulate the number of claims and loss amounts using the inversion method.

The random number to simulate the number of claims is 0.80. The random numbers to simulate the amount of claims are 0.60, 0.25, 0.70, 0.10, and 0.80.

Calculate the simulated insurance claims for one year.

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112.

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Answers

80. 3.002
81. 1996.90
82. 1.5
83. 0.55
84. 6
85. 703
86. 2.7386
87. 7.5
88. 0.6798
89. 3772.21
90. 156.25
91.
    a. 0.01867
    b. 0.22313 ± 0.03659
92. 0.000144
93. (1.50426, 1.93574)
94. 0.1264
95. (1/3,0.2447)
96. All Statements are false.
97. 0.2534, 0.61, Cannot reject,
98. B
99. 0.5
100. 0.1818
101. $\chi^2 = 5.5100$; critical value = 4.605; Reject H_0
102. $\chi^2 = 18.5$; critical value = 7.779; Reject H_0
103. 6.6586
104. T = 2.197; critical value = 4.605
105. False; True; True
106. 714.42
107. 1.6
108. 2
109. 1105.36
110. 14,400
111. 630.79
112.