STAT 479
Quiz 1
Fall 2022

September 13, 2022

1. X is distributed as a zero modified Poisson distribution with $\lambda = 2$ and $p_0^M = 0.4$.

   Calculate $e(2)$.

   Solution:

$$e(2) = \frac{E[X] - E[x \wedge 2]}{1 - F(2)}$$

$$p_0^M = 0.4$$

$$p_1^M = p_1^T (1 - p_0^M) = \left[\frac{\lambda}{e^{\lambda} - 1}\right](1 - p_0^M) = \left[\frac{2}{e^2 - 1}\right](0.6) = 0.18782$$

$$p_2^M = p_2^T (1 - p_0^M) = \left[\frac{\lambda^2}{2!(e^{\lambda} - 1)}\right](1 - p_0^M) = \left[\frac{2^2}{2!(e^2 - 1)}\right](0.6) = 0.18782$$

$$1 - F(2) = 1 - p_0^M - p_1^M - p_2^M = 1 - 0.4 - 0.18782 - 0.18782 = 0.22436$$

$$E[X] = E[X^T](1 - p_0^M) = \left[\frac{\lambda}{1 - e^{-\lambda}}\right](1 - p_0^M) = \left[\frac{2}{1 - e^{-2}}\right](0.6) = 1.38782$$

$$E[X \wedge 2] = 0(p_0^M) + 1(p_1^M) + 2(1 - p_0^M - p_1^M)$$

$$= (0)(0.4) + (1)(0.18782) + (2)(1 - 0.4 - 0.18782) = 1.01218$$

$$e(2) = \frac{E[X] - E[x \wedge 2]}{1 - F(2)} = \frac{1.38782 - 1.01218}{0.22436} = 1.6743$$

2. X is distributed as a Pareto distribution with parameters of θ and α. You are also given that $E[X]=2000$ and $Var[X]=8,000,000$.

Calculate $TVaR_{0.9}(X)$.

Solution:

$$E[X]=2000=\frac{\theta}{\alpha-1} \qquad Var[X]=8,000,000=\frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)}$$

$$\frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)}=\frac{\theta^2}{(\alpha-1)^2}\bullet\frac{\alpha}{\alpha-2}=(2000)^2\left[\frac{\alpha}{\alpha-2}\right]=8,000,000$$

$$==>\frac{\alpha}{\alpha-2}=2==>\alpha=2\alpha-4==>\alpha=4$$

$$E[X]=2000=\frac{\theta}{\alpha-1}=\frac{\theta}{3}==>\theta=6000$$

$$TVaR_{0.9}=\pi_{0.9}+\frac{\theta(1-0.9)^{-1/\alpha}}{\alpha-1}$$

$$F(\pi_{0.9})=0.9==>1-\left(\frac{\theta}{\theta+\pi_{0.9}}\right)^{\alpha}=0.9==>0.1=\left(\frac{6000}{6000+\pi_{0.9}}\right)^{4}==>\pi_{0.9}=4669.68$$

$$TVaR_{0.9}=\pi_{0.9}+\frac{\theta(1-0.9)^{-1/\alpha}}{\alpha-1}=4669.68+\frac{6000(1-0.9)^{-1/4}}{4-1}=8226$$