Homework Problems
Stat 479

Chapter 2

1. Model 1 in the table of six models handed out in class is a uniform distribution from 0 to 100. Determine what the table entries would be for a generalized uniform distribution covering the range from a to b where a < b. In other words, what is $F(x)$, the support, type of distribution, $S(x)$, $f(x)$, $p(x)$, $h(x)$ and the mode.

Solution:

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & \text{for } a \le x \le b \\ 1 & \text{for } x > b \end{cases}$$

$Support \implies a \le x \le b$

$Type \implies continuous$

$$S(x) = 1 - F(x) = \begin{cases} 1 & \text{for } x < a \\ 1 - \dfrac{x-a}{b-a} = \dfrac{b-a-x+a}{b-a} = \dfrac{b-x}{b-a} & \text{for } a \le x \le b \\ 0 & \text{for } x > b \end{cases}$$

$$f(x) = \frac{1}{b-a} \qquad \text{for } a \le x \le b$$

$p(x) = undefined$

$$h(x) = \frac{f(x)}{S(x)} = \frac{\dfrac{1}{b-a}}{\dfrac{b-x}{b-a}} = \frac{1}{b-x}$$

Mode ==> All points are a mode or there is no mode.

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2. Let X be a discrete random variable with probability function $p(x) = 2(1/3)^x$ for $x = 1, 2, 3, \ldots$

What is the probability that X is odd?

Solution:

$$p(1)=2\left(\frac{1}{3}\right) \quad p(3)=2\left(\frac{1}{3}\right)^3 \quad p(5)=2\left(\frac{1}{3}\right)^5 \quad \ldots$$

$$\Pr[\text{X is odd}] = p(1)+p(3)+p(5)+\ldots = 2\left[\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^5+\ldots\right]$$

$$=2\left[\frac{\frac{1}{3}-0}{1-\frac{1}{9}}\right]=\frac{3}{4}$$

3. * For a distribution where $x \geq 2$, you are given:

- The hazard rate function: $h(x)=\dfrac{z^2}{2x}$, for $x \geq 2$
- $F(5)=0.84$.

Calculate z .

Solution:

$$S(b)=e^{-\int_{-\infty}^{b} h(x)dx} \qquad \text{and} \qquad F(b)=1-S(b)$$

$$\Longrightarrow F(5)=1-S(5)=1-e^{-\int_2^5 \frac{z^2}{2x}dx}=0.84$$

$$\Longrightarrow e^{-\int_2^5 \frac{z^2}{2x}dx}=0.16 \Longrightarrow e^{-\int_2^5 \frac{z^2}{2x}dx}=e^{\left[-\frac{z^2}{2}\bullet\ln x\right]_2^5}=e^{\left[-\frac{z^2}{2}\bullet\ln\frac{5}{2}\right]}=0.16$$

$$\Longrightarrow \ln\left(e^{\left[-\frac{z^2}{2}\bullet\ln\frac{5}{2}\right]}\right)=\ln(0.16) \Longrightarrow \left[-\frac{z^2}{2}\bullet\ln\frac{5}{2}\right]=\ln(0.16) \Longrightarrow z=2$$

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4. $F_X(t) = \dfrac{t^2 - 1}{9999}$ for $1 < t < 100$. Calculate $f_X(50)$.

Solution:

$$f(t) = \frac{d}{dt}F(t) = \frac{d}{dt}\frac{t^2-1}{9999} = \frac{2t}{9999} \Longrightarrow f(50) = \frac{100}{9999}$$

5. You are given that the random variable X is distributed as a Weibull distribution with parameters $\theta = 3$ and $\tau = 0.5$.

Calculate:

a. $\Pr[X \le 5]$

Solution:

$$F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\tau}} = 1 - e^{-\left(\frac{x}{3}\right)^{0.5}}$$

$$\Pr[X \le 5] = F(5) = 1 - e^{-\left(\frac{5}{3}\right)^{0.5}} = 0.725$$

b. $\Pr[3 \le X \le 5]$

Solution:

$$\Pr[3 \le X \le 5] = F(5) - F(3) = 1 - e^{-\left(\frac{5}{3}\right)^{0.5}} - \left(1 - e^{-\left(\frac{3}{3}\right)^{0.5}}\right) = 0.093$$

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6. You are given that the random variable X is distributed as a Geometric distribution with parameters $\beta = 3$.

Calculate:

a. $\Pr[X \leq 5]$

Solution:

$$p_k = \frac{\beta^k}{(1+\beta)^{k+1}} = \frac{3^k}{4^{k+1}} = (0.25)(0.75)^k$$

$$p_0 = 0.25 \quad p_1 = 0.25(0.75) \quad p_2 = 0.25(0.75)^2 \quad \ldots \quad p_5 = 0.25(0.75)^5$$

$$\Pr[X \leq 5] = p_0 + p_1 + \ldots + p_5 = (0.25)\left[1 + 0.75 + \ldots + (0.75)^5\right]$$

$$= (0.25)\left[\frac{1-(0.75)^6}{1-0.75}\right] = 1-(0.75)^6 = 0.82202$$

b. $\Pr[3 \leq X \leq 5]$

Solution:

$$\Pr[3 \leq X \leq 5] = p_3 + p_4 + p_5 = (0.25)\left[(0.75)^3 + (0.75)^4 + (0.75)^5\right] = 0.24390$$

7. A random variable X has a density function $f(x) = \dfrac{4x}{\left(1+x^2\right)^3}$, for $x > 0$.

Determine the mode of X.

Solution:

$f'(x) = 0$ will give us the mode

$$f'(x) = \frac{4}{(1+x^2)^3} + \frac{4x(-3)(2x)}{(1+x^2)^4} = \frac{4}{(1+x^2)^3} - \frac{24x^2}{(1+x^2)^4} = 0$$

$$\Longrightarrow 4(1+x^2) - 24x^2 = 0 \Longrightarrow 4 - 20x^2 = 0$$

$$\Longrightarrow 4 = 20x^2 \Longrightarrow x^2 = 0.2 \Longrightarrow x = \sqrt{0.2} \quad \text{since } x > 0$$